

Thermal Physics

1. Thermodynamics: - Thermodynamics systems, Macroscopic and

Microscope Variables, Thermodynamical Equilibrium, Thermodynamical State, Zeroth law of thermodynamics and concept of temperature.

Heat and work and their path-dependence, Thermal processes, First Law of thermodynamics and internal energy, Jule's law, Applications of first law, Carnot cycle, Carnot Engine and Refrigerator, Reversible and Irreversible processes, Carnot's Theorem, Thermodynamical scale of temperature, Clausius Theorem, Clausius Inequality, Entropy, Calculation of entropy in various processes, Entropy and unavailable energy, Physical significance of entropy, Second law of thermodynamics.

Conditions for natural changes, Thermodynamic potentials and Maxwell's equation, Applications of Maxwell's equations, Joule thomson effect, Inversion Temperature.

Change of phase. First and second order phase transitions, and Ehrenfest's equations.

Kinetic Theory of Gases :- Maxwell-Boltzmann law of distribution of molecular velocities, Evaluation of r.m.s. velocity and average and most probable speeds, Mean free Path, Phenomenon

Conduction of Heat :- Forcier equation of one-dimensional flow of heat and its steady-state solution Periodic flow of heat (only sinusoidal heat current).

Radiation :- Radiation as electromagnetic waves. Emissive and Absorptive powers, radiation in a hollow enclosure, Black-body radiation, Kirchhoff's Law, Intensity and energy density, Pressure and energy density, Stefan-Boltzmann law, Solar constant and temperature of sun, Temperature of Non-black bodies, Distribution of energy in the spectrum of black body radiation, Adiabatic expansion of black body radiation, Wien's distribution law, Wien's displacement law, Wien's formula, Rayleigh-jenai's law, Planck's law.

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(Q2.) Show that enthalpy remains constant in a reversible isobaric adiabatic process. (20/4)

Q3. State the Kelvin-Planck statement of the second law of thermodynamics: (2014, 16) 2

Q4. State zeroth law of thermodynamics and use it to establish the concept of empirical temperature.

~~Q5~~ In a certain process, 500 cal. of heat is supplied to a
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system and at the same time 100 J of work is done on the system. What is the increase in the internal energy of the system? (2014, 17)

Q6. Describe Carnot's engine and find its efficiency.
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Q7. Define thermodynamic potentials and use them to obtain Maxwell's thermodynamical relations.

Or Apply conditions of exact differentials for thermodynamic potential to derive Maxwell's relations. (2017) (2011, 14, 16)

Q8. Define Entropy. Derive an expression for entropy of an ideal gas. (2014, 17, 18)

Q9. Explain first order phase transition. (2014, 16, 19)

Q10. Write short notes on second order phase transition.

Q11. Calculate under what pressure water will boil at 120°C (2012, 14, 16, 17, 19)

if the change in specific volume when 1 gm of water is converted into steam is 1676 cm^3 . Given that : Latent heat of steam = 540 cal/g , $J = 4.2 \times 10^7 \text{ ergs/cal}$, 1 atm pressure = 10^6 dynes/cm^2 .
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Q9. Derive the Maxwell Boltzmann distribution law for speed of molecules of gas. (2019) $\dots 64$

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CHAPTER - 1

Thermodynamics

Q1. Prove that heat given to any system (or taken from it) in an isochoric process is equal to the change in its internal energy. (2014)

Ans. In an isochoric process, the volume is remain unchanged if heat is added or removed from the system, therefore, work done is zero.

Now, according to first law of thermodynamics.

$$\Delta Q = \Delta U + \Delta W$$

Since for a isochoric process, $\Delta W = 0$

$$\therefore \Delta Q = \Delta U + 0$$

$$\Delta Q = \Delta U$$

Where, ΔQ is the heat added to the system and ΔU is the increase in internal energy of the system.

So, from equation (ii), heat given to the system (or taken from it) in an isochoric process is equal to change in its internal energy.

Q2. Show that enthalpy remains constant in a reversible isobaric adiabatic process. (2014)

Ans. Entropy of a system is given by

$$\Delta S = \frac{\Delta Q}{T} + S_{\text{gen}}$$

Where, ΔQ is the heat exchanged
 T is the absolute temperature of system or surrounding.

For a reversible adiabatic process,

$$\Delta Q = 0 \text{ and } S_{\text{gen}} = 0$$

$$\therefore \Delta S = 0$$

Now, From first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

$$\Rightarrow \Delta Q = \Delta H + V dp$$

$$\Rightarrow T dS = \Delta H + V dp$$

For an isobaric process, $dp = 0$

$$\therefore T dS = \Delta H$$

...(i)

...(ii)

...(1)

...(2)

...(3)

From eq. (1) and (3),

$$\Delta H = 0$$

$$\Rightarrow H = \text{constant}$$

Therefore, Enthalpy change is zero or enthalpy remains constant in a reversible isobaric adiabatic process.

Q3. State the Kelvin-Planck statement of the second law of thermodynamics. (2014, 16)

Ans. According to Kelvin-Planck statement—It is impossible to construct a device which, operating in a cycle, has sole effect of extracting heat from a reservoir and performing an equivalent amount of work.

Q4. State zeroth law of thermodynamics and use it to establish the concept of empirical temperature. (2013, 14, 16, 19)

Ans. Zeroth law was formulated by R.H. Fowler in 1931. "It states that if two systems are separately in thermal equilibrium with a third system then those two systems must be in thermal equilibrium with each other."

We consider a reference state C. Let pressure and volume of gas be P_C and V_C . If we choose a particular value of P_A and demand thermal equilibrium between A and C. It is determined by V_A . It is expressed as—

$$f_{AC}(P_A, V_A; P_C, V_C) = 0$$

...(1)

Similarly, equilibrium state between B & C demands,

$$f_{BC}(P_B, V_B; P_C, V_C) = 0$$

...(2)

We can solve equation (1) and (2), to give

$$V_C = f_{AC}(P_A, V_A; P_C)$$

$$V_C = f_{BC}(P_B, V_B; P_C)$$

$$\text{Therefore, } f_{AC}(P_A, V_A; P_C) = f_{BC}(P_B, V_B; P_C)$$

$$\text{or } f_{AC} - f_{BC} = 0$$

(3)

If A and B are also in equilibrium with each other then,

$$f_{AB}(P_A, V_A; P_B, V_B) = 0$$

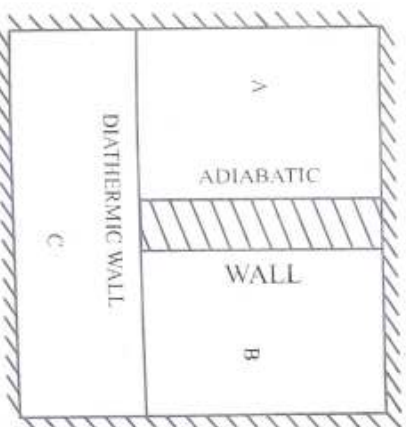
...(4)

From zeroth law equation (3) must agree with equation (4). In other words, the extra coordinates V_C must drop out from equation (3). It must read

$$f_A(P_A, V_A) - f_B(P_B, V_B) = 0$$

...(5)

In general, we can say that two or more systems are in thermal equilibrium if there exists a function f_i of its variables X_i, Y_i which have a common value for all system. Thus—



$$f_A(X_A, Y_A) = f_B(X_B, Y_B) = \text{common value}$$

The common value T of these functions is the empirical temperature. Thus we can write—

$$f(X, Y) = T$$

...(6)

Where T is same for all systems. Equation is called the equation of state and T is the empirical temperature.

Q5. In a certain process, 500 cal. of heat is supplied to a system and at the same time 100 J of work is done on the system. What is the increase in the internal energy of the system? (2014, 17)

$$\text{Ans. } dQ = +500 \text{ Cal.}$$

$$= +2092 \text{ Joule}$$

$$dW = -100 \text{ J}$$

From, 1st Law of thermodynamics,

$$dQ = dU + dW$$

$$dU = dQ - dW$$

$$= 2092 - (-100)$$

$$= 2192 \text{ J}$$

$$dQ = 500 \text{ Cal.}$$

$$= 500 \times 4.18$$

$$dQ = 2095 \text{ Joule}$$

$$dW = -100 \text{ J}$$

$$dQ = dU + dW$$

$$dU = dQ - dW$$

Q6. Describe Carnot's engine and find its efficiency. (2016, 18, 19)

Ans. A heat engine is a practical arrangement to convert heat into mechanical work. Sadi Carnot conceived and ideal theoretical engine free from all the imperfections of actual engines and hence never realized in actual practice. His imaginary engine is, however taken as a standard against which the performance of actual engines is judged.

The ideal Carnot's engine consists of :

- A cylinder with perfectly non-conducting walls but perfectly conducting base containing air (which is supposed to behave like a perfect gas) as the working substance and fitted with a perfectly insulating and frictionless piston upon which weights can be placed.
- A hot body of infinitely large heat capacity maintained at a constant high temperature T_1 absolute serving as the source of heat.
- A cold body of infinitely large heat capacity maintained at a lower constant temperature T_2 absolute serving as the sink of heat.
- Perfectly insulating platform serving as a stand for the cylinder.

The cylinder may be placed on any of the three bodies and may be moved from one to the other without friction, i.e., without doing any work.

Efficiency of the cycle—

$$\text{Efficiency, } \eta = \frac{\text{Useful output}}{\text{Input}}$$

$$\eta = \frac{W}{H_1}$$

Heat supplied from the source from A to B

$$H_1 = RT_1 \log \frac{V_2}{V_1}$$

Heat rejected to the sink

$$H_2 = RT_2 \log \frac{V_2}{V_1}$$

$$\therefore \eta = \frac{W}{H_1} = \frac{H_1 - H_2}{H_1}$$

$$\eta = \frac{R(T_1 - T_2) \log \left[\frac{V_2}{V_1} \right]}{RT_1 \log \left[\frac{V_2}{V_1} \right]}$$

$$\eta = 1 - \frac{T_2}{T_1}$$

Q7. Define thermodynamic potentials and use them to obtain Maxwell's thermodynamical relations. (2011, 14, 16)
Or Apply conditions of exact differentials for thermodynamic potential to derive Maxwell's relations. (2017)

Ans. Thermodynamic potentials—

(1) **Internal Energy, U**—According to first law of thermodynamics, there is a certain function U , of the variables which characterises the system and this function may be called intrinsic energy or internal energy.

$$dU = dQ - dW$$

(2) **Helmholtz's function, F**—It is another characteristic property of the system and depends only on its thermodynamic state.

$$F = U - TS$$

(3) **Enthalpy, H**—Enthalpy is an extensive thermodynamic property and is given by the symbol H .

$$H = U + PV$$

(4) **Gibbs Potential, G**—Like Helmholtz free energy, the Gibbs's free energy is also a characteristic property of an open system and depends only on the thermodynamic state of the system.

$$G = H - TS.$$

Maxwell's Relations from thermodynamic potentials—

(1) **Internal Energy**—The internal energy U is a function whose change is independent of path followed between two states.

$$dU = dQ - dW$$

Now, $dQ = TdS$ from second law of thermodynamics.

$$dU = TdS - pdV$$

Taking partial differential with respect to S and V , we get

$$\left(\frac{\partial U}{\partial S} \right)_V = T \text{ and } \left(\frac{\partial U}{\partial V} \right)_S = -P$$

Since dU is perfect differential. Hence

$$\frac{\partial^2 U}{\partial V \partial S} = \frac{\partial}{\partial V} \left[\frac{\partial U}{\partial S} \right]_V = \frac{\partial}{\partial S} \left[\frac{\partial U}{\partial V} \right]_S$$

$$\frac{\partial}{\partial V} (T) = \frac{\partial}{\partial S} (-P)$$

$$\left(\frac{\partial T}{\partial V} \right)_S = - \left(\frac{\partial P}{\partial S} \right)_V$$

It is first thermodynamical relation of Maxwell.

(2) **Helmholtz function**—From first and second law of thermodynamics

$$dU = TdS - PdV$$

For isothermal process we can write

$$TdS = d(TS),$$

$$dU = d(TS) - PdV$$

$$dU - d(TS) = -PdV$$

$$d(U - TS) = -PdV$$

$$\text{Now, } U - TS = F,$$

$$dF = dU - TdS - SdT$$

From first law,

$$dU = TdS - PdV$$

$$dF = (TdS - PdV) - SdT$$

$$dF = -PdV - SdT$$

Taking partial differential of F

$$\left(\frac{\partial F}{\partial V}\right)_T = -P \text{ and } \left(\frac{\partial F}{\partial T}\right)_V = -S$$

Since F is a perfect differential

$$\frac{\partial^2 F}{\partial T \partial V} = \frac{\partial}{\partial T} \left[\frac{\partial F}{\partial V} \right]_T = \frac{\partial}{\partial V} \left(\frac{\partial F}{\partial T} \right)_V$$

$$\frac{\partial}{\partial T} (-P) = \frac{\partial}{\partial V} (-S)$$

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

It is second thermodynamical relation of Maxwell.

(3) **Enthalpy, H** —It is an extensive thermodynamic property, called total heat function.

$$H = U + pV$$

$$dH = dU + pdV + Vdp$$

from first law,

$$dH = (TdS - pdV) + pdV + Vdp$$

$$dH = TdS + Vdp$$

Partial differential of H treating S and P as independent variables,

$$\left(\frac{\partial H}{\partial S}\right)_P = T \text{ and } \left(\frac{\partial H}{\partial P}\right)_S = V$$

$$\frac{\partial^2 H}{\partial P \partial S} = \frac{\partial^2 H}{\partial S \partial P}$$

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

It is the third thermodynamical Maxwell relation.

