

DEPARTMENT OF MATHEMATICS
UNIVERSITY OF ALLAHABAD, PRAYAGRAJ

Learning Outcomes and Programme Outcomes (POs) for Bachelor's/ Undergraduate's degree:

The Bachelor's/Undergraduate's degree in Mathematics qualifies students who have acquired a systematic and coherent body of knowledge of mathematical principles and techniques. Graduates at this level are expected to demonstrate the ability to apply logical reasoning, analytical skills, and core mathematical theories to solve defined problems. This foundation prepares them for professional practice, higher education, and lifelong learning in a variety of scientific and technical contexts.

Descriptor	Knowledge and understanding: The graduates should be able to demonstrate the acquisition of:
P.O. 1.	comprehensive, factual, theoretical, and specialized knowledge in broad multidisciplinary contexts with depth in the underlying principles and theories relating to one or more fields of learning.
P.O. 2.	knowledge of the current and emerging issues and developments within the chosen field(s) of learning.
Descriptor	Application of knowledge and skills: The graduates should be able to demonstrate the ability to:
P.O. 3.	apply the acquired theoretical knowledge to gather and analyze quantitative/qualitative data to assess the appropriateness of different approaches to solving problems.
P.O. 4.	Employ the right approach to generate solutions to problems related to the chosen fields of learning.
Descriptor	Generic learning outcomes: The graduates should be able to demonstrate the ability to:
P.O. 5.	pursue self-paced and self-directed learning to upgrade knowledge and skills that will help adapt to changing demands of the workplace and pursue higher level of education and training.
P.O. 6.	Critically evaluate evidence for taking actions to generate solutions to specific problems associated with the chosen fields of learning based on empirical evidence.
P.O. 7.	exercise judgement across a broad range of functions based on empirical evidence, for determining personal and/or group actions to generate solutions to specific problems associated with the chosen fields of learning.
Descriptor	Employability and job-ready skills, and entrepreneurship skills and capabilities/ qualities and mindset: The graduates should be able to demonstrate the acquisition of:
P.O. 8.	knowledge and essential skills set and competence that are necessary to take up a professional job relating to the chosen field of learning and professional practice,

P.O. 9.	entrepreneurship skills and mindset required for setting up and running an economic enterprise or pursuing self-employment, requiring the exercise of full personal responsibility for the outputs of own work, and full responsibility for the output of the group.
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Programme Specific Outcomes (PSOs):

The undergraduate course in Mathematics is a programme that equips students with a robust foundation in mathematical principles, analytical reasoning, and problem-solving skills. It aims to foster a deep appreciation for the discipline and its applications in diverse fields. Hence, at the end of the undergraduate course in the Mathematics Programme, the following are the programme-specific outcomes:

PSO1 The Core Theorem: The learner will demonstrate comprehensive knowledge and understanding of the fundamental axioms and theorems across core mathematical areas, including Algebra, Calculus, Differential Equations, and Real & Complex Analysis.

PSO2 The Mathematician's Mindset: The learner will cultivate a rigorous, inquisitive, and resilient intellectual attitude, demonstrating precision, logical integrity, and perseverance in deconstructing and solving complex problems.

PSO3 The Language of Logic: The learner will master the art of mathematical discourse, capable of constructing clear, logical arguments and articulating complex ideas with precision, both in written proof and oral explanation.

PSO4 The Applied Solution: The learner will be proficient in translating real-world problems into mathematical models and deploying a powerful toolkit of analytical, computational, and statistical techniques to arrive at innovative and effective solutions.

**DEPARTMENT OF MATHEMATICS
FACULTY OF SCIENCE
UNIVERSITY OF ALLAHABAD, PRAYAGRAJ**

Undergraduate Course Structure

B.A./B.Sc. Part-I

There shall be three papers, each consisting of 5 units. Each unit will have 14 lectures scheduled and hence 70 lectures per paper. Each paper will have 50 marks to its credit. One question with an alternate will be asked from each Unit. Students will have to attempt all questions.

PAPER I: GEOMETRY

Course Objectives:

This course will enable the prospective teacher educator to:

- Analyze and solve problems related to conic sections (parabola, ellipse, hyperbola) using the polar coordinate system, including finding polar equations of the chord, tangent, normal, and related circles/asymptotes.
- Formulate and apply vector techniques to determine and manipulate equations of planes and straight lines in three dimensions, including calculations involving angles, shortest distance, perpendicular length, and conditions for coplanarity.
- Describe and analyze the properties of spheres, including finding the equation of a sphere, determining intersections (plane section and intersection of two spheres), and applying concepts like tangent planes, radical plane, and orthogonal systems of spheres.
- Examine the geometric properties of cones and cylinders, including finding their equations, understanding tangent lines and planes, and exploring the concept of generating lines for ruled surfaces like the hyperboloid of one sheet and the hyperbolic paraboloid.
- Comprehend and utilize the standard equations and geometric features of central conicoids (e.g., ellipsoid) and paraboloids, particularly concerning tangent planes, polar lines, enveloping shapes, diametral planes, and the properties of conjugate diameters.

Course Learning Outcomes (COs)

After successful completion of this course, the student will be able to:

CO1: Analyze and solve spatial geometry problems involving straight lines and planes using vector techniques, including calculating angles, lengths of perpendiculars, shortest distance between skew lines, and finding equations for planes defined by specific conditions.

CO2: Manipulate the different forms of line equations (symmetrical, non-symmetrical) and determine conditions for geometric relationships such as coplanar lines and the orthogonal projection of points and lines onto a plane.

CO3: Determine the equation of a sphere and analyze its geometric properties, including plane sections, intersection of two spheres, tangent planes, radical planes, and the conditions for orthogonal systems and coaxial systems of spheres.

CO4: Derive and analyze the equations and tangent properties of cones and cylinders (including right circular types). Furthermore, describe and utilize the concept of generating lines (rulings) for surfaces like the hyperboloid of one sheet and the hyperbolic paraboloid.

CO5: Identify the standard equations and analyze the properties of central conicoid and paraboloids (e.g., ellipsoid). This includes finding tangent planes, polar planes, enveloping shapes, and analyzing advanced concepts like the locus of the intersection of three mutually perpendicular tangent planes and properties of conjugate diameters.

