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Unsolved Paper

BSC-2nd year

[2011-22(23)]

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KEY POINT AU BSC-2

B.A./B.Sc. (Part-II) Exam., 2011

(Three Years Degree Course)

MATHEMATICS

(FIRST PAPER)

(Linear Algebra)

Time Allowed : Three Hours] [Maximum Marks :50

Note : Answer **all** questions. **All** questions carry equal marks.

1. Prove or disprove the following :

(i) The subspace generated by the subset $S = \{(x,y) : x^2 + y^2 = 1\}$ of the vector space/ IR^2 (IR) is a circle.

(ii) The dimension of the linear sum of two subspaces $w_1 = \{(x,y,z)\} : x + y - 2z = 0\}$ and $w_2 = \{ (x,y,z) : x = y = z\}$ of the vector space IR^3 (IR) is 3.

(iii) If W_1 and W_2 are subspaces of three dimensional vector space v such that dim $W_1 = 1$, dim $W_2 = 2$ and $W_1 \not\subseteq W_2$, then $V = W_1 \oplus W_2$.

(iv) The coset of the subspace $W = \{(x,y) : 2x + 3y = 0\}$ of the vector space IR^2 (IR) determined by $a = (1,1) \in IR^2$ is $W+a = \{x,y\} : 2x+3y=5\}$.

OR

(a) (i) If {a, b} is a lineraly independent subset of a vector space V ($\mathbb{C}$), then prove that {a, b, ia, ib} will be linearly independent subset of V (IR).

(ii) Prove that the subset $S = \{(1,0,0), (1,2,3), (1,4,9)$ generates the vector space IR^3 (IR).

(b) If W_1 and W_2 are two subspaces of a vector space V over a field F such that $V = W_1 \oplus W_2$ and $\{x_1, x_2, ..., x_n\}$ is a basis of W_1, then prove that $\{W_2 + x_1, W_2, + X_2, --, W_2 + x_n\}$ will be a basis of V/W_2.

2. Prove or disprove the following :

(i) If $T : V_1 \to V_2$ be a linear transformation where dim $V_1 >$ dim V_2, then there exists a non - zero vector x in v_1 such that $T(x) = 0_2$ where O_2 is zero vector in V_2.

(ii) If $T : IR^2 \to IR$ and $S : IR^2 \to IR$ are two linear transformations such that Ker T = Ker S and I_m T = I_m S, then T = S.

(iii) If $T : V_1 \to V_2$ is a linear transformation such that rank T = 2 and dim $V_2 = 5$, then nullity of the transpose of will be 2.

(iv) If $T : V \to V$ is a linear operator such that $T^5 = T^4$, then ker $T^4 \cap I_m T^4$ is a singleton.

OR

(a) Let $T_1 : V_1 \to V_2$ and $T_2 : V_2 \to V_3$ be linear transformations , where

V_1, V_2 and V_3 are finite dimensional vector spaces over a filed. Then prove that rank $(T_2 T_1) \geq$ rank T_1 + rank T_2 – dim V_2.

(b) Define annihilator of a non - empty subset of a vector space. If W be a subspace of the vector space IR^4 (IR) spanned by subset $s = \{(1, 2, -3, 4), (0, 1, 4, -1,\}$, then find a basis of the annihilator of W.

3. Prove or disprove the following :

(i) It is not possible to find two non-square real matrices A and B such that A.B and B.A both are defined and A.B. But B.A ≠ 0.

(ii) If A is a 6×6 matrix such that rank A = 5, then rank of adj adj A will be 1.

(iii) If A is an n-square matrix such that $A^5 = 0$, then $I_n - A$ is invertible.

(iv) It is possible to construct 3×3 real matrices A and B such that

$$A.B. - B.A. = \begin{bmatrix} 0 & -2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & -4 \end{bmatrix}$$

OR

(a) Write the condition under which a system of m non-homogeneous linear equations in n unkowns written in matrix form A.X.=B is

(i) inconsistent

(ii) consistent with unique solution and

(iii) Consistent with infinite solutions, and hence find the values of λ and μ for which the following system of linear equations over filed IR is (i) inconsistent (ii) consistent with unique solution and (iii) Consistnet with infinite solutions :

$x + 2y + 3z = 5,\ 3x - y + 2z = 12,\ 3x - y + \lambda z = \mu.$

(b) Express the matrix $A = \begin{bmatrix} 3 & 4 & -1 \\ 2 & 7 & -3 \\ 5 & -2 & 0 \end{bmatrix}$ as a product of elementary matrices.