(4) **Gibb's potential, G** —This function is such that it remains constant if a thermodynamic process remains isobaric as well as isothermal.

$$G = U + pV - TS$$

$$dG = dU + pdV + Vdp - TdS - SdT$$

$$\text{Using } dU = TdS - pdV$$

$$dG = Vdp - SdT$$

Taking P and T as independent variables, the partial differentials will be,

$$\left(\frac{\partial G}{\partial P}\right)_T = V \text{ and } \left(\frac{\partial G}{\partial T}\right)_P = -S$$

G is an exact differential,

$$\frac{\partial^2 G}{\partial T \partial P} = \left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T$$

$$\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T$$

It is fourth thermodynamical relation of Maxwell.

Q8. Define Entropy. Derive an expression for entropy of an ideal gas. (2014, 17, 18)

Ans. The concept of entropy was first introduced by Clausius in 1854, while working on the formulation and application of the second law of thermodynamics. It is very important thermodynamic quantity and has proved very useful in the study of behaviour of heat engines.

In general, if Q is the amount of heat absorbed or rejected at a temperature T in going from one adiabatic to the other, then

$$\frac{Q}{T} = \text{constant.}$$

This constant ratio is called the change in entropy between the states represented by the two adiabatics.

Hence, entropy of a system is a function of the thermodynamical coordinate defining the state of system viz., the pressure, temperature, volume or internal energy, and its change between two states is equal

to integral of the quantity dQ/T between the states along any reversible path joining them dS is an exact differential as it is differential of an actual function.

Entropy for ideal gas—Let us consider unit mass of perfect gas occupying Volume V at pressure P and Temperature T . Let the quantity of heat dQ be given to the gas, then by first law of thermodynamics, $dQ = dU + dW$

Now, if C_V is the specific heat of the gas at constant volume, dT is the rise in temperature, dV the change in volume,

$$dU = C_V dT \text{ and } dW = PdV$$

$$\therefore dQ = C_V dT + PdV$$

$$dS = \frac{dQ}{T}$$

$$dS = \frac{C_V dT}{T} + \frac{PdV}{T}$$

$$S = \int C_V \frac{dT}{T} + \int \frac{P}{T} dV \quad \dots(1)$$

$$T = \frac{PV}{R} \text{ and } dT = \frac{PdV + VdP}{R}$$

from (1),

$$\begin{aligned} S &= \int C_V \left(\frac{PdV + VdP}{RT} \right) + \int \frac{PdV}{T} \\ &= \int C_V \left(\frac{PdV + VdP}{PV} \right) + \int \frac{PdV}{PV} \cdot R \end{aligned}$$

$$S = \int C_V \left(\frac{dV}{V} + \frac{dP}{P} \right) + \int (C_P - C_V) \frac{dV}{V}$$

$$S = \int C_V \frac{dV}{V} + \int C_P \frac{dV}{V}$$

$$S = C_V \log \frac{V_2}{V_1} + C_P \log \frac{V_2}{V_1}$$

Q9. Explain first order phase transition. (2014, 16, 19)

Ans. Suppose, m_1 and m_2 are the masses in the liquid and vapour phases and g_1 and g_2 are the specific values of the Gibbs's potential in the two phases. Then for the whole system,

$$G = m_1 g_1 + m_2 g_2 \quad \dots(1)$$

If a small quantity of liquid changes to vapour, differentiating eq. (1),

$$\delta G = \delta m_1 g_1 + \delta m_2 g_2 \quad \dots(2)$$

As the change takes place at constant temperature and pressure, the process is isothermal and isobaric.

$$\therefore \delta G = 0 \quad \dots(3)$$

from equation (2) and (3)

$$\delta m_1 g_1 + \delta m_2 g_2 = 0$$

$$\text{But, } \delta m_1 = -\delta m_2$$

$$\therefore g_1 = g_2$$

Let the temperature of the system be increased from T to $T + dt$.

For equilibrium

$$g_1 + dg_1 = g_2 + dg_2$$

$$\Rightarrow dg_1 = dg_2$$

If the condition of saturation is satisfied

$$\left[\frac{dg_1}{dT} \right]_{\text{sat.}} = \left[\frac{dg_2}{dT} \right]_{\text{sat.}} \quad \dots(4)$$

The pressure also changes from p to $p + dp$

$$\therefore dg_1 = \left[\frac{\delta g_1}{\delta T} \right]_p dT + \left[\frac{\delta g_1}{\delta P} \right]_T dp$$

$$\text{or } \frac{\delta g_1}{\delta T} = \left[\frac{\delta g_1}{\delta T} \right]_p + \left[\frac{\delta g_1}{\delta P} \right]_T \cdot \left[\frac{\delta P}{\delta T} \right] \quad \dots(5)$$

But, for unit mass,

$$dg = VdP - SdT$$

$$\therefore \left[\frac{\delta g}{\delta P} \right]_T = V \text{ and } \left[\frac{\delta g}{\delta T} \right]_P = -S$$

Substituting these values in equation (5),

$$\left[\frac{\delta g_1}{\delta T} \right]_{\text{sat.}} = -S_1 + V_1 \left[\frac{\partial P}{\partial T} \right]_{\text{sat.}}$$

Similarly,

$$\left[\frac{\delta g_2}{\delta T} \right]_{\text{sat.}} = -S_2 + V_2 \left[\frac{\partial P}{\partial T} \right]_{\text{sat.}}$$

Substituting these values in equation (4),

$$-S_1 + V_1 \left[\frac{dP}{dT} \right] = -S_2 + V_2 \left[\frac{dP}{dT} \right]$$

$$\text{But } S_2 - S_1 = \frac{\delta H}{T} = \frac{L}{T}$$

$$\therefore \left[\frac{dP}{dT} \right]_{\text{sat}} = \left[\frac{S_2 - S_1}{V_2 - V_1} \right]$$

$$\left[\frac{dP}{dT} \right]_{\text{sat}} = \frac{L}{T(V_2 - V_1)}$$

This is known as Clausius-Clapeyron heat equation.

Q10. Write short notes on second order phase transition.
(2012, 14, 16, 17, 19)

Ans. The phase transition during which there is no transference of heat and there is no change in volume, such transition is known as second order phase transition.

Suppose m_1 and m_2 are the masses of liquid and vapour phase and g_1 and g_2 are the specific value of Gibbs's potential in the two phase. Then for whole system,

$$G = m_1 g_1 + m_2 g_2 \quad \dots (1)$$

If a small quantity of liquid changes to vapour,

$$\delta G = \delta m_1 g_1 + \delta m_2 g_2 \quad \dots (2)$$

As the change takes place at constant temperature and pressure.

$$\therefore \delta G = 0 \quad \dots (3)$$

from equation (2) and (3)

$$\delta m_1 g_1 + \delta m_2 g_2 = 0$$

But, $\delta m_1 = \delta m_2$

$$\therefore g_1 = g_2$$

If Y and δ are the specific volume and entropy. Then $S_1 = S_2$ and $V_1 = V_2$

$$\left(\frac{\partial g_2}{\partial T} \right)_p - \left(\frac{\partial g_1}{\partial T} \right)_p = -(S_2 - S_1) < 0$$

$$\text{and } \left(\frac{\partial g_2}{\partial P} \right)_T - \left(\frac{\partial g_1}{\partial P} \right)_T = -(V_2 - V_1) > 0$$

$$\text{Then, } \left(\frac{\partial g_2}{\partial T} \right)_p - \left(\frac{\partial g_1}{\partial T} \right)_p = -(S_2 - S_1) = 0$$

$$\text{or } \left(\frac{\partial g_1}{\partial T} \right)_p = \left(\frac{\partial g_2}{\partial T} \right)_p$$

$$\text{and } \left(\frac{\partial g_2}{\partial P} \right)_T - \left(\frac{\partial g_1}{\partial P} \right)_T = -(V_2 - V_1) = 0$$

$$\text{or } \left(\frac{\partial g_1}{\partial P} \right)_T = \left(\frac{\partial g_2}{\partial P} \right)_T$$

If k is isothermal compressibility and α is the coefficient of volume expansion at constant pressure, then

$$\frac{C_p}{T} = \left(\frac{\partial S}{\partial T} \right)_p = \frac{\partial}{\partial T} \left[- \left(\frac{\partial g}{\partial T} \right)_p \right] = - \left[\frac{\partial^2 g}{\partial T^2} \right]_p$$

$$kV = - \left(\frac{\partial V}{\partial P} \right)_T = - \frac{\partial}{\partial P} \left[\left(\frac{\partial g}{\partial P} \right)_T \right] = \left(\frac{\partial^2 g}{\partial P^2} \right)_T$$

$$\alpha V = \left(\frac{\partial V}{\partial T} \right)_p = \frac{\partial}{\partial T} \left[\left(\frac{\partial g}{\partial P} \right)_T \right] = \frac{\partial^2 g}{\partial T \partial P}$$

$$\therefore \left[\frac{\partial^2 g_2}{\partial T^2} \right]_p - \left[\frac{\partial^2 g_1}{\partial T^2} \right]_p = \frac{C_p}{T} - \frac{C_p}{T}$$

$$\left[\frac{\partial^2 g_2}{\partial P^2} \right]_T - \left[\frac{\partial^2 g_1}{\partial P^2} \right]_T = V(k_2 - k_1)$$

$$\frac{\partial^2 g_2}{\partial T \partial P} - \frac{\partial^2 g_1}{\partial T \partial P} = V(\alpha_2 - \alpha_1)$$

For second order phase transition,

$$S_1 = S_2$$

$$S_1 + dS_1 = (S_2 + dS_2)$$

at $T + dT$ and $P + dP$.

$$\text{But, } dS = \left(\frac{\partial S}{\partial T} \right)_p dT + \left(\frac{\partial S}{\partial P} \right)_T dP$$

$$\left(\frac{\partial S}{\partial P} \right)_T = - \left(\frac{\partial V}{\partial T} \right)_p \text{ for Maxwell's relation.}$$

$$\therefore dS = \frac{C_P}{T} dT - \left(\frac{\partial V}{\partial T} \right)_P dP$$

$$dS = \frac{C_P}{T} dT - (\alpha V) dP$$

$$\therefore \frac{C_P}{T} dT - \alpha_1 V dP = \frac{C_P}{T} dT - V \alpha_2 dP$$

$$\frac{dP}{dT} = \frac{C_P - C_A}{T_1 (\alpha_2 - \alpha_1)} \quad \dots(4)$$

and similarly,

$$V_1 = V_2$$

$$dV_1 = dV_2$$

$$\therefore dV = \left(\frac{\partial V}{\partial T} \right)_P dT + \left(\frac{\partial V}{\partial P} \right)_T dP$$

$$\partial V = V \alpha dT - V k dP$$

$$\text{As } dV_1 = dV_2$$

$$V_1 \alpha_1 dT - V_1 k_1 dP = V_2 \alpha_2 dT - V_2 k_2 dP$$

$$\frac{dP}{dT} = \frac{\alpha_2 - \alpha_1}{k_2 - k_1} \quad \dots(5)$$

Equation (4) and (5) are known as Ehrenfest's equations.

Q11. Calculate under what pressure water will boil at 120°C if the change in specific volume when 1 gm of water is converted into steam is 1676 cm³. Given that : Latent heat of steam = 540 cal/g, J = 4.2 × 10⁷ ergs/cal.

1 atm pressure = 10⁶ dynes/cm².

(2014)

Ans. from Maxwell's second relation,

$$\left(\frac{\partial S}{\partial V} \right)_T = \left(\frac{\partial P}{\partial T} \right)_V$$

$$T \left(\frac{\partial S}{\partial V} \right)_T = T \left(\frac{\partial P}{\partial T} \right)_V$$

$$\text{or } \left(\frac{\partial Q}{\partial V} \right)_T = T \left(\frac{\partial P}{\partial T} \right)_V$$

Here, $\partial Q = 540$ cal, = 540 × 4.2 × 10⁷ ergs.

$$\partial V = 1676 \text{ cc, } T = 373 \text{ K}$$

$$dT = 50 \text{ K, } dP = ?$$

$$\frac{540 \times 4.2 \times 10^7}{1676} = 373 \left(\frac{\partial P}{50} \right)$$

$$\partial P = \frac{50 \times 540 \times 4.2 \times 10^7}{373 \times 1676}$$

$$\text{or } \partial P = 1.815 \times 10^6 \text{ dynes/cm}^2$$

$$= 1.815 \text{ atm.}$$

Hence, pressure required at which water would boils at 150°C.

$$P = 1.815 + 1 = 2.815 \text{ atm.}$$

Q12. Write short notes on Joule-Thompson effect.

(2014, 16, 17, 19)

Ans. The principle of Joule-Thomson experiment consists in forcing a gas at a constant pressure through a porous plug thermally insulated from outside. The porous plug consists of a compressed porous material like cotton, wool or silk fibres having a number of fine holes or pores and is thus equivalent to a number of narrow orifices in parallel. The gas, on passing through the plug under a large drop of pressure, gets throttled, i.e., its molecules get further apart from one another, so that the gases has to be do some internal work against inter-molecular attractions, if these exist.

Let us consider a simple arrangement. Let LL' is a cylinder of non-conducting walls having a porous plug C fitted with two non-conducting piston A and B . A fixed mass of a gas having volume V_1 is filled between piston A and porous plug.

To calculate the work done when gas passes through the pores from high pressure side, we can assume that the gas is pressed forward by means of an imaginary piston A and the gas pushes another imaginary piston B on the other side of plug at constant pressure P_2 . The Piston A takes the dotted piston A' and the piston B' takes the position B when whole of the gas has passed to the other side to the plug.

It is the area of cross-section of the piston is ' a ' and ' d ' is the distance moved by the piston A then the external work done on the gas on higher pressure side will be—

$$W_1 = P_1 a d_1 = P_1 V_1$$

Similarly, external work done by the gas on the lower pressure

side of the plug

$$H_2 = P_2 \cdot d_2 = P_2 V_2$$

Where, d_2 is the distance moved by the piston B_2 and V_2 is the volume of the gas at pressure P_2 having same mass at higher pressure.

Net external work done by the gas is—

$$\Delta W = W_2 - W_1 = P_2 V_2 - P_1 V_1 \quad \dots(1)$$

Using first law of thermodynamics—

$$\Delta U + \Delta W = 0 \text{ or } \Delta W = -\Delta U$$

Hence, external work is done on the expense of its own internal energy.