Mapping of Course Outcomes:

CO/PSO	PSO1	PSO2	PSO3	PSO4	PO1	PO4	PO6	PO7
CO1	✓	✓	✓		✓			
CO2	✓	✓	✓		✓	✓	✓	
CO3	✓	✓	✓		✓	✓		
CO4	✓	✓	✓		✓	✓	✓	
CO5	✓	✓		✓	✓	✓		✓

Course Contents:

Unit – 1: Polar Coordinates

Polar equation of a parabola, ellipse, and hyperbola when focus is taken as pole, Polar equations of the chord joining two points, tangent, normal, polar (chord of contact), pair of tangents, asymptotes, director circle, and auxiliary circle of a conic.

Unit – 2: Straight Lines & Planes (Using Vector technique)

Normal form of equation, intercept form of equation, and general equation of a plane, plane passing through three points, angle between two planes, two sides of a plane, Length of perpendicular from a point to a plane, Bisectors of angles between two planes, Planes passing through the line of intersection of two planes.

Symmetrical and non-symmetrical form of equations of a line, transformation of non-symmetrical form to symmetrical form, Planes passing through a line, coplanar lines, The shortest distance between two lines, Length of perpendicular from a point to a line, Orthogonal projection of a point and a line on a plane, Lines intersecting two lines, intersection of three planes, volume of a tetrahedron, pair of planes.

Unit – 3: SPHERES

Equation of a sphere, Plane section of a sphere, and intersection of two spheres, spheres passing through a circle, tangent plane, plane of contact, polar lines, angle of intersection of two spheres, power of a point, radical plane, line, centre of spheres, coaxial system of spheres, Orthogonal systems of spheres,

Unit – 4:

(a) CONES & CYLINDERS

Cones and cylinders with given base, intersection of a cone and a plane passing through the vertex of the cone, tangent lines and planes, reciprocal cones, normal plane passing through a generator of the cone, right circular cones and cylinders.

(b) GENERATING LINES

Ruled surfaces, generating lines of a hyperboloid of one sheet and hyperbolic paraboloid and its properties, generators through a point on the principal elliptic section of hyperboloid of one sheet, (θ, ϕ) point on hyperboloid of one sheet, and equations of generators at (θ, ϕ) point.

Unit – 5: CENTRAL CONICOIDS & PARABOLOIDS

Standard equations of central conicoids and paraboloids, tangent lines and planes, polar planes and polar lines, enveloping cones and cylinders, section with given centre, diametral planes of central conicoids and paraboloids, Locus of point of intersection of three mutually perpendicular tangent planes to central conicoids and paraboloids, Normal drawn from a point to a central conicoid and a paraboloid, Cone passing through the normals drawn from a point to central conicoid and paraboloid, Conjugate diameters of an ellipsoid and its properties.

Books recommended:

1. R. S. Gupta and R. D. Pathak: Conic Sections.
2. Mata Ambar Tiwari and R. S. Sengar: A course in Vector Analysis and its Applications.

3. N. Saran and R. S. Gupta: Analytical Geometry of Three Dimensions.

PAPER II: ELEMENTARY ANALYSIS

Course Objectives:

This course will enable the prospective teacher educator to:

- Apply the principles of mathematical logic (statements, connectives, tautologies, quantifiers) and set theory (relations, partitions, functions) to analyze mathematical arguments and understand the axiomatic structure of the real number system ($\mathbb{R}$) as a complete ordered field.
- Demonstrate a rigorous understanding of the properties of integers (induction, division algorithm, Fundamental Theorem of Arithmetic) and the formal theory of real sequences, including determining convergence/divergence and applying the Cauchy's general principle of convergence and algebra of limits.
- Analyze the concepts of limit of a function and continuity for real-valued functions, utilizing the formal ϵ - δ and sequential definitions, and apply theorems like the Intermediate Value Property and boundedness theorem for continuous functions on closed intervals.
- Evaluate the differentiability of functions, apply the rules of differentiation (including the Chain Rule), and utilize fundamental theorems of differential calculus, such as Rolle's Theorem and Lagrange's and Cauchy's Mean Value Theorems, to deduce function properties like monotonicity.
- Employ higher-order derivatives and theorems like Leibnitz's Theorem and Taylor's/Maclaurin's Theorems for approximations and analysis; solve optimization problems (maxima and minima), apply L'Hospital's rule for indeterminate forms, and analyze curve properties like convexity, concavity, and curvature.

Course Learning Outcomes (COs)

After successful completion of this course, the student will be able to:

CO1: Students will be able to analyze and construct logical arguments using connectives, tautologies, and quantifiers, and demonstrate a clear understanding of sets, relations (equivalence/order), functions, partitions, and the axiomatic structure of $\mathbb{R}$ as a complete ordered field.

CO2: Students will be able to prove results using principles of mathematical induction, apply number theory concepts like the Division and Euclidean Algorithms and the Fundamental Theorem of Arithmetic, and rigorously determine the convergence/divergence of real sequences using formal definitions, the Cauchy criterion, and the algebra of limits.

CO3: Students will be able to calculate the limit of a function using both the $\epsilon - \delta$ definition and sequential characterization, test for continuity, and apply key theorems like the Intermediate Value Property to characterize the behavior of continuous functions on closed intervals.

CO4: Students will be able to determine the differentiability of a function, apply the Chain Rule, and utilize the fundamental theorems of differential calculus.

CO5: Students will be able to apply Leibnitz's Theorem and Taylor's/Maclaurin's Theorems for polynomial approximations, solve problems involving maxima and minima, evaluate indeterminate forms using L'Hospital's rule, and analyze curve geometry.

Mapping of Course Outcomes:

CO/PSO	PSO1	PSO2	PSO3	PSO4	PO1	PO4	PO6	PO7
CO1	✓	✓	✓		✓			
CO2	✓	✓	✓		✓	✓	✓	
CO3	✓	✓	✓		✓	✓		
CO4	✓	✓	✓		✓	✓	✓	
CO5	✓	✓		✓	✓	✓		✓

Course Contents:

Unit – 1:

Statements, Connectives (Conjunction, Disjunction, Negation, Conditional and Biconditional Joins) Statement formulas, Tautologies, Implication and equivalences, Statement Functions, Quantifiers, Sets, Relations, Equivalence and Order relations, Partitions, Functions, Direct and Inverse images of subsets under maps.