If U_1 and U_2 be the internal energies of the gas in the initial and final states, then—

$$\Delta W = -(U_2 - U_1),$$

from equation (1),

$$\begin{aligned} -(U_2 - U_1) &= P_2 V_2 - P_1 V_1 \\ -U_2 + U_1 &= P_2 V_2 - P_1 V_1 \\ U_1 + P_1 V_1 &= U_2 + P_2 V_2 \\ H_1 &= H_2 \end{aligned}$$

The quantity ($U + PV$) which is conserved, is called enthalpy or total heat function and denoted by H . This is why Joule-Thomson effect is sometimes referred to as the iso-enthalpic effect.

Q13. Write short notes on absolute temperature scale.

(2011, 14, 17, 18)

Ans. If a temperature scale defined utilizing property of working of Carnot's engine, it is an absolute scale of temperature because it does not depend upon the particular property of any substance as in the case of other thermodynamic scales.

Suppose an engine works between temperatures T_1 and T_2 . Here H_1 is heat absorbed at T_1 and H_2 is the heat rejected at T_2 .

$$\eta = f(T_1, T_2)$$

$$\eta = \frac{H_1 - H_2}{H_1} = f(T_1, T_2)$$

$$1 - \frac{H_2}{H_1} = f(T_1, T_2).$$

$$\frac{H_1}{H_2} = \frac{1}{1 - f(T_1, T_2)} = F(T_1, T_2) \quad \dots(1)$$

Similarly, if engine works between T_2 and T_3 and H_2 is heat absorbed at T_2 and H_3 is heat rejected at T_3 .

$$\frac{H_2}{H_3} = F(T_2, T_3) \quad \dots(2)$$

If the engine works b/w T_1 and T_3 , then—

$$\frac{H_1}{H_3} = F(T_1, T_3) \quad \dots(3)$$

From equation (1), (2) and (3)

$$\frac{H_1}{H_3} = \frac{H_1}{H_2} \times \frac{H_2}{H_3}.$$

$$F(T_1, T_3) = F(T_1, T_2) \times F(T_2, T_3)$$

$$F(T_1, T_3) = \frac{F(T_1, T_2)}{F(T_2, T_3)}$$

This relation can be satisfied only if—

$$F(T_1, T_2) = \frac{\phi(T_1)}{\phi(T_2)}$$

$$F(T_2, T_3) = \frac{\phi(T_2)}{\phi(T_3)}$$

$$F(T_3, T_1) = \frac{\phi(T_1)}{\phi(T_3)}$$

Where, ϕ is another function

$$\therefore \frac{H_1}{H_2} = F(T_1, T_2) = \frac{\phi(T_1)}{\phi(T_2)}$$

The expression on the right hand side represents the ratio of the two kelvin temperatures and this can be denoted by T_1/T_2 .

$$\therefore \frac{H_1}{H_2} = \frac{T_1}{T_2} \quad \dots(4)$$

The relation (4) is used to represent a new scale & this does not depend upon the property of working substance

$$\eta = \frac{H_1 - H_2}{H_1} = \frac{T_1 - T_2}{T_1}$$

$$\eta = 1 - \frac{T_2}{T_1}$$

Q14. Explain quasi-static thermodynamic process. (2015, 16)

Ans. A quasi-static process is defined as the process in which the deviation from the thermodynamic equilibrium is infinitesimal and all the states through which the system passes during a quasi-static process can be considered as equilibrium state.

In actual practice, many process closely approach a quasi-static process and may be treated as such with no significant error. Consider the expansion of a gas in a closed cylinder fitted with a piston. Initially weights are on the piston and the pressure of the gas weights are small and are taken off slowly one by one, the process can be considered quasi-static. If however, all the weights are removed at once, expansion takes place suddenly and it will be a non-equilibrium process. The system will not be in equilibrium at any time during the process.

A quasi-static process is an ideal concept that is applicable to all thermodynamic system including electric and magnetic systems. It should be noted that conditions for such a process can never be satisfied rigorously in practice.

Q15. A carnot engine has 50% efficiency, if the temperature of the sink is 7°C for increasing efficiency to 0.7, by how many degrees, the temperature of the source be increased? (2015)

Ans. Case I : $\eta = 50\% = 0.5$

$$T_2 = 273 + 7 = 280 \text{ K}$$

$$T_1 = ?$$

$$\eta = 1 - \frac{T_2}{T_1}$$

$$0.5 = 1 - \frac{280}{T_1}$$

$$T_1 = 560 \text{ K}$$

$$T_2 = 7^\circ\text{C}$$

$$T_1 = 280^\circ\text{C}$$

$$\eta = 1 - \frac{T_2}{T_1}$$

$$0.5 = 1 - \frac{280}{T_1}$$

$$T_1 = 560^\circ\text{C}$$

Case II : $\eta' = 70\% = 0.7$
 $T_2 = 280 \text{ K}$

$$T_1' = ?$$

$$\eta' = 1 - \frac{T_2}{T_1'}$$

$$0.7 = 1 - \frac{280}{T_1'}$$

$$T_1' = 840 \text{ K}$$

Increase in temperature = $840 - 560 = 280 \text{ K}$.

Q16. Discuss the variation of boiling and melting points with pressure. (2015, 17)

Ans. **Effect on boiling points**—When a liquid boils *i.e.* changes from liquid to gaseous state, there is an increase in its volume so that

$V_2 > V_1$ or the quantity $(V_2 - V_1)$ is positive. Hence, $\frac{dP}{dT}$ is a positive

quantity which means the boiling point of a liquid rises with increase in pressure or vice-versa. Thus a liquid will boil at lower temperature under reduced pressure. Hence, water will boil at lower temperature under reduced pressure. Hence, water will boil at a temperature less than 100°C when the atmospheric pressure is less than 76 cm of mercury of normal pressure.

Effects on melting point—When a solid melts, there may be an increase in volume as in the case of certain substances like wax and sulphur or there may be decrease in volume as in the case of certain substance like ice, gallium and bismuth.

(i) When $V_2 > V_1$ (as for wax and sulphur) dP/dT is a +ve quantity. This means that melting point of such substances rises with increase in pressure.

(ii) When $V_2 < V_1$ (as for ice, gallium and bismuth), $(V_2 - V_1)$ is a negative quantity. Hence dP/dT is also negative which means that melting point of such substance decreases with increase in pressure. Thus ice will melt at a lower temperature than 0°C when pressure is higher than normal pressure, 76 cm of mercury.

Q17. Obtain an expression for change in temperature of gases under going joule-Thomson expansion. Explain, why usually hydrogen and helium show heating effect upon suffering joule-Thomson expansion? (2015)

Ans. When a gas under constant pressure is forced through an insulated porous plug to a region of lower constant pressure, its temperature changes. This is called the joule-Thomson effect.

The magnitude and sign of this temperature change depends upon the initial temperature of gas and proportional to the difference of pressure between two sides of plug.

Let us consider 1 gm mole of real gas is allowed to expand from P_1, V_1 to P_2, V_2 . And temperature changes from T_1 to T_2 . Net external work done by gas

$$= P_2 V_2 - P_1 V_1.$$

Now, an internal work is also done by the gas in overcoming the force of molecular attraction.

$$\int_{V_1}^{V_2} P dV = \int_{V_1}^{V_2} \frac{a}{V^2} dV = \frac{a}{V_1} - \frac{a}{V_2},$$

Hence total work done by the gas—

$$W = (P_2 V_2 - P_1 V_1) + \frac{a}{V_1} - \frac{a}{V_2}.$$

From Vander Waal's equation for a gas—

$$\left[P + \frac{a}{V^2} \right] (V - b) = RT,$$

$$PV + \frac{a}{V} - Pb - \frac{ab}{V^2} = RT.$$

$$PV = RT + Pb - \frac{a}{V} \quad \left\{ \text{neglecting } \frac{a}{V^2} \right\}$$

Now,

$$W = \left(RT_2 + P_2 b - \frac{a}{V_2} \right) - \left(RT_1 + P_1 b - \frac{a}{V_1} \right) + \frac{a}{V_1} - \frac{a}{V_2}$$

$$W = R(T_2 - T_1) - b(P_1 - P_2) + \frac{2aP}{RT_1} - \frac{2aP}{RT_2}$$

$$W = R(T_2 - T_1) - b(P_1 - P_2) + \frac{2aP}{RT_1} - \frac{2aP}{RT_2}$$

$$W = -RdT - b(P_1 - P_2) + \frac{2a}{RT}(P_1 - P_2).$$

$$\{ T_1 - T_2 = dT \}.$$

$$W = (P_1 - P_2) \left[\frac{2a}{RT} - b \right] - RdT$$

If C_v is specific heat of gas at constant volume, then heat loss by the gas to do work is $C_v dT$.

$$C_v dT = (P_1 - P_2) \left[\frac{2a}{RT} - b \right] - RdT$$

$$(C_v + R) dT = (P_1 - P_2) \left[\frac{2a}{RT} - b \right]$$

$$C_p dT = (P_1 - P_2) \left[\frac{2a}{RT} - b \right]$$

Hence, fall in temperature

$$dT = \frac{(P_1 - P_2)}{C_p} \left[\frac{2a}{RT} - b \right]$$

The gas will be heated if its initial temperature is more than the inversion temperature. The inversion temperature of hydrogen and helium are much below than room temperature and hence ordinary temperature these gases show a heating effect.

If these gases are cooled below their temperature of inversion and then passes through the porous plug, they will also show a cooling effect.

Q18. Using Maxwell's thermodynamical relation, obtain Clausius-Clapeyron equation. (2015)

Ans. Maxwell's second relation of thermodynamic

$$\left(\frac{\partial S}{\partial V} \right)_T = \left(\frac{\partial P}{\partial T} \right)_V$$

$$T \left(\frac{\partial S}{\partial V} \right)_T = T \left(\frac{\partial P}{\partial T} \right)_V$$

$$\left(\frac{\partial Q}{\partial V} \right)_T = T \left(\frac{\partial P}{\partial T} \right)_V$$

$$\{ TdS = dQ \}.$$

Change in heat brings change in volume due to change of state only, thus heat added to the system must be latent heat.

$$\delta Q = L \text{ and } \delta T = T_2 - T_1$$

$$\therefore \frac{L}{T_2 - T_1} = T \left(\frac{\partial P}{\partial T} \right)_V$$

$$\left(\frac{\partial P}{\partial T} \right) = \frac{L}{T (T_2 - T_1)}$$

This is the Clausius-Clapeyron equation.

Q19. Show that there is increase in entropy for a thermodynamic system undergoing an irreversible process, while a reversible process is an isentropic process. (2015)

Ans. Irreversible process—Suppose the working substance is an engine performs an irreversible cycle of changes, absorbing an amount of heat Q_1 at a temperature T_1 from the source and rejecting the quantity of heat Q_2 at a temperature T_2 of the sink. Then efficiency of this cycle is—

$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

According to Carnot's theorem, this efficiency is less than that of a reversible engine working between the same two temperature T_1 and T_2 .

$$\eta = 1 - \frac{T_2}{T_1}$$

Thus,

$$1 - \frac{Q_2}{Q_1} < 1 - \frac{T_2}{T_1}$$

$$\frac{Q_2}{Q_1} > \frac{T_2}{T_1}$$

$$\frac{Q_2}{T_2} > \frac{Q_1}{T_1}$$

$$\frac{Q_2}{T_2} - \frac{Q_1}{T_1} > 0$$

Considering the whole system, the source losses entropy by amount $\frac{Q_1}{T_1}$ and sink gains entropy by amount $\frac{Q_2}{T_2}$.

$$\therefore \Delta S = \frac{Q_2}{T_2} - \frac{Q_1}{T_1}$$

Which is greater than zero or positive. Thus there is an increase in entropy of the system during irreversible process. Therefore, the entropy of a system increases in all the irreversible processes.

Reversible process—Let us consider a Carnot engine, in which heat Q_1 is absorbed at temperature T_1 and is +ve and heat Q_2 is rejected at temperature T_2 and is -ve.

The gain in entropy during absorption of heat $Q_1 = \frac{Q_1}{T_1}$.

Loss in entropy during rejection of heat $Q_2 = \frac{Q_2}{T_2}$.

Thus net gain in entropy in whole cycle $= \frac{Q_1}{T_1} - \frac{Q_2}{T_2}$.

but in an complete reversible Carnot's cycle $\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$.

$$\therefore \frac{Q_1}{T_1} - \frac{Q_2}{T_2} = 0.$$

It means total change in entropy of working substance is zero. Thus in a cycle of reversible process the entropy of system remains unchanged or the change in entropy of the system is zero.

Q20. 1 kg of water at 0°C is mixed with 1 kg of water at 100°C, resulting a 2 kg of water at 50°C. Find change in entropy due to mixing. (2015)

Ans. Change in entropy of universe due to mixing

$$= 2mC_p \log_e \left[\frac{T_1 + T_2}{2\sqrt{T_1 T_2}} \right]$$

$$\Delta S = 2 \times 1 \times 1 \log_e \left[\frac{273 + 373}{2\sqrt{273 \times 373}} \right]$$

$$\Delta S = 1.294 \text{ kJ/K.}$$

Q21. Write short note on specific heat of saturated vapour. (2015)

Ans. The latent heat of vapourisation of water of temperature T is given by—

$$L = 800 - 0.705 T$$

$$\frac{L}{T} = \frac{800}{T} - 0.705$$

$$\frac{dL}{dT} = -0.705$$

By clausius equation,

$$C_{\text{vap}} - C_{\text{liq}} = \frac{dL}{dT} - \frac{L}{T}$$

$$\Rightarrow C_{\text{vap}} - C_{\text{liq}} = -0.705 - \frac{800}{T} + 0.705$$

$$C_{\text{vap}} - C_{\text{liq}} = -\frac{800}{T}$$

Since, for water $C_{\text{liq}} = 1$.

$$C_{\text{vap}} = 1 - \frac{800}{T}$$

$$\text{at } 0^\circ\text{C}, C_{\text{vap}} = 1 - 2.9034 = -1.9304$$

$$\text{at } 100^\circ\text{C}, C_{\text{vap}} = 1 - 2.1440 = -1.1440$$

Thus, specific heat of saturated vapour is a negative quantity. The meaning of saturated vapour of 100°C is -1.1440 means, if a gram of water vapour at 100°C and 760 mm pressure has its pressure increased to the vapour pressure corresponding to 101°C and if at the same time 1.144 calories are withdrawn, then the water vapour will remain saturated at 101°C .