Axiomatic introduction of IR as a complete ordered Field, Existence of square roots of positive real numbers.

Unit – 2: Properties of Integers and Sequences

Natural numbers, First Principle of induction, Well Ordering property of N, Second Principle of Induction, Integers and rational numbers, Archimedean Property, Rational and Irrational Density Theorems, Division and Euclidean Algorithm in Z, Primes, Fundamental Theorem of Arithmetic, Irrationality of surds.

Intervals in IR, Real sequences and their algebra, Limit of a sequence, Bounded, convergent, monotone and Cauchy sequences, Cauchy's general principle of convergence, Algebra of limits (passage to limits under addition, multiplication, inversions and inequality), Divergence of sequences.

Unit – 3: Limits and Continuity

Real valued Functions of one variable, their graphs and algebra, Neighbourhoods of a point and limit points of subsets of $\mathbb{R}$, Limit of a function at limit point of its domain, sequential characterization of limit of a function, Algebra of limits. One sided limits, Limit of a function as $x \rightarrow \pm \infty$, Infinite limits.

Continuity, Local Boundedness and local maintenance of sign, Boundedness and intermediate value properties of continuous functions over closed intervals, Image of a closed interval under continuous maps.

Unit – 4: Differentiability

Differentiability of a function at a point and its geometrical interpretation, algebra of differentiable functions, Chain rule of differentiation, sign of derivatives and monotonicity of functions, Interior Extremum Theorem, Rolle's Theorem, Lagrange's Mean value Theorem, Cauchy's Mean value Theorem.

Unit – 5: Applications of Derivatives

Higher derivatives, Leibnitz Theorem, Taylor's and Maclaurin's Theorem with Lagrange's and Cauchy's Forms of remainders, Maxima and minima, Local extremum points (necessary and sufficient conditions), Indeterminate Forms, L'Hospital's rule, Convexity and concavity, Points of inflexion, curvature of curves in explicit form $y = f(x)$, Darboux's Theorem on intermediate value property of derivatives.

Books recommended:

1. Sherbert & Bartle: Introduction to Real Analysis.
2. N. N. Bhattacharya: Elementary Analysis.
3. Thomas & Finney: Calculus & Analytic Geometry.
4. T.M. Apostol: Calculus Vol. 1.
5. Smith & Albrecht: Fundamental Concepts in Analysis.
6. S. C. Malik: Mathematical Analysis.
7. Arora & Bansilal: Real Analysis.

PAPER III: DIFFERENTIAL EQUATIONS AND VECTOR CALCULUS

Course Objectives:

This course will enable the prospective teacher educator to:

- Explain the origin and formation of ordinary differential equations (ODEs), classify them by order and degree, and solve various types of first-order ODEs.
- Apply analytical and iterative methods, such as Picard's iteration method, to find solutions; model and solve problems from geometry (e.g., orthogonal trajectories) and mechanics (e.g., Newton's law of cooling, escape velocity); and solve higher-degree first-order equations to find general and singular solutions.
- Analyze the structure of homogeneous linear differential equations, determine linear dependence/independence of solutions, and solve linear ODEs with constant coefficients.
- Apply linear ODEs to model and analyze physical systems, specifically damped and forced oscillations, and solve simple linear systems of differential equations; concurrently, calculate derivatives of vector functions and understand the fundamental concepts of scalar and vector fields, gradient, divergence, and curl, and basic vector identities.
- Utilize various coordinate systems, including orthogonal curvilinear, spherical polar, and cylindrical polar coordinates, and evaluate double, line, surface, and volume integrals. Furthermore, apply the concepts from the fundamental integral theorems of vector calculus—Green's, Stokes', and Gauss' Divergence Theorems (using their statements only) to simple problems.

Course Learning Outcomes (COs)

After successful completion of this course, the student will be able to:

CO1: Classify first-order ODEs and accurately solve standard types including separable, exact, linear, and those reducible to these forms, using techniques like integrating factors.

CO2: Apply differential equations to model and solve real-world problems in geometry (orthogonal trajectories) and mechanics (growth/decay, motion in a resisting medium), and find solutions to advanced first-order equations like Clairaut's equation.

CO3: Determine the linear independence of solutions and solve homogeneous and non-homogeneous linear ODEs with constant coefficients using established methods: undetermined coefficients, variation of parameters, and the operator method.

CO4: Apply ODEs and Understand Vector Fields: Analyze physical phenomena like damped and forced oscillations using linear systems of ODEs, and demonstrate a conceptual and computational understanding of scalar/vector fields and the differential operators: gradient, divergence, and curl.

CO5: Integrate functions over various regions using double, line, surface, and volume integrals, utilizing coordinate systems like spherical and cylindrical polar coordinates, and state and apply the fundamental theorems of vector calculus (Green's, Stokes', and Gauss's Theorems) to simple problems.

Mapping of Course Outcomes:

CO/PSO	PSO1	PSO2	PSO3	PSO4	PO1	PO4	PO7
C01	✓	✓		✓	✓	✓	
C02	✓	✓		✓	✓	✓	✓
C03	✓	✓	✓	✓	✓	✓	
C04	✓	✓	✓	✓	✓	✓	✓
C05	✓	✓		✓	✓	✓	

Course Contents:

Unit – 1:

Origin, concept and formation of an ordinary differential equation and its solution, initial-value problems and statement of Existence and Uniqueness Theorems, order and degree of differential equation, First order equations, separable equations, equations reducible to separable form, exact equations.

Integrating factors, linear equations, Geometrical meaning of a differential equation and its solution, isoclines and direction field.

Unit – 2:

Picard's iteration method, Curves under given geometrical conditions, orthogonal trajectories, mechanical applications (Newton's law of cooling, growth and decay problems, motions in a resisting medium, escape velocity). Equations solvable for p , x or y , Clairaut's equations, singular solutions.