Q22. Write short note on equivalence of Kelvin-Planck and Clausius statements of second law of thermodynamics. (2015, 18)

Ans. Clausius statement—No process is possible whose sole result is the transfer of heat from a colder body to a hotter body.

Kelvin statement—No process is possible whose sole result is the complete conversion of heat into work.

For stabilisation between kelvin and clausius statement of second law of thermodynamic we prove that if Kelvin (K) statement false $\Rightarrow$ Clausius statement (C) is false.

Let K is false. Then we can draw heat Q from a heat reservoir at temperature T_2 and convert it entirely into work W ($= Q$), with no

other effect. Next we convert this work into heat Q by means of friction and deliver it to a reservoir at temperature $T_1 > T_2$ with no other effect. The total result of this two step process is the transfer of heat Q amount from a cooler body to a hot body with no other effect. Therefore C is false.

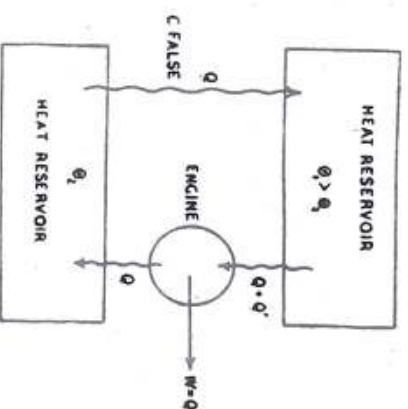


Fig. Equivalence of Clausius and Kelvin statements.

Proof that C false $\Rightarrow K$ false. Let us define an engine to be thermodynamic system that can under go a cyclic process in which the system does the following things—

- (1) It absorbs heat Q_1 from hot reservoir at T_1 .
- (2) It performs a work W .
- (3) It rejects heat Q_2 to cold reservoir at T_2 .

Let us operate the above engine which absorbs ($Q + Q'$) heat from the reservoir, converts Q' into work W and rejects Q amount heat to cold reservoir of T_2 which is again transferred to hot reservoir. Hence the total result is that Q' is extracted from single reservoir and entirely converted into work with no other effect. Hence K is false.

Q23. Write short notes on TdS equation and their application. (2015)

Ans. First TdS equation—The entropy S of pure substance can be taken as a function of temperature and volume.

$$dS = \left(\frac{\partial S}{\partial T} \right)_V dT + \left(\frac{\partial S}{\partial V} \right)_T dV$$

multiply both side by T

$$TdS = T \left[\left(\frac{\partial S}{\partial T} \right)_V dT + T \left(\frac{\partial S}{\partial V} \right)_T dV \right] \dots (1)$$

$$\text{But, } C_p = T \left[\frac{\partial S}{\partial T} \right]_p$$

and from maxwell relation,

$$\left[\frac{\partial S}{\partial T} \right]_p = \left[\frac{\partial P}{\partial T} \right]_p$$

Substituting these values in equation (1)

$$TdS = C_p dT + T \left[\frac{\partial P}{\partial T} \right]_p dV$$

Second Tds Equation—

The entropy S of a pure substance can also be regarded as a function of temperature & pressure,

$$\therefore dS = \left[\frac{\partial S}{\partial T} \right]_p dT + \left[\frac{\partial S}{\partial P} \right]_T dP$$

Multiply both sides with T

$$TdS = T \left[\frac{\partial S}{\partial T} \right]_p dT + T \left[\frac{\partial S}{\partial P} \right]_T dP \quad \dots(2)$$

$$\text{But } C_p = T \left[\frac{\partial S}{\partial T} \right]_p$$

& from maxwell's relations—

$$\left[\frac{\partial S}{\partial P} \right]_T = - \left[\frac{\partial V}{\partial T} \right]_p$$

Substituting in equation (2)

$$TdS = C_p dT - T \left[\frac{\partial V}{\partial T} \right]_p dP$$

The Tds equations are used to derive latent heat equation, calculation of heat evolved during reversible isothermal change of pressure, etc.

Q24. Obtain an expression for the work done by an ideal gas when it expands adiabatically.

Ans. During adiabatic process, the gas expands from volume V_1

and V_2 . Work done for an increase in volume dV ,
 $dW = P dV$

$$W = \int_{V_1}^{V_2} P dV.$$

During an adiabatic process—

$$PV^\gamma = \text{constant} = K.$$

$$P = \frac{K}{V^\gamma}$$

$$W = \int_{V_1}^{V_2} \frac{K}{V^\gamma} dV = \frac{K}{1-\gamma} \left[\frac{1}{V_2^{\gamma-1}} - \frac{1}{V_1^{\gamma-1}} \right].$$

$$W = \frac{1}{1-\gamma} \left[\frac{k}{V_2^{\gamma-1}} - \frac{k}{V_1^{\gamma-1}} \right].$$

$$W = \frac{1}{1-\gamma} \left[\frac{P_2 V_2^\gamma}{V_2^{\gamma-1}} - \frac{P_1 V_1^\gamma}{V_1^{\gamma-1}} \right].$$

$$W = \frac{1}{1-\gamma} [P_2 V_2 - P_1 V_1].$$

Q25. Applying the first law of thermodynamics derive the equation $C_p - C_v = R$ for an ideal gas. (2016, 19)

Ans. The internal energy of a system is a single valued function of the state variables, i.e., pressure, temperature, volume, etc. In the case of a gas any two of the variables P, V, T are sufficient to define completely its state. If V and T are chosen as the independent variables—

$$U = f(V, T)$$

differentiating,

$$dU = \left(\frac{\partial U}{\partial T} \right)_V dT + \left(\frac{\partial U}{\partial V} \right)_T dV \quad \dots(1)$$

from first law of thermodynamics,

$$\delta H = dU + P dV \quad \dots(ii)$$

from eq. (i) and (ii),

$$\delta H = \left(\frac{\partial U}{\partial T} \right)_V dT + \left(\frac{\partial U}{\partial V} \right)_T dV + P dV$$

$$\frac{\delta H}{\delta T} = \left(\frac{\partial U}{\partial T} \right)_V + \left[P + \left(\frac{\partial U}{\partial V} \right)_T \right] \frac{dV}{dT} \quad \dots (iii)$$

If gas is heated at constant volume,

$$\left(\frac{\delta H}{\delta T} \right) = C_V$$

$$\text{and } \frac{dV}{dT} = 0.$$

$$\therefore \left(\frac{\delta H}{\delta T} \right)_V = \left(\frac{\partial U}{\partial T} \right)_V = C_V$$

When gas is heated at constant pressure:

$$\left(\frac{\delta H}{\delta T} \right)_P = C_P$$

from equation (iii),

$$C_P = \left(\frac{\partial U}{\partial T} \right)_V + \left[P + \left(\frac{\partial U}{\partial V} \right)_T \right] \left(\frac{\partial V}{\partial T} \right)_P$$

$$C_P = C_V + \left[P + \left(\frac{\partial U}{\partial V} \right)_T \right] \left(\frac{\partial V}{\partial T} \right)_P \quad \dots (iv)$$

From Joule's experiment, for an ideal gas on opening the stopcock,

no work was done and no heat transfer took place.

$$\text{So, } \delta H = 0 = dU + 0$$

$$\text{Therefore, } dU = 0.$$

$$\left(\frac{\partial U}{\partial V} \right)_T = 0.$$

From ideal gas equation

$$PV = RT$$

$$P \left(\frac{\partial V}{\partial T} \right)_P = R$$

$$\therefore C_P - C_V = P \left[\frac{\partial V}{\partial T} \right]_P + \left[\frac{\partial U}{\partial V} \right]_T \left[\frac{\partial V}{\partial T} \right]_P$$

$$\text{But, } \left(\frac{\partial U}{\partial V} \right)_T = 0$$

$$\therefore C_P - C_V = P \left[\frac{\partial V}{\partial T} \right]_P = R.$$

$$\Rightarrow C_P - C_V = R.$$

Q26. The efficiency of a carnot engine is $1/6$. If the temperature of the sink is decreased by 65°C , the efficiency becomes $1/3$. Calculate the temperature of source & sink. (2016, 18)

$$\text{Ans, } \eta_1 = \frac{1}{6}$$

$$\eta_1 = \frac{1}{6} = 1 - \frac{T_2}{T_1}$$

...(i)

$$\eta_2 = \frac{1}{3}$$

$$\eta_2 = \frac{1}{3} = 1 - \frac{T_2 - 65}{T_1}$$

...(ii)

Subtracting equation (i) from (ii).

$$\frac{1}{3} - \frac{1}{6} = 1 - \frac{T_2 - 65}{T_1} - 1 + \frac{T_2}{T_1}$$

$$\frac{1}{6} = \frac{T_2 - T_2 + 65}{T_1}$$

$$T_1 = 390 \text{ K} = 117^\circ\text{C}.$$

from equation (i),

$$\frac{1}{6} = 1 - \frac{T_2}{390}$$

$$T_2 = 390 - \frac{390}{6} = 325 \text{ K} = 52^\circ\text{C}.$$

$$T_2 = 52^\circ\text{C}.$$

Q27. Establish the concept of Entropy and disorder. (2016)

Ans. With an increase in entropy, the thermal agitation and hence disorder of the molecule of substance increase, i.e., growth of entropy

implies a transition from order to disorder. Thus the principle of increase in entropy is intimately connected with the less ordered state of affairs. Entropy of a substance is therefore said to be a measure of the degree of disorder prevailing among molecules, just as temperature is a measure of the degree of hotness of a substance.

The entropy of any isolated system increases and approaches, more or less rapidly, to the inert state of maximum entropy. We may recognize this fundamental law of physics to be an inherent tendency of nature to proceed from a more ordered state to a less ordered one or from a less disordered to a more disordered state, or we may state in other words that the ultimate destiny of the universe is not order but chaos or disorder.

Q28. 1 kg of water at 273 K is brought in contact with a heat reservoir at 373 K. What is the change in entropy of water as its temperature reaches 373 K? (2016)

Ans. Increase in entropy when the temperature of 1000 g of water raised from 273 to 373 K

$$dS = \int_{T_1}^{T_2} \frac{dH}{T} = ms \times 2.3026 \log_{10} \frac{T_2}{T_1}$$

$$dS = 1000 \times 1 \times 2.3026 \log_{10} \frac{373}{273}$$

$$dS = 312 \text{ cal/K}$$

Q29. What do you understand by the internal energy of a system? State first law of thermodynamics and use it to derive a relation between the changes in volume, temperature and heat. (2017)

Ans. **Internal Energy**—It can be defined as the energy possessed by a body or by a system due to molecular motion and molecular configuration.

First law of thermodynamics—If the quantity of heat supplied to a system be δQ , the amount of external work done be δW and increase in internal energy of the molecules be dU . Then,

$$\delta Q = dU + \delta W$$

This represents first law of thermodynamics.

$$\text{Now, } \delta Q = dU + \delta W$$

$$dU = dQ - dW$$

$$dU = TdS - PdV$$

$$\therefore dQ = TdS \text{ and } dW = PdV \}$$

At constant volume, $dV = 0$

$$dU = TdS.$$

$$\left[\frac{dU}{dS} \right]_V = T$$

Correct ans

differentiating equation (1), w.r.t. volume at constant entropy

$$\frac{d^2 U}{dS dV} = \left[\frac{\partial T}{\partial V} \right]_S$$

....(2)

At constant entropy, $dS = 0$

$$dU = -PdV$$

$$\left[\frac{dU}{dV} \right]_S = -P$$

....(3)

differentiating equation (3), w.r.t. entropy at constant volume

$$\frac{d^2 U}{dS dV} = - \left[\frac{\partial P}{\partial S} \right]_V$$

....(4)

from equation (2) and (4) :

$$\left[\frac{dT}{dV} \right]_S = - \left[\frac{\partial P}{\partial S} \right]_V$$

Q30. Write short notes on joules law. (2017)

Ans. Joules law state as follow—

The internal energy of a perfect gas is a function of the absolute

temperature only.

$$\text{i.e., } U = f(T).$$

According to non-flow energy equation,

$$dQ = dU + dW$$

$$dW = 0, \text{ since volume remain constant}$$

$$\therefore dQ = dU$$

At constant volume for perfect gas, for 1 kg

$$dQ = C_V dT$$

and integrating

$$u = C_V T + K$$

According to joule's law $U = f(T)$, which means that internal energy varies linearly with absolute temperature. Internal energy can be made zero at any arbitrary reference temperature.

For a perfect gas it can be assumed that $U = 0$ when $T = 0$, hence k is zero.

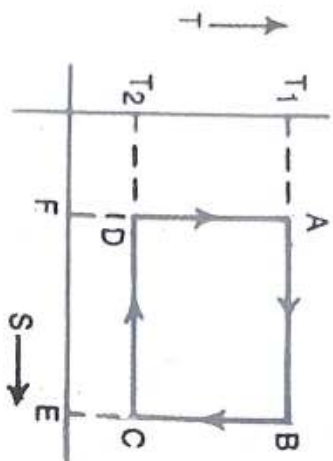
for perfect gas

$$u = C_V T$$

$$\text{For mass } m, U = mC_V T$$

Q31. What is T-S diagram. Draw T-S diagram for Carnot-cycle. (2012, 18)

Ans. For a Carnot cycle, it consists of two isothermal and two adiabatics. The isothermal and represented by horizontal straight lines (parallel to entropy axis) and adiabatics by vertical straight lines (parallel to temperature axis) on T-S diagram.



Line AB represents the isothermal expansion at a constant temperature T_1 of the source, the vertical line BC is the adiabatic expansion during which there is no change in entropy but a fall of temperature from T_1 to T_2 , the temperature of sink.

Line CD is the second isothermal representing compression at constant temperature T_2 and DA is the final adiabatic compression involving a rise of temperature from T_2 to T_1 entropy remaining the same.

The amount of heat energy absorbed in isothermal expansion AB is given by the area under AB i.e., $ABEF$. Heat rejected in isothermal compression CD is given by the area $CDFE$.

$$\text{Total heat absorbed} = \text{Area } ABEF$$

$$= T_1 (S_2 - S_1)$$

$$\text{Heat rejected} = \text{Area } CDEF$$

$$= T_2 (S_2 - S_1)$$

$$\text{Heat energy converted into work} = (T_1 - T_2) (S_2 - S_1)$$

$$\text{Efficiency } \eta = \frac{\text{Work output}}{\text{Heat input}}$$

$$\eta = \frac{(T_1 - T_2)(S_2 - S_1)}{T_1(S_2 - S_1)}$$

$$\eta = 1 - \frac{T_2}{T_1}$$

Q32. If 4 kg of water at 10°C is mixed with 8 kg of water at 40°C, then find the resulting change in entropy for such system $C_V = 1 \text{ cal/gm/}^\circ\text{C}$. (2018)

Ans. Change in entropy

$$S = mC_V \ln \frac{T_2}{T_1}$$

Find temperature of mixture,

$$T = \frac{40 + 10}{2} = 25^\circ\text{C}$$

Now, change in entropy, when 4 kg of water reaches 25°C.

$$S_1 = 4 \times 10^3 \times 1 \ln \frac{298}{283}$$

$$S_1 = 0.202 \times 10^3 \text{ cal/}^\circ\text{C}$$

Change in entropy when 8 kg water change from 40°C to 25°C

$$S_2 = 8 \times 10^3 \times 1 \ln \frac{298}{313}$$

$$S_2 = -0.343 \times 10^3 \text{ Cal/}^\circ\text{C}$$

Thus total entropy change of system.

$$\Delta S = -0.141 \times 10^3 \text{ Cal/}^\circ\text{C}$$

Q33. Prove that :

$$\frac{E_S}{E_T} = \frac{C_P}{C_V}$$

Where, E_S is adiabatic bulk modulus and E_T is isothermal bulk modulus. (2018)

Ans. The bulk modulus is defined as the ratio of the excess pressure and the corresponding volume strain.

$$E = \frac{\Delta P}{\left(-\frac{\Delta V}{V}\right)} = -\frac{\Delta P V}{\Delta V}.$$

If ΔP is positive, ΔV will be negative and vice-versa. The negative sign in our definition of bulk modulus ensure that E is positive.

The bulk modulus of gas depends on the pressure. Under isothermal condition the bulk modulus is equal to its pressure.

Under adiabatic condition, the bulk modulus is equal to γP , where $\gamma = C_p/C_v$ and is the ratio of specific heat at constant pressure to constant volume.

$$E_T = P$$

$$E_S = \gamma P = \frac{C_p}{C_v} P.$$

$$\frac{E_S}{E_T} = \frac{C_p}{C_v}.$$

Q34. Derive expression for joule thomson coefficient μ for Vander Waal's gas. (2018)

$$\mu = \frac{1}{C_p} \left\{ \frac{2a}{RT} - b \right\}$$

Ans. In joule-thomson experiment, enthalpy remains constant.

$$H = U + PV = \text{constant.}$$

$$dH = dU + PdV + VdP = 0$$

$$\therefore dQ = dU + PdV$$

$$dQ = TdS.$$

$$\therefore dH = TdS + VdP = 0$$

Consider entropy as function of P and T .

$$\text{i.e., } S = f(P, T)$$

$$dS = \left(\frac{\partial S}{\partial T} \right)_P dT + \left(\frac{\partial S}{\partial P} \right)_T dP \quad \dots(1)$$

from (1) and (2),

$$T = \left[\frac{\partial S}{\partial T} \right]_P dT + \left[T \left(\frac{\partial S}{\partial P} \right)_T + V \right] dP = 0 \quad \dots(2)$$

$$\dots(3)$$

$$\text{and } C_p = \left(\frac{\partial Q}{\partial T} \right)_P = T \left[\frac{\partial S}{\partial T} \right]_P \quad \dots(4)$$

from Maxwell's eq.

$$\left(\frac{\partial S}{\partial P} \right)_T = - \left(\frac{\partial V}{\partial T} \right)_P$$

$$C_p dT = \left[T \left(\frac{\partial V}{\partial T} \right)_P - V \right] dP$$

$$dT = \frac{1}{C_p} \left[T \left(\frac{\partial V}{\partial T} \right)_P - V \right] dP$$

Joule-Thomson coefficient,

$$\mu = \left(\frac{\partial T}{\partial P} \right) = \frac{1}{C_p} \left[T \left(\frac{\partial V}{\partial T} \right)_P - V \right]$$

for Vander Waal's gas equation of state,

$$\left(P + \frac{a}{V^2} \right) (V - b) = RT \quad \dots(5)$$

$$PV - Pb + \frac{a}{V} - \frac{ab}{V^2} = RT$$

neglecting, $\frac{ab}{V^2}$,

$$PV - Pb + \frac{a}{V} = RT$$

differentiating both sides with T , keeping P as constant.

$$P \left(\frac{dV}{dT} \right)_P - \frac{a}{V^2} \left(\frac{dV}{dT} \right) = R.$$

$$\left(\frac{dV}{dT} \right)_P = \frac{R}{\left[P - \frac{a}{V^2} \right]} \quad \dots(6)$$

from equation (5),

$$P = \frac{RT}{V - b} - \frac{a}{V^2} \quad \dots(7)$$

from equation (6) and (7)

$$\left(\frac{dT}{dT}\right)_P = \frac{P(V-b)}{RT - V}$$

$$T \left[\frac{dT}{dT} \right]_P = (V-b) \left[1 - \frac{2a}{V RT} \right]^{-1}$$

$$\left(\frac{dT}{dT}\right)_P = \frac{1}{T} \left\{ (V-b) + \frac{2a}{RT} \right\}$$

$$\therefore \mu = \frac{1}{C_P} \left\{ T \left(\frac{\partial V}{\partial T} \right)_P - V \right\}$$

$$\mu = \frac{1}{C_P} \left\{ (V-b) + \frac{2a}{RT} - V \right\}$$

$$\mu = \frac{1}{C_P} \left\{ \frac{2a}{RT} - b \right\}$$

Q35. Prove that,

$$C_P - C_V = -TE\alpha^2 V.$$

(2018)

$$\text{Ans. } C_P = \left[\frac{\partial H}{\partial T} \right]_P = T \left[\frac{\partial S}{\partial T} \right]_P$$

$$C_V = \left[\frac{\partial H}{\partial T} \right]_V = T \left[\frac{\partial S}{\partial T} \right]_V$$

Considering S as a function of temperature and volume.

$$dS = \left[\frac{\partial S}{\partial T} \right]_V dT + \left[\frac{\partial S}{\partial V} \right]_T dV$$

$$\left[\frac{\partial S}{\partial T} \right]_P = \left[\frac{\partial S}{\partial T} \right]_V \left[\frac{\partial T}{\partial T} \right] + \left[\frac{\partial S}{\partial V} \right]_T \left[\frac{\partial V}{\partial T} \right]_P$$

$$\text{But, } \left[\frac{\partial S}{\partial V} \right]_T = \left[\frac{\partial P}{\partial T} \right]_V$$

$$\therefore T \left[\frac{\partial S}{\partial V} \right]_P = T \left[\frac{\partial S}{\partial T} \right]_V + T \left[\frac{\partial P}{\partial T} \right]_V \left[\frac{\partial V}{\partial T} \right]_P$$

$$C_P = C_V + T \left[\frac{\partial P}{\partial T} \right]_V \left[\frac{\partial V}{\partial T} \right]_P$$

Now, from gas equation,

$$P = f(V, T)$$

$$dP = \left[\frac{\partial P}{\partial T} \right]_V dT + \left[\frac{\partial P}{\partial V} \right]_T dV$$

$$\frac{dP}{dT} = \left[\frac{\partial P}{\partial T} \right]_V + \left[\frac{\partial P}{\partial V} \right]_T \left[\frac{\partial V}{\partial T} \right]_P$$

At constant pressure,

$$dP = 0$$

$$\left[\frac{\partial P}{\partial T} \right]_V = - \left[\frac{\partial P}{\partial V} \right]_T \left[\frac{\partial V}{\partial T} \right]_P$$

$$\therefore C_P - C_V = -T \left[\frac{\partial P}{\partial V} \right]_T \left[\frac{\partial V}{\partial T} \right]_P^2$$

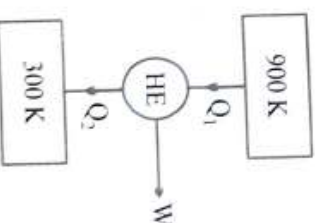
$$\text{But, } E = V \left[\frac{\partial P}{\partial V} \right]_T \text{ and } \alpha = \frac{1}{V} \left[\frac{\partial V}{\partial T} \right]_P$$

$$\therefore C_P - C_V = -TE\alpha^2 V.$$

Q36. A carnot engine takes 1000 calories heat at 627°C and rejects a part at 27°C. Find its efficiency, work done and heat rejected. (2019)

$$\text{Ans. } T_1 = 627 + 273 = 900 \text{ K}$$

$$T_2 = 27 + 273 = 300 \text{ K}$$



$$\eta = 1 - \frac{T_2}{T_1}$$

$$\eta = 1 - \frac{300}{900}$$

$$\eta = \frac{600}{900} = 0.666 \text{ or } 66.67\%$$

$$\eta = 0.667 = \frac{W}{Q_1}$$

$$W = 0.667 \times 1000$$

$$W = 667 \text{ cal.}$$

$$W = Q_1 - Q_2$$

$$Q_2 = 1000 - 667$$

$$Q_2 = 333 \text{ cal.}$$

[$\because Q_1 = 1000 \text{ cal.}$]

CHAPTER - 2

Kinetic Theory of Gases

Q1. Prove that mean free path ' λ ' in any gas is given by—

$$\lambda = \frac{m}{\sqrt{2}(\pi\sigma^2\rho)}$$

Where, σ and ρ are the molecular diameter and mass density respectively. (2014)

Ans. The mean free path is defined as the average distance traversed by a molecule between two successive collisions.

Let us consider a gas processing n molecules per unit volume. Let us assume that only one molecule which is under consideration is in motion while all other are at rest.

If σ is the diameter of each molecule, then the moving molecule will collide with all those molecules, whose centres lie within a distance σ from its centre.

If v is the velocity of the moving molecule then in one second it will collide with all molecules whose centre lie in a cylinder of radius σ and length v , i.e., within a volume $(\pi\sigma^2v)$.

Then the number of collision made by the moving molecules in one second is given by—

$$N = \pi\sigma^2vn.$$

The distance moved in one second = v and the number of collisions in one second = $\pi\sigma^2v \times n$.

$\therefore$ mean free path

$$\lambda = \frac{\text{Total distance traversed in one second}}{\text{Number of collision suffered by the molecules in one second.}}$$

$$\lambda = \frac{v}{\pi\sigma^2vn}$$

$$\Rightarrow \lambda = \frac{1}{\pi\sigma^2n}$$

This equation is deduced by Clausius.

Maxwell assumed the realistic assumption that all the molecules are moving with all possible velocities in all possible directions and found result by applying distribution law of molecular speeds.

The exact result obtain by him is—

$$\lambda = \frac{1}{\sqrt{2} \pi \sigma^2 n}$$

Let m be the mass the each molecules then $m \times n = \rho$, where, ρ is the density of molecules.

$$\therefore \lambda = \frac{m}{\sqrt{2} \sigma^2 \pi n \times m}$$

$$\lambda = \frac{m}{\sqrt{2} (\pi \sigma^2 \rho)}$$

Q2. Write notes on maxwell's distribution of molecular speed of gas. (2014, 17, 19)

Ans. Let us consider a zone of cells, i.e., group of cells in the phase space corresponding to the range in speed from v to $v + dv$. If g_i represents number of cells in the zone, then the number of particles in its zone, each having energy close to—

$$\epsilon_i = \frac{1}{2} m v^2,$$

By Maxwell's-Boltzmann law,

$$N_i = A g_i e^{-\beta \epsilon_i}$$

...(1)

$$\text{where, } \beta = \frac{1}{kT}.$$

As the number of molecules with speeds lying between v and $v + dv$ will be proportional to dv , we may write,

$$N_i = N_V dv,$$

Where, N_V is known as the distribution function with regards to speeds, so that equation (1) becomes,

$$N_V dv = A g_i e^{-\beta \epsilon_i}$$

We have,

...(1)

$$g_i = \frac{\text{Volume of zone in phase space}}{\text{Volume of one cell}}$$

Volume of zone in phase space

$$= \iiint dx dy dz dv_x dv_y dv_z$$

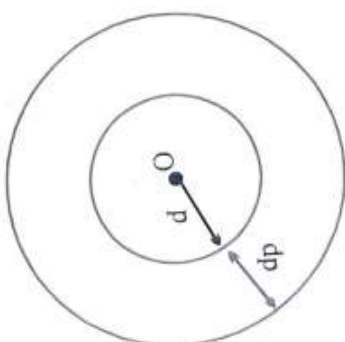
If h^3 is the volume of one cell, then

$$g_i = \frac{\iiint dx dy dz dP_x dP_y dP_z}{h^3} \quad \dots(3)$$

If V is the volume occupied by gas, we must have,

$$V = \iiint dx dy dz \quad \dots(4)$$

Let us imagine that P_x, P_y, P_z are along three mutually perpendicular axes and draw a sphere of radius P . Then the surface of sphere will contain all the points for which $P_x^2 + P_y^2 + P_z^2 = P^2$.



Now, draw another sphere of radius $P + dP$.

The volume between two spheres will contain all the molecules in momentum space having momentum between P and $P + dP$.