Unit – 3:

Homogeneous linear differential equations, linearly dependent and independent solutions, fundamental system of solutions, equations with constant coefficients, Methods of undetermined coefficients, variation of parameters and operators for determining particular integrals, Euler-Cauchy equations.

Unit – 4:

- (a) Applications to damped and forced oscillations, linear systems of differential equations with constant coefficients.
- (b) Derivatives of a vector function of a single scalar variable, scalar and vector fields, gradient, divergence, and curl, vector identities.

Unit – 5:

Orthogonal curvilinear coordinates, spherical polar and cylindrical polar coordinates, double integrals. Line, surface, and volume integrals, Green's, Stokes',

and Gauss's Theorems (statements only) and their simple applications.

Books recommended:

1. R. S. Sengar: Ordinary Differential Equations.
2. E. Kreyszig: Advanced Engineering Mathematics.
3. B. Rai & D. P. Choudhury: Ordinary Differential Equations: An Introduction.
4. Mata Ambar Tiwari & R. S. Sengar: A Course in Vector Analysis and its Applications.
5. G. F. Simmons: Differential Equations.

B.A. /B.Sc. Part-II

There shall be three papers, each consisting of 5 units. Each unit will have 14 lectures scheduled and hence 70 lectures per paper. Each paper will have 50 marks to its credit. One question with an alternate will be asked from each Unit. Students will have to attempt all questions.

Paper-I: LINEAR ALGEBRA

Course Objectives:

Upon completing this course, students will be enabled to:

- Build a foundational understanding of abstract algebraic structures, starting with fields and progressing to the axiomatic theory of vector spaces and their fundamental properties like basis, dimension, and subspaces.
- Analyze the structure and properties of linear transformations, connecting them to their matrix representations and exploring advanced concepts such as dual spaces, annihilators, and the algebra of linear maps.
- Master the interplay between linear transformations and matrices, including the effect of change of basis, and apply these concepts to solve systems of linear equations and compute determinants.
- Investigate the geometric structure of vector spaces through inner products, leading to concepts of orthogonality, and analyze linear transformations via their eigenvalues, eigenvectors, and canonical forms.
- Classify bilinear and quadratic forms, and apply spectral theorems to diagonalize matrices, thereby solving problems related to the classification of conic sections and quadric surfaces.

Course Learning Outcomes (COs)

After successful completion of this course, the student will be able to:

CO1: Define and provide examples of fields and vector spaces, determine linear independence, and find the basis and dimension of a vector space and its quotient spaces.

CO2: Analyze linear transformations by finding their rank, nullity, and matrix representations, and perform operations in the dual space, including working with the transpose of a transformation.

CO3: Solve systems of linear equations, determine the consistency of a system, find the inverse of a matrix, and compute determinants using cofactors and adjoint matrices.

CO4: Construct orthonormal bases using the Gram-Schmidt process, find eigenvalues and eigenvectors, and verify the Cayley-Hamilton theorem for a given linear transformation or matrix.

CO5: Diagonalize matrices with distinct eigenvalues, orthogonally reduce real symmetric matrices, and classify bilinear forms and second-degree equations using Sylvester's law of inertia.

Mapping of Course Outcomes:

CO/PSO	PSO1	PSO2	PSO3	PSO4	PO1	PO4	PO6	PO7
CO1	✓	✓	✓		✓	✓		
CO2	✓	✓	✓		✓	✓		
CO3	✓	✓		✓	✓	✓		
CO4	✓	✓	✓	✓	✓	✓	✓	
CO5	✓	✓		✓	✓	✓	✓	✓

Course Contents:

Unit – 1:

Definition of a field, Examples $\mathbb{R}$, $\mathbb{C}$, $\mathbb{Q}$ and $\mathbb{Z}_p$ of fields. Vector Space: definition, examples and basic properties, Subspaces, subspace generated by a subset, Linear dependence and independence of a subset, Basis and Dimension of a vector space, Sum and direct sum of subspaces, Quotient spaces.

Unit – 2:

Linear transformations and Isomorphism of vector spaces, Null space, range space, rank and nullity of linear transformations, Fundamental theorem of homomorphism, Rank nullity Theorem, $\text{Hom}(V, W)$ as a vector space. $\dim \text{Hom}(V, W) = \dim V \cdot \dim W$. Dual space V^* and transpose of a linear transformation, Annihilator of a subset of a vector space.

Unit – 3:

Matrix representation of linear transformation. Algebra of matrices, Effect of change of basis on matrix representation. Equivalent and similar matrices, Rank and nullity of a matrix, System of linear equations, their consistency and inconsistency, Reduction of a matrix to normal form and algorithms to find the inverse (if it exists), Definition of Determinant and statement of properties (without proof), Co-factor, Adjoint of a matrix and its properties.

Unit – 4:

Inner product space: Definition and Examples, Cauchy Schwarz inequality, Norm of a vector, Notion of angle, Orthogonality, Orthonormal set and basis, Gram-Schmidt orthonormalization, Bessel's Inequality, Orthogonal complement, Adjoint of a linear transformation, inner product and distance preserving transformations (orthogonal and Unitary), Hermitian, skew Hermitian, Symmetric and Skew Symmetric matrices. Invariant Subspaces, Eigenvalues, Eigenvectors, Characteristic polynomials of a Linear transformation.

Unit – 5:

Diagonalization of a matrix with distinct Eigenvalues, Orthogonal (Unitary) reduction of real Symmetric (Hermitian) matrices, Cayley-Hamilton Theorem.

Bilinear forms, Equivalence of bilinear forms, matrix representation of Bilinear forms, Quadratic forms, Classification of Symmetric, Skew Symmetric bilinear forms over $\mathbb{R}$ and $\mathbb{C}$. Sylvester's law of inertia, Classification of curves and surfaces represented by equations of 2nd degree.

Books recommended:

1. K. Hoffman and R. Kunze: Linear Algebra.
2. I. N. Herstein: Topics in Algebra, 2nd Edition.
3. Serge Lang: Introduction to Linear Algebra 2nd Edition.
4. Ramji Lal: Algebra Vol. II.
5. M. Artin: Algebra.