Volume between two spheres

$$\begin{aligned} &= \iiint dP_x dP_y dP_z = \frac{4}{3} \pi (P + dP)^3 - \frac{4}{3} \pi P^3 \\ &= \frac{4}{3} \pi [(P + dP)^3 - P^3] = \frac{4}{3} \pi (3P^2 dP) \\ &= 4\pi P^2 dP. \end{aligned}$$

Substituting these values in equation (3).

$$g_i = \frac{V (4\pi P^2 dP)}{h^3} = \frac{4\pi V P^2 dP}{h^3} \quad \dots(5)$$

from equation (2) and (5) and $\epsilon_i = \frac{1}{2} m v^2$,

$$N_V dv = \frac{4\pi V}{h^3} P^2 e^{-\beta m v^2 / 2} dv$$

$$N_V dV = \frac{4\pi h^3 m^3}{h^3} V^2 e^{-\beta m V^2/2} dV$$

$$\text{Put, } \frac{4\pi h^3 m^3}{h^3} = R, \beta = \frac{1}{kT},$$

$$N_V dV = B V^2 e^{-mV^2/2kT} dV$$

Integrating equation (6) from 0 to ∞ .

$$N = \int_0^{\infty} N_V dV = B \int_0^{\infty} V^2 e^{-\beta m V^2/2} dV$$

$$\beta = \left[\frac{1}{4} \sqrt{\frac{8\pi}{\beta^3 m^3}} \right] = N$$

$$B = 4N \sqrt{\frac{\beta^3 m^3}{8\pi}}$$

$$B = 4N \sqrt{\left[\frac{1}{kT} \right]^3 \frac{m^3}{8\pi}}$$

$$B = 4\pi N \left[\frac{m}{2\pi kT} \right]^{3/2}$$

Substituting the value of B in equation (6), Maxwell's distribution law of speed of molecules may be expressed as—

$$N_V = 4\pi N \left[\frac{m}{2\pi kT} \right]^{3/2} V^2 e^{-mV^2/2kT}$$

Q3. Write short note on viscosity of gases.

(2015)

Ans. Consider a gas flowing along horizontal surface. The layer in contact with the wall of the surface is at rest. The velocity of layer increases with increase in distance from fixed layer. Viscosity is defined as the tangential force per unit area required to maintain a unit velocity gradient.

$$F = -\eta A \frac{du}{dx}$$

...(1)

Here η is the coefficient of viscosity and A is the area of the layer and du/dx is the velocity gradient.

The velocity of a molecule at any layer at a height x above the fixed layer $= x \times \frac{du}{dx}$.

If mean free path of molecule is λ , then distance for first collision to occur $= (x - \lambda)$. The average velocity of these molecules

$$= (x - \lambda) \frac{du}{dx}.$$

Now, the molecules are moving at random in all directions.

Approximately it can be assumed that about one-third of the molecules are moving along each axis. As the molecule in a given space will be free to move in any direction, one-sixth can be assumed to be moving in one direction and other one-sixth in opposite direction.

Let C be the rms velocity of the molecules, n be the number of molecules per cc and m the mass of each molecule. Then, the number of molecules crossing unit area in one direction in one second $= \frac{nC}{6}$.

$$\text{Mass of the molecules} = \frac{m n C}{6} = M$$

$$\text{Momentum in opposite direction} = M \left\{ (x - (-\lambda)) \frac{du}{dx} \right\}.$$

Total change in momentum in one second.

$$F = M(x - \lambda) \frac{du}{dx} - M(x + \lambda) \frac{du}{dx}.$$

$$F = -2M\lambda \frac{du}{dx}.$$

$$F = -2 \left[\frac{1}{6} m n C \right] \lambda \frac{du}{dx}.$$

$$F = -\frac{1}{3} m n C \lambda \frac{du}{dx}$$

...(2)

From equation (1) and (2)

$$\eta A \frac{du}{dx} = \frac{1}{3} m n C \lambda \frac{du}{dx}$$

$$A = 1 \text{ sq cm}$$

$$\therefore \eta = \frac{1}{3} m n c \lambda$$

$$\text{But } \lambda = \frac{1}{\sqrt{2} \pi \sigma^2 n}$$

$$\Rightarrow \eta = \frac{1}{3} m n C \frac{1}{\sqrt{2} \pi \sigma^2 n}$$

$$\eta = \frac{mC}{3\sqrt{2}\pi\sigma^2} \quad \dots(3)$$

$$\text{Now } C \propto \sqrt{T}$$

$$\therefore \eta \propto \sqrt{T}.$$

The coefficient of viscosity of gas is directly proportional to square root of its temperature in kelvin (k).

Q4. Obtain expression for most probable, average speed and root mean square, speed of gas molecules using Maxwell Boltzmann distribution law. (2014, 15, 17)

Ans. Average speed—Let there be N molecules, each having velocity v_i , then the average speed ($\bar{v}$) is defined as—

$$\langle v \rangle = \frac{\sum_i N_i v_i}{\sum_i N_i} = \frac{1}{N} \sum_i N_i v_i$$

Where, N is the total number of molecules. As the speed of molecules is approximately continuous, we may replace summation by integral, i.e.,

$$\bar{v} = \frac{1}{N} \int_0^\infty v dN = \frac{1}{N} \int_0^\infty v 4\pi N \left[\frac{m}{2\pi kT} \right]^{3/2} \times v^2 e^{-mv^2/kT} dv$$

$$\bar{v} = 4\pi \left[\frac{m}{2\pi kT} \right]^{3/2} \int_0^\infty v^3 e^{-mv^2/2kT} dv$$

$$\bar{v} = 4\pi \left[\frac{m}{2\pi kT} \right]^{3/2} \left[\frac{2(kT)^2}{m^2} \right]$$

$$\bar{v} = \sqrt{\frac{8kT}{m\pi}}$$

Most probable speed—For most probable speed, the probability P_V will be maximum, Maxwell's distribution law of speed of molecules may be expressed as—

$$N_v dv = 4\pi N \left[\frac{m}{2\pi kT} \right]^{3/2} v^2 e^{-mv^2/2kT} \times dv$$

$$\frac{N_v}{N} dv = 4\pi \left[\frac{m}{2\pi kT} \right]^{3/2} v^2 e^{-mv^2/2kT} dv$$

According to definition of probability, this is the probability of molecules to possess the speed between V and $V + dV$ i.e.,

$$P_V dV = 4\pi \left[\frac{m}{2\pi kT} \right]^{3/2} V^2 e^{-mV^2/2kT} dV \quad \dots(1)$$

Now, differentiating w.r.t V and equate it to zero,

$$4\pi \left[\frac{m}{2\pi kT} \right]^{3/2} \left[2V e^{-mV^2/2kT} - \frac{m}{kT} V^3 e^{-mV^2/2kT} \right] = 0$$

$$2V e^{-mV^2/2kT} = \frac{m}{kT} V^3 e^{-mV^2/2kT}$$

$$V^2 = \frac{2kT}{m}$$

$$V = \sqrt{\frac{2kT}{m}}$$

Root mean square speed—The mean square speed—

$$\overline{v^2} = \frac{1}{N} \int_0^\infty v^2 dN$$

$$\overline{v^2} = \frac{1}{N} \int_0^\infty v^2 4\pi N \left[\frac{m}{2\pi kT} \right]^{3/2} v^2 e^{-mv^2/2kT} dv$$

$$\overline{v^2} = 4\pi \left[\frac{m}{2\pi kT} \right]^{3/2} \int_0^\infty v^4 e^{-mv^2/2kT} dv$$

$$\bar{v}^2 = 4\pi \left[\frac{m}{2\pi kT} \right]^{3/2} \left[\frac{3}{8} \sqrt{\frac{32\pi (kT)^5}{m^5}} \right]$$

$$\bar{v}^2 = \frac{3kT}{m}$$

$$\left\{ \because \int_0^\infty 4\pi v^2 e^{-mv^2/2kT} dv = \frac{3}{8} \sqrt{\frac{32\pi (kT)^5}{m^5}} \right\}$$

$$rms = \sqrt{\bar{v}^2} = \sqrt{\frac{3kT}{m}}$$

Q5. Derive an expression for the coefficient of thermal conductivity of a gas from kinetic theory. Discuss its variation with temperature and pressure. (2017, 19)

Ans. The thermal conductivity of gas is given by—

$$k = \frac{1}{3} \rho \bar{\lambda} C_v \bar{v}$$

where, ρ = density

$\bar{\lambda}$ = mean free path

C_v = Specific heat at constant volume

$\bar{v}$ = average velocity.

$$k = \frac{1}{3\sqrt{2}} \cdot \frac{mC_v \bar{v}}{\pi \sigma^2} \text{ and } \bar{v} = \sqrt{\frac{8kT}{\pi m}}$$

$$\therefore k = \frac{1}{3\sqrt{2}} \cdot \frac{mC_v}{\pi \sigma^2} \sqrt{\frac{8kT}{\pi m}}$$

$$k = \frac{2}{3} \frac{C_v}{\sigma^2} \sqrt{\frac{mkT}{\pi^3}}$$

Thus, $k \propto \sqrt{T}$.

Thermal conductivity of gas is directly proportional to square root of absolute temperature and independent of pressure.

Q6. Show that for a gas coefficient of viscosity is given by

$\eta = \frac{1}{3} \rho \bar{C} \bar{\lambda}$, when ρ , $\bar{C}$, $\bar{\lambda}$ are density, mean speed and mean free path respectively. (2018)

Ans. Let G be the momentum of each molecule in the plane and $\frac{dG}{dz}$ the momentum gradient upward direction perpendicular to plane (MN).

Let each of the planes AB and CD be at a distance $\bar{\lambda}$ from MN,

where $\bar{\lambda}$ is the mean free path.

Then momentum of each molecule at plane CD

$$= G + \bar{\lambda} \frac{dG}{dz}$$

and momentum of each molecule at plane AB

$$= G - \bar{\lambda} \frac{dG}{dz}$$

Therefore, the momentum carried downwards by the molecules crossing unit area of the plane MN per second from plane CD.

$$= \frac{1}{6} n \bar{C} \left[G - \bar{\lambda} \frac{dG}{dz} \right]$$

Thus net momentum transferred downward per unit area of the plane MN per second.

$$= \frac{1}{6} n \bar{C} \left[G + \bar{\lambda} \frac{dG}{dz} \right] - \frac{1}{6} n \bar{C} \left[G - \bar{\lambda} \frac{dG}{dz} \right]$$

$$= \frac{1}{3} \bar{C} \bar{\lambda} n \frac{dG}{dz} = \frac{1}{3} n \bar{C} \bar{\lambda} \frac{d}{dz} (mu)$$

$$= \frac{1}{3} m n \bar{C} \bar{\lambda} \frac{du}{dz}$$

Thus coefficient of viscosity is defined as tangential force per unit area between two layers required to maintain a unit velocity gradient in it, i.e.,

Coefficient of viscosity, $\eta = \frac{\text{tangential stress}}{\text{velocity gradient}}$

$$\eta = \frac{1}{3} m n \bar{C} \bar{\lambda} \left[\frac{du}{dz} \right]$$

$$\eta = \frac{1}{3} m \bar{C} \bar{\lambda}$$

$$\eta = \frac{1}{3} \rho \bar{C} \bar{\lambda}$$

Q7. For ideal gas show that ratio of rms speed and most probable

speed is $\sqrt{\frac{3}{2}}$.

(2018)

Ans. For ideal gas.

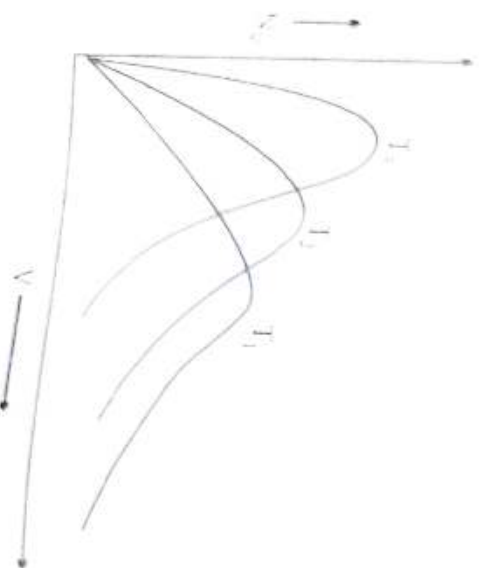
root mean square velocity = $\sqrt{\frac{3kT}{m}} = v_{rms}$

most probable speed = $\sqrt{\frac{2kT}{m}} = v_{mp}$

$$\frac{v_{rms}}{v_{mp}} = \sqrt{\frac{3}{2}}.$$

Q8. Show Maxwell's Boltzman distribution of molecular velocity at different temperature $T_1 > T_2 > T_3$. How total number of molecules can be obtained at a given temperature using velocity distribution curves? (2015)

Ans. Maxwell's distribution law of speed of molecules may be expressed as—



$$N_V dV = 4\pi N \left[\frac{m}{2\pi kT} \right]^{3/2} V^2 e^{-mV^2/2kT} dV$$

This equation represents the effect of temperature on the speed distribution of given number of molecules. The graph of N_V against V according to equation at different temperatures ($T_1 > T_2 > T_3$).

The higher is the temperature, higher is speed possessed by the molecules. However area under all the curves are same.

$$\frac{dN}{dV} = 4\pi N \left[\frac{m}{2\pi kT} \right]^{3/2} V^2 e^{-mV^2/2kT}$$

$$\frac{dN}{dV} = \frac{4N}{\sqrt{\pi}} \left[\frac{m}{2\pi kT} \right]^{3/2} V^2 e^{-mV^2/2kT}$$

taking $\frac{mV^2}{2} = E$, mean KE of a molecule

$$\frac{dN}{dV} = \frac{4N}{\sqrt{\pi}} \left[\frac{m}{2\pi kT} \right]^{3/2} V^2 e^{-E/kT}.$$

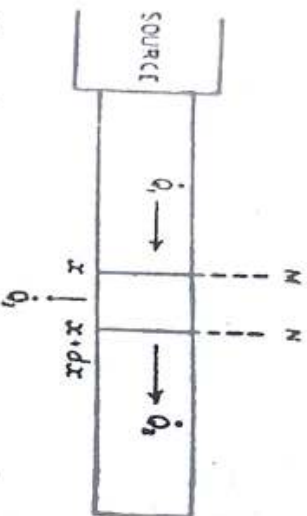
Thus, Maxwell's law of distribution of velocities show that the mean kinetic energy of all the molecules of a gas remains constant at a fixed temperature, though at any instant the molecules are moving with different velocities.

Conduction of Heat

Q1. Drive Fourier's differential equation for rectilinear flow of heat long a metal bar. Obtain an expression for the temperature distribution when rod is :
(2017)

- (i) Open
(ii) Covered.

Ans. Consider the case of a long bar of a good conductor being heated by a source at one end. At a distance x from the hot end, let θ be the temperature above the surrounding and $d\theta/dx$ the temperature gradient at any instant.



Consider an element MN of thickness dx at x . The heat flowing per second into the element at M is :

$$\dot{Q}_1 = -kA \frac{d\theta}{dx}$$

....(1)

and that flowing out at N is :

$$\dot{Q}_2 = -kA \frac{d}{dx}(\theta + d\theta).$$

$$\dot{Q}_2 = -kA \frac{d\theta}{dx} - kA \frac{d}{dx} \left(\frac{\partial \theta}{\partial x} \right) dx.$$

$$\dot{Q}_2 = -kA \frac{d\theta}{dx} - kA \frac{\partial^2 \theta}{\partial x^2} dx$$

....(2)

The gain of heat by the element MN per second is—

$$\dot{Q}_1 - \dot{Q}_2 = kA \frac{\partial^2 \theta}{\partial x^2} dx$$

....(3)

If the bar is insulated so that no heat is lost to the surroundings, we have $\dot{Q}_3 = 0$. Due to the absorption of heat $\dot{Q}_1 - \dot{Q}_2$ the temperature of the element increases. If ρ is the density, C_p the specific heat per unit mass and $\partial\theta/\partial t$ the rate of rise of temperature, then we have from the law

$$kA \frac{\partial^2 \theta}{\partial x^2} dx = \rho A dx C_p \frac{\partial \theta}{\partial t}.$$

$$\frac{\partial \theta}{\partial t} = \frac{k}{\rho C_p} \frac{\partial^2 \theta}{\partial x^2}$$

....(4)

The quantity $\frac{k}{\rho C_p}$ is called the thermal diffusivity and k is a

property of the substance. If the bar is exposed to the surroundings, then a part $\dot{Q}_3$ of the heat $\dot{Q}_1 - \dot{Q}_2$ will be emitted per second as radiation by the surface to the surroundings and only the remaining part would be absorbed by the element MN . Let E be the emissive power of the surface, p is perimeter and θ , the excess temperature over that of surroundings. The heat lost per second by the surface area pdx of the element MN is :

$$\dot{Q}_3 = E\theta pdx.$$

Now, equation (3) becomes,

$$kA \frac{\partial^2 \theta}{\partial x^2} dx = \rho A dx C_p \frac{\partial \theta}{\partial t} + E\theta pdx.$$

$$\frac{\partial \theta}{\partial t} = \frac{k}{\rho C_p} \frac{\partial^2 \theta}{\partial x^2} - \frac{Ep}{\rho A C_p} \theta$$

....(5)

This is the Fourier equation of one dimensional flow of heat. It is often written as :

$$\frac{\partial \theta}{\partial t} = b \frac{\partial^2 \theta}{\partial x^2} - \mu \theta$$

....(6)

$$b = \frac{k}{\rho C_p}, \mu = \frac{Ep}{\rho A C_p}$$

For steady state solution of differential equation

$$\frac{\partial \theta}{\partial t} = 0.$$

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So, equation (6), becomes

$$\frac{d^2\theta}{dx^2} = m^2\theta$$

$$m^2 = \frac{\mu}{h} = \frac{EP}{kK}$$

(i) Open—Steady state with loss of heat from sides—

Its solution is of the form—

$$\theta = a e^{\mu x} + b e^{-\mu x}$$

At great distances from the hot end, $x \rightarrow \infty$, the temperature approaches that of surroundings, that is, $\theta = O$. Hence $a = O$ and $\theta = b e^{-\mu x}$.

(ii) Covered—Steady state with no loss of heat from sides.

In this case, $\mu = O$.

$$\frac{d^2\theta}{dx^2} = O$$

Whose obvious solution is

$$\theta = ax + b,$$

The constant a and b are to be determined from boundary conditions—

$$\theta = \theta_2 \text{ at } x = O$$

$$\theta = \theta_1 \text{ at } x = L$$

Thus, $b = \theta_2$ and $a = (\theta_1 - \theta_2) / L$. Then the temperature at any point x along the bar is given by—

$$\theta = \theta_2 - \frac{(\theta_2 - \theta_1)}{L} x.$$

Q2. Write short note on periodic flow of heat. (2011, 14, 16)

Ans. The periodic heating and cooling of the metal bar is never be in steady state.

Let us assume that a rod is covered so that there is no loss of heat from the sides. The temperature θ at any point x from the end A is determined by the Fourier equation—

$$\frac{\partial \theta(x,t)}{\partial t} = h \frac{\partial^2 \theta(x,t)}{\partial x^2}$$

....(1)

Where, $b = \frac{k}{\rho C_p}$, thermal diffusivity of bar. The alternate heating and cooling of the end of rod can be represented by a sinusoidal function of time.

$$\theta(t) \propto \sin wt.$$

$$\theta(t) \propto \text{Im}[e^{iwt}].$$

$$\begin{aligned} \theta(x, t) &= \theta(x) \cdot \theta(t) \\ &= \text{Im}[\theta(x) e^{iwt}] \end{aligned}$$

We have,

$$i m \theta(x) = b \frac{d^2 \theta(x)}{dx^2}$$

$$\frac{d^2 \theta(x)}{dx^2} = a^2 \theta(x) \quad \dots(1)$$

$$\text{where, } a^2 = \frac{i w}{b} = \frac{w}{b} e^{i\pi/2}.$$

Solution of equation (1),

$$\theta(x) = A e^{ax} + B e^{-ax}$$

$$a = \sqrt{\frac{i w}{b}} = \sqrt{\frac{w}{b}} e^{i\pi/4}.$$

Since, $x = \infty$, temperature is finite.

$$A = O$$

$$\text{Thus, } \theta(x) = B e^{-ax}.$$

$$\begin{aligned} \theta(x) &= \theta(O) e^{-ax} = \theta(O) \exp \left[-\sqrt{\frac{w}{b}} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) x \right] \\ &= \theta(O) \exp \left[-\sqrt{\frac{w}{2b}} (1 + i) x \right] \\ &= \theta(O) \exp \left[-\sqrt{\frac{w}{2b}} x \right] \exp \left[-i \sqrt{\frac{w}{2b}} x \right] \\ &= \theta(O) \exp \left[-\sqrt{\frac{w}{2b}} x \right] \exp \left[-i \sqrt{\frac{w}{2b}} x \right]. \end{aligned}$$

The complete solution can now be expressed as—

CHAPTER - 4

Radiation

$$\theta(x, t) = \ln[\theta(O)e^{-ax}e^{iwt}].$$

$$\theta(x, t) = \ln[\theta(O) \exp\left\{-\sqrt{\frac{w}{2b}}x\right\} \exp\left\{i\left(wt - \sqrt{\frac{w}{2b}}x\right)\right\}].$$

$$\theta(x, t) = \theta(O) \exp\left\{-\sqrt{\frac{w}{2b}}x\right\} \sin\left(wt - \sqrt{\frac{w}{2b}}x\right)$$

It represents a damped simple harmonic wave, the damping factor

$$\text{being } \exp\left\{-\sqrt{\frac{w}{2b}}x\right\}.$$

The periodic part of solution can be written as—

$$\begin{aligned} \sin\left(wt - \sqrt{\frac{w}{2b}}x\right) &= \sin\left(\frac{2\pi}{\tau}t - \sqrt{\frac{\pi}{\tau b}}x\right). \\ &= \sin\frac{2\pi}{\lambda}\left(\frac{\lambda}{\tau}t - \frac{\lambda}{2}\sqrt{\frac{1}{\pi\tau b}}x\right) \end{aligned}$$

It is of the form $\sin\frac{2\pi}{\lambda}(vt - x)$.

$$\text{if } v = \frac{\lambda}{\tau} \text{ and } \frac{\lambda}{2}\sqrt{\frac{1}{\pi\tau b}} = 1.$$

Q1. State and prove Kirchhoff's Law for thermal radiation. What is its importance? (2012, 14, 16)

Ans. Kirchhoff's Law—It states that the ratio of the emissive power (e_λ) to the absorptive power (a_λ) for the radiation of a particular wavelength and at a particular temperature is constant for all bodies. Moreover this ratio is equal to the emissive power of a perfectly black body.

Suppose e_λ and a_λ are the emissive and absorptive powers of a body. Let Q be the quantity of heat radiations incident on the surface in one second. Q_1 is the quantity of heat absorbed by the surface and Q_2 is the quantity of heat reflected by it.

As the body radiates heat, at all temperature the heat emitted by it in **one second** = $e_\lambda d\lambda$.

Total heat falling on the surface = Q ...(1)

Total heat given out by surface = $(Q - Q_1) + e_\lambda d\lambda$(2)

from equation (1) and (2),

$$(Q - Q_1) + e_\lambda d\lambda = Q.$$

$$\text{But } a_\lambda = \frac{Q_1}{Q}.$$

$$Q_1 = a_\lambda Q$$

$$Q - a_\lambda Q + e_\lambda d\lambda = Q$$

$$\Rightarrow \frac{e_\lambda}{a_\lambda} = \frac{Q}{d\lambda} = \text{constant} \quad \dots(3)$$

For a perfectly black body, emissive power = E_λ , and absorptive power $a_\lambda = 1$.

$$\therefore \frac{E_\lambda}{1} = \frac{Q}{d\lambda} \quad \dots(4)$$

from equation (3) and (4),

$$\frac{e_\lambda}{a_\lambda} = E_\lambda = \text{constant}.$$

i.e., the ratio of the emission power to absorptive power is equal to emissive power of a perfectly black-body at a given temperature which is Kirchhoff's law.

If emissive power e_λ of a body greater, Kirchhoff's law tells that corresponding absorptive power a_λ must be greater, i.e., from Kirchhoff's law it follows that good absorbers of radiation are also good emitter of radiation.

Kirchoff's law established that the atoms of each element give a spectrum which is characteristic of that element. Thus the presence of characteristic spectral lines in the spectrum of an element indicates the presence of the element in the substance of which spectrum is taken.

Q2. Derive Planck's distribution law for Black body radiation. (2012, 14, 15, 17, 18, 19)

Ans. Planck found an empirical formula to explain the experimentally observed distribution of energy in the spectrum of a black body. The formula may be deduced using the following postulates.

A black body radiation chamber is filled up not only with radiation, but also with simple harmonic oscillators or resonators of the molecular dimensions, which cannot have many value of energy, but only energies given by—

$$E = nh\nu$$

where, ν is the frequency of oscillator

h is the Planck's constant

n is a number that can take only integral values (0, 1, 2, 3....)

The oscillators cannot radiate or absorb energy continuously, but an oscillator of frequency ν can be only radiate or absorb energy in units or quanta of magnitude $h\nu$. The average energy of Planck's oscillator—

$$\bar{e} = \frac{h\nu}{e^{h\nu/kT} - 1}$$

The number of resonators per unit volume in the frequency range ν to $\nu + d\nu$ is given by—

$$N = \frac{8\pi\nu^2}{C^3} d\nu$$

The energy density belonging to range $d\nu$ can be obtained by multiplying the average energy of a Planck's oscillator by the number of resonators per unit volume, in the frequency range ν and $\nu + d\nu$

$$E_\nu d\nu = \frac{8\pi\nu^2}{C^3} \cdot \frac{h\nu}{e^{h\nu/kT} - 1} d\nu$$

Where, $E_\nu d\nu$ is the energy density, i.e., total energy per unit volume belonging to range, $d\nu$. This law is called Planck's distribution law.

Q3. If Wein's constant $b = 0.3 \text{ cm} \cdot \text{K}$. Calculate the temperature of sun whose radiations has maximum energy at wavelength $\lambda = 5500 \text{ \AA}$. (2014, 16)

$$\text{Ans. } \lambda_m T = \text{constant} = 0.3 \text{ cm K} = 0.3 \times 10^{-2} \text{ mK}$$

$$\lambda_m = 5500 \text{ \AA} = 5500 \times 10^{-10} \text{ m}$$

$$\therefore \text{Temperature of body, } T_s = \frac{0.3 \times 10^{-2}}{5500 \times 10^{-10}}$$

$$T_s = 5454 \text{ K}$$

Q4. If a black body at a temperature of 6174 K emits 4700 Å with maximum energy. Calculate the temperature at which it will emit a wavelength of $1.4 \times 10^{-5} \text{ m}$ with maximum energy. (2015)

Ans. According to Wein's displacement law—

$$\lambda_m T = \text{constant}$$

$$(\lambda_m T)_1 = (\lambda_m T)_2$$

$$4700 \times 10^{-10} \times 6174 = 1.4 \times 10^{-5} \times T_2$$

$$T_2 = \frac{4700 \times 10^{-10} \times 6174}{1.4 \times 10^{-5}}$$

$$T_2 = 207.27 \text{ K}$$

Q5. Establish that black radiation remains black even after undergoing adiabatic expansion. Radius of a spherical black body is doubled during adiabatic expansion. If the initial temperature is 300°C, find the temperature after the expansion. (2015)

Ans. In this case no exchange of heat between the radiation and the surroundings. As a consequence of change in volume, the density of radiation in the enclosure would change. Since the density of

radiation is function of temperature only, the temperature of radiation would necessarily change. In order to be able to apply the laws of black body radiation to the radiation undergoing adiabatic change of volume, we have to prove that the radiation, in the process of change, maintains the spectral energy distribution characteristic of a black body, that is, the radiation remains completely black.