Paper-II: Analysis

Course Objectives:

Upon completing this course, students will be enabled to:

- Master the convergence tests for infinite series, including those for positive-termed, absolutely, and conditionally convergent series, and understand the implications of rearranging terms.
- Build a rigorous understanding of the Riemann integral based on upper and lower sums, and explore the fundamental theorems connecting integration and differentiation.
- Extend the concept of integration to improper integrals over unbounded intervals and with unbounded integrands, analyzing their convergence using various tests.
- Transition from concrete analysis on the real line to the abstract framework of metric spaces, generalizing concepts of convergence, openness, and closedness.
- Synthesize the concepts of convergence from sequences, series, and integrals under the unifying structure of Cauchy sequences and completeness in metric spaces.

Course Learning Outcomes (COs)

After successful completion of this course, the student will be able to:

CO1: Master the convergence tests for infinite series, including those for positive-termed, absolutely, and conditionally convergent series, and understand the implications of rearranging terms.

CO2: Build a rigorous understanding of the Riemann integral based on upper and lower sums, and explore the fundamental theorems connecting integration and differentiation.

CO3: Extend the concept of integration to improper integrals over unbounded intervals and with unbounded integrands, analyzing their convergence using various tests.

CO4: Transition from concrete analysis on the real line to the abstract framework of metric spaces, generalizing concepts of convergence, openness, and closedness.

CO5: Synthesize the concepts of convergence from sequences, series, and integrals under the unifying structure of Cauchy sequences and completeness in metric spaces.

Mapping of Course Outcomes:

CO/PSO	PSO1	PSO2	PSO3	PSO4	PO2	PO5	PO8	PO9
CO1	✓	✓		✓		✓		
CO2	✓	✓	✓			✓		
CO3	✓	✓	✓		✓		✓	
CO4	✓	✓		✓	✓			
CO5	✓	✓	✓		✓	✓	✓	✓

Course Contents:

Unit – 1: Series of Positive Terms

Convergence of Infinite series, addition and removal of terms, n th term test, Cauchy's criterion, Series of positive terms, Comparison Theorem, Comparison Test, Ratio and Root Tests, Comparison of ratios, Raabe's test, Logarithmic ratio test, Cauchy's Condensation test, De Morgan & Bertrand's test, higher logarithmic ratio test.

Unit – 2: General Series

Convergence of arbitrary infinite series, Alternating Series, Absolute and conditional, Abel's and Dirichlet's test, rearrangement of series, Dirichlet's and Riemann's Theorem on rearrangement of absolutely and conditionally convergent series, convolution (Cauchy) product of series, Mertens' Theorem.

Unit – 3: Riemann's Theory of Integration

Step functions and their integrals, upper and lower integrals of a bounded function, Integrable functions, Riemann's condition of integrability, Properties of integrals, Mean Value Theorem for Integrals, Differentiations of functions defined by integrals, Fundamental Theorem of integral calculus, primitives, change of variables, Second Mean Value Theorem, Integral Test.

Unit – 4: Convergence of Improper Integrals

Integrals over infinite intervals with bounded integrands, Convergence of such integrals, Necessary and sufficient conditions, Case of positive integrands, comparison test, μ -test, absolute convergence, convergence of integrals of product of two functions, Abel's and Dirichlet's test Convergence of integrals with unbounded integrands.

Unit – 5: Metric Spaces

Definition and examples of Metric spaces, Open balls, interior, boundary, closure and accumulation points, Open and closed sets, Interior and closure of sets, Limits of sequences in Metric spaces, Cauchy sequences, Completeness.

Books recommended:

1. S. C. Malik: Mathematical Analysis.
2. Smith & Albrecht: Fundamental Concepts of Analysis.
3. G. F. Simmons: Topology & Modern Analysis.
4. D. Widder: Advanced Calculus.
5. Arora & Bansi Lal: Real Analysis.
6. Shanti Narayan: Real Analysis.
7. T. M. Apostol: Mathematical Analysis.

PAPER III: MECHANICS

Course Objectives:

Upon completing this course, students will be enabled to:

- Analyze static systems using the principles of common forces, virtual work, and assess the stability of equilibrium configurations.
- Formulate and solve the problems related to rectilinear motion for a particle, including the cases of simple harmonic motion, motion under inverse square law, resisting medium motion, and varying mass problems.
- Express the velocity and acceleration of a particle in various coordinate systems (Cartesian, polar, and tangential & normal) and apply these to determine the trajectory of a particle under a given force.
- Investigate the dynamics of constrained motion, specifically analysing the motion of a particle in a vertical circle and along a cycloidal path.
- Deduce the fundamental properties of motion under central forces, including the conservation of angular momentum and Kepler's Laws of planetary motion, using the concepts of areal velocity and orbital equations.

Course Learning Outcomes (COs)

After successful completion of this course, the student will be able to:

CO1: Apply the principle of virtual work to solve statics problems and determine the stability of equilibrium configurations.

CO2: Formulate and solve the differential equations governing rectilinear motion for various cases, including simple harmonic motion, motion under the inverse square law, and motion in a resisting medium.

CO3: Compute velocity and acceleration vectors in Cartesian, polar, and tangential-normal coordinates and derive the path of a particle from a given force field.

CO4: Analyze the forces and motion of a particle constrained to move in a vertical circle and along a cycloid.

CO5: Apply the conservation of angular momentum to central force motion, derive the differential equation of the orbit (the p - r equation), and explain Kepler's laws using the theory of central forces.

Mapping of Course Outcomes:

CO/PSO	PSO1	PSO2	PSO3	PSO4	PO3	PO4	PO8	PO9
CO1	✓	✓		✓	✓	✓	✓	
CO2	✓	✓		✓	✓	✓	✓	
CO3	✓	✓	✓	✓		✓	✓	
CO4	✓	✓		✓	✓	✓	✓	
CO5	✓	✓	✓	✓		✓	✓	✓

Course Contents:

Unit – 1: Statics

Common centenary, virtual work and stability of equilibrium.

Unit – 2: Dynamics of a Particle

Rectilinear Motion: Simple Harmonic Motion including the cases of horizontal and vertical elastic strings, Motion under inverse square law, Motion in resisting media, Motion of varying mass.

Unit – 3:

Motion in plane: Kinematics and kinetics of motion, velocity and acceleration in Cartesian, polar and tangential & normal coordinates, determination of path under a given force.

Unit – 4:

Constrained Motion: Motion in a vertical circle, Cycloidal motion.

Unit – 5:

Motion under Central Forces: Conservation of angular momentum, areal velocity, differential equation to the path of a particle moving under a central attractive force, (p, r) equation to the path, energy equation, Kepler's laws of planetary motion.

Books recommended:

1. S. L. Loney: Statics.
2. P. L. Srivastava: Elementary Dynamics.
3. R. S. Verma: Statics.

B.A. /B.Sc. Part-III

There shall be 4 papers each consisting of 5 units. Each unit will have 14 lectures scheduled and hence 70 lectures per paper. The first three papers will be of 60 marks credit and the fourth will be of 45 marks credit. One question with an alternative will be asked from each unit. Students will have to attempt all questions.

Paper-I: Abstract Algebra

Course Objectives:

Upon completing this course, students will be enabled to:

- Acquire a fundamental understanding of the core structures in abstract algebra, namely groups, rings, and fields, and their defining properties.
- Develop the ability to analyze and prove basic theorems concerning group theory, including Lagrange's Theorem and the Isomorphism Theorems, and apply them to classical problems in number theory.
- Explore the structure of specific, important groups such as symmetric groups, dihedral groups, and matrix groups, and understand concepts like group actions and conjugacy classes.
- Build a foundation in ring theory, investigating substructures like ideals and quotient rings, and understand the properties of polynomial rings over fields.
- Apply algebraic concepts to solve problems in number theory, including linear congruences, quadratic residues using Legendre symbols, and understand the principles behind theorems like the Chinese Remainder Theorem.

Course Learning Outcomes (COs)

After successful completion of this course, the student will be able to:

CO1: Define, identify, and provide examples of key algebraic structures such as groups, subgroups, rings, ideals, and fields.

CO2: Prove fundamental theorems and apply them to solve problems; for instance, use Lagrange's Theorem to derive Euler's and Fermat's Theorems in number theory.

CO3: Classify groups of small order, decompose permutations into cycles, and analyze group structures using concepts like normal subgroups, homomorphisms, and group actions.

CO4: Perform computations in quotient rings and polynomial rings, and determine the solvability of linear congruences and systems of congruences using the Chinese Remainder Theorem.

CO5: Calculate and apply number-theoretic functions like Euler's totient function and the Legendre symbol, and utilize the Möbius Inversion Formula.

Mapping of Course Outcomes:

CO/PSO	PSO1	PSO2	PSO3	PSO4	PO2	PO5	PO7	PO8
CO1	✓	✓	✓			✓		✓
CO2	✓	✓	✓			✓	✓	✓
CO3	✓	✓	✓		✓	✓	✓	
CO4	✓	✓	✓			✓		✓
CO5	✓	✓		✓			✓	✓

Course Contents:

Unit-1:

Semigroups, Groups: definition, elementary properties and Examples including V_8, Q_8, D_8, Z_m, U_m , Group of Symmetries of regular n-gons, Matrix groups and group of Rigid motion of R^n (Example). Homomorphisms, Isomorphisms, (To determine homomorphism between groups given above), Subgroups, Subgroup generated by a subset, Cyclic groups, order of an element. Classification of. Cyclic groups, Subgroups of above groups, Group of automorphisms, centre of a group, coset decomposition, Lagrange Theorem and applications to number Theory. (Euler's Theorem, Fermat's Theorem and Wilson's Theorem).

Unit-2:

Product of groups, Normal Subgroups, Quotient groups, Correspondence theorem, Fundamental Theorem of homomorphisms, 1st and 2nd isomorphism Theorem and their applications, commutator subgroups, Abelianizer.

Symmetric Groups, Representation of a finite group as a symmetric group and as an orthogonal group, Cycle and Transposition, decomposition of permutations, Alternating map, Even and Odd permutations, Alternating groups, Normal subgroups of S.

Unit-3:

Group action: Definition and Examples including regular and Inner Conjugation action, Orbits and isotropy groups. Class formula with special reference to Inner conjugation, Conjugacy classes and centralizer, Normalizer and Conjugacy class of subgroups, Prime power order group and its centre being non-trivial, Abelian

property of groups of order p^2 , Classification of groups of order p , Classification of groups of order upto 8.

Unit -4:

Rings, Subrings, Homomorphisms, Examples of rings including $Z_m, Z[i], Z[w]$, Matrix rings, Quaternion rings, ideals, quotient rings, Fundamental Theorem of homomorphism, Correspondence Theorem, Integral domains, characteristic of an integral domain, Division rings and Fields, Field of fractions of a commutative integral domain, Polynomial ring over Commutative ring with identity, and over fields, division algorithm, remainder and factor theorem, ideals of $F[X]$.

Unit-5:

Linear Congruences and an algorithm to find solutions of linear congruences, group of units of Z , Chinese remainder Theorem, quadratic residues, Euler's Criterion, Legendre Symbol, Gauss' Quadratic Reciprocity Theorem. Multiplicativity of Euler's phi function, Divisor function, sum of divisor function and Möbius functions, Möbius Inversion Theorem and its applications.

Books Recommended:

1. N. Herstein : Topics in Algebra, 2nd Edition, Wiley Eastern Ltd. 1988
2. M. Artin: Algebra, Prentice-Hall of India Pvt. Ltd. New Delhi, 1994.
3. J.B. Fraleigh : Narosa Publishing House, New Delhi 1998.
4. Ramji Lal : Algebra Vol. I, Shail Publication.
5. David M. Burton: Elementary Number Theory, Tata McGraw-Hill Publishing Company Ltd., New Delhi 2007.
6. I. Niven, H. S. Zuckerman, H.L. Montgomery: The Theory of Numbers, John Wiley & Sons, India, 2000.