To prove this let us take a cylinder with a piston, all the interior surface being perfect reflectors. Let the cylinder be filled with black body radiation at temperature T_1 . Let the piston moves outwards, thus increasing the volume occupied by the radiation. This has the effect of lowering the temperature of radiation. If the radiation at any temperature other than T_1 is not black body radiation, it can be made to have characteristic of black body radiation, by introducing a very small black body of negligible thermal capacity inside the cylinder. This process being irreversible causes the entropy of the radiation to increase for the radiation still remains black by removing the black body from inside the enclosure.

With the black body still inside the enclosure, move the piston inwards till the radiation occupies its initial volume. The work done during backward drive of the piston $\int PdV$ is same as that done during the forward drive.

This is because the density of radiation and therefore the pressure of radiation is the same at every stage of compression or expansion of volume. The original stage of radiation has thus reached and in the entire cycle, there has been no gain of work or heat energy though the entropy of radiation has increased. This is violation of the second law of thermodynamics. The original assumption that the irreversible process of the redistribution of energy by the introduction of black body cannot have occurred. Hence, the energy distribution must have retained the characteristic of black body radiation during the process of adiabatic change.

For an adiabatic expansion—

$$Tr = \text{constant}$$

$$T_1 V_1 = T_2 V_2$$

$$T_2 = \frac{(300 + 273) \cancel{V_1}}{(2 \cancel{V_1})}$$

$$T_2 = \frac{573}{2} = 286.5 \text{ K}$$

$$T_2 = 13.5^\circ\text{C}.$$

Q6. Deduce the Wein's Displacement law using the laws of thermodynamics. (2016)

Ans. It states that product of wavelength corresponding to maximum energy λ_m and absolute temperature T is constant. According to this λ_m decreases with increase in temperature.

The derivation of this law depends upon the change in wavelength when radiation is reflected from a moving mirror or wall. Let us imagine a spherical enclosure of perfectly reflecting wall and capable of expanding. Let it be filled with diffuse radiation of energy u at a uniform temperature T . If V is volume of enclosure, then total internal energy U is given by—

$$U = nV \quad \dots(1)$$

Let us suppose that the walls move out slowly with uniform velocity hence the radiation expands adiabatically. If dV is the change in volume and P is the radiation pressure on enclosure wall. From first law of thermodynamics, we have—

$$\begin{aligned} dQ &= dU + dW \\ dU + PdV &= 0 \quad \dots(2) \end{aligned}$$

$$\text{But } P = \frac{4}{3}u$$

$$\therefore d(uV) + \frac{4}{3}udV = 0$$

$$udV + Vdu + \frac{4}{3}udV = 0$$

$$\frac{4}{3}udV + Vdu = 0$$

$$\frac{4}{3} \frac{dV}{V} = - \frac{du}{u}$$

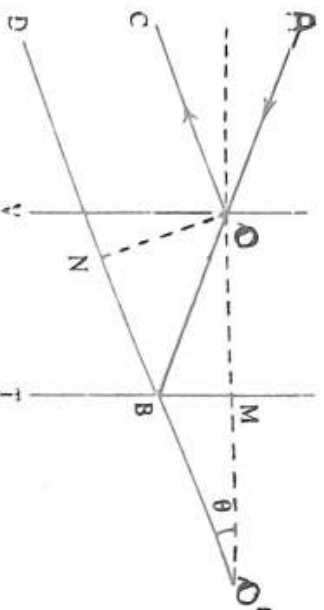
On integrating,

$$\frac{4}{3} \ln V = - \ln u + \text{constant.}$$

$$\ln u V^{4/3} = \text{constant}$$

But, $u = \sigma T^4$ from stefan's law—

$$\begin{aligned} T^4 V^{4/3} &= \text{constant} \\ TV^{1/3} &= \text{constant} \quad \dots(3) \end{aligned}$$



Let particular wave crest strike the wall at O . The crest will reflected along OC . The next will reach at O in time T , T being time period of wave motion. During this time the wall has traversed a distance.

$$OM = VT$$

Path followed by the second crest will be OBD . The increase in wavelength due to one reflection is —

$$\begin{aligned} d\lambda &= OB + BN = OB' + BN = O'N \\ &= OO' \cos \theta = 2OM \cos \theta \\ &= 2vT \cos \theta = \frac{2v\lambda}{c} \cos \theta \end{aligned} \quad \dots(4)$$

Number of reflections in time δt .

$$= \frac{C}{2r \cos \theta} \delta t = \frac{C}{2r \cos \theta} \cdot \frac{\delta r}{v}$$

Now, $\delta\lambda$ = change in wavelength in one reflection $\times$ number of reflection in time δt .

$$\begin{aligned} \delta\lambda &= \frac{2v\lambda}{C} \cos \theta \times \frac{C}{2r \cos \theta} \cdot \frac{\delta r}{v} \\ \delta\lambda &= \lambda \cdot \frac{\delta r}{r} \end{aligned} \quad \dots(5)$$

from volume of sphere, $V = \frac{4}{3} \pi r^3$

$$\frac{\delta r}{r} = \frac{1}{3} \frac{\delta V}{V} \quad \dots(6)$$

from eq. (5) and (6) —

$$\frac{\delta\lambda}{\lambda} = \frac{\delta r}{r} = \frac{1}{3} \frac{\delta V}{V} \quad \dots(7)$$

integrating, we get

$$\log \lambda = \frac{1}{3} \log V + \log k.$$

$$\lambda = kV^{1/3}$$

$$V^{1/3} = \frac{\lambda}{k} \quad \dots(8)$$

Substituting in equation (3),

$$\frac{\lambda}{k} \cdot T = \text{constant}$$

$$\lambda \cdot T = \text{constant}$$

This is wein's displacement law.

Q7. Show that for radiation in an enclosure, $P = \frac{1}{3}u$, where P is the pressure and u is the energy density. (2017)

Ans. Consider the radiation falling normally on the surface of a body.

In one second all the radiation contained in a cylinder of length c cm and unit cross sectional area. Thus on each unit area of the surface in one second cw ergs of energy would fall. This will bring w units of momentum per second, the momentum vector being directed normally towards the surface. If the waves are absorbed by the surface, it receives this momentum and consequently experiences a pressure equal to w .

Suppose the radiation in incident at an angle θ . The energy contained in the cylinder of length c and cross-sectional area unity

would now fall on a surface of area $\frac{1}{\cos \theta}$. The force per unit area of

the surface is now $w \cos \theta$ and acts in the direction of rays. Its component normal to the surface is $w \cos^2 \theta$. If the radiation is absorbed, the resulting pressure is —

$$P = w \cos^2 \theta \quad \dots(1)$$

If N is the total number of beams moving towards the surface then, total energy density in just front of the surface and the pressure P is given by,

$$u = Nw, p = \Sigma w \cos^2 \theta = w \Sigma \cos^2 \theta.$$

To calculate $\Sigma \cos^2 \theta$, we draw a hemisphere of radius ρ round a point O at the centre of the surface element. From the hemisphere we cut out a ring of area $2\pi\rho \sin \theta \rho d\theta$ by means of two cones of semiangle θ and $\theta + d\theta$ drawn from O as apex. If dN is the number of beams that cross this ring, then

$$\frac{dN}{N} = \frac{\text{area of ring}}{\text{area of hemisphere}} = \frac{2\pi\rho \sin \theta \rho d\theta}{2\pi\rho^2} = \sin \theta d\theta.$$

The value of $\cos^2 \theta$ is same of all the dN beams. Therefore, their contribution to $\Sigma \cos^2 \theta$ is $\cos^2 \theta dN$ or $N \cos^2 \theta \sin \theta d\theta$.

$$\Sigma \cos^2 \theta = \int \cos^2 \theta dN = N \int_0^{\pi/2} \cos^2 \theta \sin \theta d\theta = \frac{1}{3} N$$

and from (1), we have

$$p = \frac{1}{3} w N = \frac{1}{3} u.$$

Q8. Show that for adiabatic expansion of black body radiation $\lambda T = \text{constant}$.

(2017, 19)

Ans. Adiabatic expansion of black body radiation

$$V^{4/3} u = \text{constant}$$

$$\text{and } V^{1/3} T = \text{constant}$$

Also, for radiation of a particular wave length

$$\lambda V^{-1/3} = \text{constant}$$

$$\text{So that, } \lambda T = \text{constant}$$

This is Wein's displacement law.

Q9. Deduce Stefan's law from Planck's radiation formula.

(2017, 18, 19)

Ans. Planck's radiation formula in terms of frequency can be written as—

$$E_\nu dV = \frac{8\pi h}{C^3} \cdot \frac{V^3 d\nu}{e^{h\nu/KT} - 1} \quad \dots (i)$$

The special radiancy U_ν is related to energy density E_ν

$$U_\nu = \frac{C}{4} E_\nu \quad \dots (ii)$$

The total radiant energy over all frequencies

$$U = \int_0^\infty U_\nu dV = \frac{2\pi h}{C^2} \int_0^\infty \frac{V^3 d\nu}{e^{h\nu/KT} - 1} \quad \dots (iii)$$

Let us make substitution,

$$\frac{h\nu}{KT} = x$$

$$d\nu = \frac{kT}{h} dx.$$

Putting these values in equation (iii),

$$U = \frac{2\pi k^4 T^4}{h^3 C^2} \int_0^\infty \frac{x^3 dx}{e^x - 1}$$

$$U = \frac{2\pi^5 k^4}{15 h^3 C^2} T^4$$

$$U = \sigma T^4 \quad \left\{ \because \frac{2\pi^5}{15 h^3 C^2} = \sigma \right\}$$

Q10. Show that for adiabatic expansion of radiation $PV^{4/3} = \text{constant}$.

(2018)

Ans. Let us imagine a spherical enclosure of perfectly reflecting walls and capable of expanding. Let it be filled with diffuse radiations of energy density u at a uniform temperature T .

If V is the volume of the enclosure, then total energy of radiations is given by,

$$U = uV$$

Let us now suppose that the walls of the enclosure move outward slowly with uniform velocity so that the radiation inside it expands adiabatically. If dV is the change in volume of the enclosure, the work done by the pressure of radiation on the walls of the enclosure = PdV which is drawn from internal energy of radiation.

If dU is decrease in internal energy, then from first law of thermodynamics, we have.

$$dU + PdV = dQ = 0$$

We know, $p = \frac{1}{3}u$

$$d(uV) + \frac{1}{3}u dV = 0$$

$$u dV + V du + \frac{1}{3}u dV = 0$$

$$\frac{4}{3} \cdot \frac{dV}{V} = -\frac{du}{u}$$

integrating,

$$\frac{4}{3} \log V = -\log u + \text{constant}$$

$$\log (V^{4/3} u) = \text{constant}$$

But, $u = \sigma T^4$

$$\therefore V^{4/3} T^4 = \text{constant}$$

$$V^{1/3} T = \text{constant}$$

$$PV^{4/3} = \text{constant}$$

$$\therefore PV = RT\}$$

Q11. The emissive power of Regal star is 17000 times that of sun. If temperature of sun is 6000 K. What will be the temperature of star. (2018)

Ans. From Stefan's law,

$$E = \sigma T^4$$

$$E_{\text{sun}} = \sigma T_{\text{sun}}^4$$

$$E_{\text{star}} = \sigma T_{\text{star}}^4$$

$$\frac{E_{\text{sun}}}{E_{\text{star}}} = \left(\frac{T_{\text{sun}}}{T_{\text{star}}} \right)^4$$

$$\frac{1}{17000} = \frac{(6000)^4}{(T_{\text{star}})^4}$$

$$\therefore E_{\text{star}} = 17000 E_{\text{sun}}\}$$

$$T_{\text{star}} = (6000)(17000)^{1/4}$$

$$T_{\text{star}} = 68520 \text{ K.}$$

Q12. Write short notes on Temperature of the sun. (2019)

Ans. **Temperature of the Sun**—Knowledge of S_0 enables us to estimate the temperature of the sun. The total energy radiated per second from the surface of the sun is :

$$Q = 4\pi R_0^2 \frac{S_0}{60}$$

If r is the radius of the sun, then the total radiating surface area is $4\pi r^2$ and energy emitted per second per unit area is

$$\frac{Q}{4\pi r^2} = \frac{R_0^2}{r^2} \cdot \frac{S_0}{60}$$

By Stefan's law we must have

$$\frac{Q}{4\pi r^2} = \sigma T^4$$

where T is the surface temperature of the sun. Thus

$$T = \left(\frac{R_0^2}{\sigma r^2} \cdot \frac{S_0}{60} \right)^{1/4}$$

Here r/R_0 is the mean angular radius of the sun at the earth (angle subtended by the solar radius at the earth). Using

$$S_0 = 1.937 \text{ cal cm}^{-2} \text{ minute}^{-1}$$

$$\sigma = 5.735 \times 10^{-5} \text{ erg sec}^{-1} \text{ cm}^{-2} \text{ } ^\circ\text{K}^{-1}$$

$$r/R_0 = 959'' = 4.65 \times 10^{-3} \text{ radians}$$

we get

$$T = \left[\frac{1.937 \times 4.18 \times 10^{-7}}{60 \times (4.65)^2 \times 10^{-6} \times 5.735 \times 10^{-5}} \right]^{1/4} \cong 5742 \text{ } ^\circ\text{K}$$

where we have multiplied S_0 by $J = 4.18 \times 10^7 \text{ ergs/cal}$ to express it in ergs. This is the brightness temperature of the sun's surface.

The temperature of the sun can also be estimated by using

Wien's law

$$\lambda_m T = \text{constant} = 0.293$$

Abbott's measurements show that $\lambda_m = 4753 \text{ \AA}$ for the solar radiation. Hence

$$\frac{0.293}{7553 \times 10^{-8}} = 6060 \text{ } ^\circ\text{K}$$

This temperature, called the colour temperature, is nearly 3000°K higher than the brightness temperature. This shows that the sun does not radiate like a blackbody.