Paper II: ADVANCED ANALYSIS

Course Objectives:

Upon completing this course, students will be enabled to:

- Establish a rigorous understanding of continuity, compactness, and completeness in the general setting of metric spaces.
- Master the concepts of differentiability and higher-order differentials for functions of several variables, including the application of the Chain Rule and Taylor's Theorem.
- Apply techniques of multivariable differentiation to solve optimization problems with and without constraints using concepts of maxima, minima, and Lagrange multipliers.
- Analyze the convergence of sequences and series of functions, with a focus on uniform convergence and its implications for term-by-term integration and differentiation.
- Build a foundation in complex analysis by exploring the concepts of analyticity, the Cauchy-Riemann equations, and the properties of standard complex functions.

Course Learning Outcomes (COs)

After successful completion of this course, the student will be able to:

CO1: Prove fundamental theorems concerning continuity, compactness, and uniform continuity in metric spaces, and determine the completeness of a given metric space.

CO2: Compute partial derivatives, directional derivatives, and total differentials, and apply the Chain Rule to differentiate composite functions of several variables.

CO3: Locate and classify critical points (maxima, minima, saddle points) of multivariable functions and solve constrained optimization problems using the method of Lagrange multipliers.

CO4: Test sequences and series of functions for pointwise and uniform convergence using various tests (Weierstrass, Abel's, Dirichlet's) and manipulate power series within their interval of convergence.

CO5: Verify the analyticity of complex functions using the Cauchy-Riemann equations and represent standard complex functions like exponential and trigonometric functions in terms of their real and imaginary parts.

Mapping of Course Outcomes:

CO/PSO	PSO1	PSO2	PSO3	PSO4	PO1	PO4	PO5	PO8
CO1	✓	✓	✓		✓			
CO2	✓	✓		✓	✓	✓		✓
CO3	✓	✓	✓	✓	✓	✓		✓

CO4	✓	✓	✓		✓	✓	✓	✓
CO5	✓	✓	✓		✓		✓	

Course Contents:

Unit-1: Limits and Continuity

Limits of a function between Metric spaces, Continuity. Characterization of continuity in terms of open sets, closed sets and closures, Continuity of linear transformations and inner product function, Examples from functions between Euclidean spaces, Repeated limits.

Compact Metric spaces, Total Boundedness, Sequential compactness, Lebesgue Covering Lemma, Uniform continuity, Preservation of compactness under continuous maps, Completeness of metric spaces.

Unit-2: Functions of Several Variables

Partial derivatives, Directional derivatives, differentiability of functions of several variables, Sufficient conditions for continuity and differentiability in terms of partial derivatives, algebra of differentiable functions, Differentiability of composite functions, Chain rule of differentiation, Total differentials, Mean value Theorem for real valued functions.

Unit-3: Applications of Partial Differentiation

Homogeneous functions and Euler's Theorem, Equality of Mixed derivatives, Young's & Schwarz' Theorems, Higher differentials, Taylor's Theorem, Maxima and Minima for real valued functions of several variables, Necessary and sufficient conditions, Saddle points, Lagrange's Multipliers.

Jacobian Matrix and determinants, Implicit Function Theorem (Proof only for $f: R^2 \rightarrow R$), inverse Function Theorem (Statement only), Functional dependence.

Unit-4: Uniform Convergence

Pointwise and uniform convergence of sequences and series of functions, Necessary & sufficient conditions. (Cauchy's criterion), Weierstrass', Abel's & Dirichlet's test for uniform convergence, Term by term integration and differentiation of infinite sequences & series of functions.

Power series, Interval of convergence, uniform convergence of power series, Abel's limit theorem, Taylor and Maclaurin series for functions of one variable, Standard functions (sine, cosine, exponential, logarithmic) defined by power series and their properties.

Unit-5: Complex Analysis

Convergence of Power series over $\mathbb{C}$, Differentiability of functions $f : D \rightarrow \mathbb{C} (D \subseteq \mathbb{C})$, Analyticity, Cauchy-Riemann equations, Analyticity of functions defined by power series inside the circle of convergence, Complex standard functions $\exp z$, $\sinh z$, $\cosh z$, $\cos z$, $\sin z$, $\log z$ and their representation in terms of real functions.

Books Recommended:

1. S.C. Maik: Mathematical Analysis.
2. G.F. Simmons: Topology & Modern Analysis
3. T.M. Apostol: Mathematical Analysis.
4. D. Widder: Advanced Calculus.
5. Arora & Bansi Lal: Real Analysis.
6. Shanti Narayan: Mathematical Analysis.
7. Conway: Functions of one complex variable.

Paper III: ADVANCED MECHANICS

Course Objectives:

Upon completing this course, students will be enabled to:

- Formulate and solve problems in three-dimensional statics using vector techniques, including the concepts of wrenches, null lines, and conjugate systems.
- Analyze the rotational motion of rigid bodies by applying concepts of moment of inertia, principal axes, and the momental ellipsoid.
- Apply D'Alembert's principle and related methods to derive and solve equations of motion for rigid bodies, including rotation about a fixed axis.
- Describe fluid motion using both Lagrangian and Eulerian approaches and derive the fundamental equations of hydrodynamics, such as the equation of continuity and Bernoulli's equation.
- Model two-dimensional, irrotational fluid flow using the concepts of velocity potential, stream function, complex potential, and the method of images.

Course Learning Outcomes (COs)

After successful completion of this course, the student will be able to:

CO1: Determine the resultant wrench of a three-dimensional force system and analyze properties like the central axis, null lines, and null planes.

CO2: Calculate moments and products of inertia for standard bodies and determine the principal axes of a rigid body.

CO3: Apply D'Alembert's principle to formulate equations of motion and analyze the kinetics of a rigid body rotating about a fixed axis, including finding reactions at the axis.

CO4: Express the velocity and acceleration of a particle in cylindrical and spherical coordinates and apply the equation of continuity and Bernoulli's theorem to fluid flow problems.

CO5: Construct solutions for two-dimensional flow fields using complex potentials and apply the method of images to model the flow due to sources and doublets near boundaries.

Mapping of Course Outcomes:

CO/PSO	PSO1	PSO2	PSO3	PSO4	PO1	PO3	PO6	PO9
CO1	✓	✓		✓	✓	✓		✓
CO2	✓	✓		✓	✓	✓	✓	
CO3	✓	✓	✓	✓	✓	✓	✓	
CO4	✓	✓		✓	✓	✓	✓	✓
CO5	✓	✓	✓	✓	✓	✓		✓

Course Contents:

Unit-1: Statics (Using Vector Technique)

Forces in three dimensions, Line Coordinates of a line, Central Axis and Wrench, Resultant wrench of two wrenches, Null line, Null plane, Null point with respect to a system of forces, conjugate forces and conjugate lines.

DYNAMICS

Unit-2: Moment of Inertia

Moment and product of inertia of some standard bodies, Principal axes, Momental Ellipsoid of a body.

Unit-3:

(a) D'Alembert's principle

The general equation of motion, Motion of the Centre of inertia and motion relative to the centre of inertia.

(b) Motion about a fixed Axis

Moment of the effective forces about the axis of rotation, moment of momentum about the axis of rotation, kinetic energy of the body rotating about a fixed axis, equation of motion about the axis of rotation, reactions of the axis of rotation.

(c) Velocity and acceleration of a moving particle in cylindrical and spherical polar coordinates.

HYDRODYNAMICS

Unit – 4:

(a) Lagrangian and Eulerian approaches, Equation of continuity in different coordinate systems, boundary surfaces, velocity potential, stream lines.

(b) Euler's equation of motion, steady motion, Bernoulli's equation, and Impulsive motion.

Unit-5:

Motion in two dimensions, stream function, irrotational motion, complex potential, sources and

sinks, doublets, image system of a simple source with respect to plane, a circle and a sphere, image

system of a doublet with regard to a plane, a circle and a sphere, Circle Theorem.

Books Recommended:

1. Mata Ambar Tiwari & R.S. Sengar: A Course in Vector Analysis and its Applications.
2. S.L. Loney: Dynamics of a Particle and of rigid bodies.
3. A.S. Ramsey: A Treatise on Hydrodynamics.

Paper IV: NUMERICAL METHODS

Course Objectives:

Upon completing this course, students will be enabled to:

- Understand the necessity of numerical techniques and the impact of computational errors, and apply iterative methods to find solutions to algebraic and transcendental equations.
- Construct and utilize various interpolation formulas (Newton, Lagrange, Gauss, etc.) to estimate function values and perform inverse interpolation.
- Develop models using cubic spline interpolation and apply numerical techniques for approximating derivatives and evaluating definite integrals.

- Solve initial value problems for ordinary differential equations using single-step (e.g., Runge-Kutta) and multi-step (e.g., Predictor-Corrector) methods.
- Analyze systems of linear equations and eigenvalue problems by implementing direct and iterative numerical algorithms, and perform curve fitting using the least squares approach.

Course Learning Outcomes (COs)

After successful completion of this course, the student will be able to:

CO1: Apply root-finding algorithms such as Bisection, Newton-Raphson, and Regula-Falsi to solve arbitrary equations and analyze the impact of floating-point representation.

CO2: Employ finite difference operators to construct difference tables and apply appropriate interpolation formulas (forward, backward, central, Lagrange) for data approximation.

CO3: Implement cubic spline interpolation and use numerical differentiation/integration rules (Trapezoidal, Simpson's) to solve calculus problems.

CO4: Obtain numerical solutions to first and second-order ordinary differential equations using methods like Euler, Improved Euler, Runge-Kutta, and Predictor-Corrector pairs.

CO5: Solve systems of linear equations using factorization and iterative methods (Gauss-Seidel), fit curves to data via Least Squares, and estimate eigenvalues using the Power method and Gerschgorin's theorem.

Mapping of Course Outcomes:

CO/PSO	PSO1	PSO2	PSO3	PSO4	PO2	PO4	PO6	PO7
CO1	✓	✓		✓	✓	✓	✓	✓
CO2	✓	✓	✓	✓		✓	✓	
CO3	✓	✓		✓		✓	✓	✓
CO4	✓	✓	✓	✓	✓	✓	✓	
CO5	✓	✓		✓	✓	✓	✓	✓

Course Contents:

Unit- 1:

Need for numerical methods, floating point representation, rounding off rules. Solution of arbitrary equations: Fixed point iteration method, Bisection method, Method of false position (Regula-Falsi), Newton-Raphson's method.

Unit-2:

Interpolation: Operators: Δ , ∇ , μ , δ , E and D and their relationship, forward, backward and central difference- tables, Gregory-Newton Forward and Backward interpolation formulas, Lagrange's and Newton's Divided difference interpolation formulas, inverse interpolation, Formulas based on central differences: Gauss', Stirling's, Bessel's and Everett's interpolation formulas.

Unit-3:

Cubic spline interpolation, clamped and natural splines. Numerical Differentiation and Integration: Differentiation using interpolation formulas, Quadrature formulas, Trapezoidal and Simpson's One-third and Three-Eighths rules.

Unit-4:

Numerical Methods for O.D.E.'s: First order equations, Incremental Methods: Euler's, Taylor series and Improved Euler methods, Runge-Kutta method, Multistep methods, Predictor corrector pairs: Adam-Bashforth-Moulton's and Milne's formulas, Second order equations: Taylor series and Runge-Kutta-Nystrom Method.

Unit- 5: Numerical Linear Algebra

Solution of System of equations: Gauss Elimination, LU-factorisation, Cholesky's method, Gauss-Jacobi and Gauss Seidel iteration formulas, Curve fitting by Least Square method, Estimation of eigenvalues by Gerschgorin circles, Determination of eigenvalues and eigenvectors by iteration (Power method).

Books Recommended:

1. E. Kreyszig: Advanced Engineering Mathematics
2. K. Sankara Rao: Numerical Methods for Scientists and Engineers.
3. S.S. Sastry: Introductory Methods of Numerical Analysis
4. Jain, Iyengar & Jain Numerical Methods for Scientific and Engineering Computation
5. F.B. Hildebrand: Numerical Analysis