4. Prove or disprove the following :

(i) If a and b are two linearly independent vectors of an inner product space (V, < >) such that ||a|| = 2 and ||b|| = 3, then |< a, b >| = 6.

(ii) If a subspace W of a finite dimensional inner product space V over IR is invariant under a symmetric linear transformation T : V → V, then $W^{\perp}$ is also invariant under T.

(iii) Eigen values of linear operator T on IR^2 which takes the circle $\{(x,y): x^2 + y^2 = 1\}$ to the ellipse $\left\{(x,y): \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\right\}$ are a and b.

(iv) If A is an n-square real skew-symmetric matrix such that $A^2 = -I_n$, then n is even.

OR

(a) Define adjoint of a linear operator on an inner product space and find the adjoint of the linear operator $T : \mathbb{C}^3 \to \mathbb{C}^3$ given by $T[x,y,z)] = (x + iy - z, ix - y + z, -x + y + iz), \forall (x, y, z) \in \mathbb{C}^3$.

(b) Define eigen values and eigen vectors of a linear operator on a vector space. Find eigen values and eigen vectors of the linear operator T : $M_2 \to M_2$ defined by

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} 2c & a+c \\ b-2c & d \end{bmatrix}, \forall \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_2$$

Where M_2 is the vector space of 2×2 real matrices.

5. Prove or disprove the following :

(i) Real matrix $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ is diagonalizable.

(ii) Let A be a diagonalizable n-square real matrix having 0 and 1 its only eigen values. If the null space of A has dimension r, then the characteristic polynomial of A will be $\lambda^r (1-\lambda)^{n-r}$.

(iii) If f is a biline ar form on a real vector space V such that $f[(x,y)] = 0, \forall x \in V$, then f is a skew-symmetric.

(iv) The bilinear form f of a real quadratic form q over a vector space V is given by $f[(x,y)] = q(x + y) - q(x) - q(y)$; $x, y \in V$.

OR

(a) If A is a real matrix, then show that all the eigen values of A^t. A are non-negative real numbers. Deduce that there exits a real symmetric matrix B such that $B^2 = A^t.A$.

(b) Reduce the equation $y^2 + zx + 2y + 2 = 0$ to standard form by a suitable orthogonal change of variables and identify the conicoid represnted by it.

MATHEMATICS (SECOND PAPER), 2011

Time Allowed : Three Hours] **[Maximum Marks** : 50

Note : Attempt **all** the question. Each question carries 10 marks.

1. (a) When do we say that an infinite series $\sum_{n=1}^{\infty} a_n$ converges and has the sum S? Prove that a series of positive terms converges if and only if the sequence of its partial sums is bounded above.

OR

(a') Test the convergence of the series

(i) $\sum_{n=1}^{\infty}\left(n^{1/n}-1\right)^n$

(ii) $1+x+\frac{x^2}{2}+\frac{x^3}{3}+\ldots\ldots(x>0)$

(b) Prove that the series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ diverges if $-\infty < p \le 1$

OR

(b') Test the convergence of the series

$\sum_{n=1}^{\infty} x^{1+\frac{1}{2}+\ldots\ldots+\frac{1}{n}}$ $(x>0)$

2. (a) Prove that if the sequence $\{a_1 + \ldots + a_n\}$ is bounded and the sequence $\{b_n\}$ be monotonically decreasing and converging to zero then the series $\sum_{n=1}^{\infty} a_n b_n$ converges.

OR

(a') Test the convergence of the series $\Sigma\frac{(-1)^{n+1}}{n}\left(1+\frac{1}{n}\right)^{-n}$.

(b) Define absolute and conditional convergence of infinite series. Prove that every absolutely convergent series is convergent. Prove that $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^p}$ is conditionally convergent if $0 < p \le 1$.

OR

(b') If the two series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge to the sums A and B respectively and also the series $\sum_{n=1}^{\infty} C_n$ converges to C where $C_n = \sum_{k=0}^{n} a_k b_{n-k}$ then prove that $C = AB$.

3. (a) If a function f is bounded on [a,b] prove that $\int_a^b f(x)dx \le \overline{\int_a^b} f(x)dx$, where the quantities above are lower and upper integrals of f over [a, b] respectively.

OR

(a') Evaluate the integral $\int_{-1}^{3}[x^2-3]dx$ where [y] denotes the largest integer $\le y$.

(b) If f and g are bounded and integrable over [a, b]. Prove that fg given by (fg) (x) = f (x) g (x) is also integrable over [a,b] and if further, f is differentiable over [a, b], prove that

$$\int_a^b f(x)g(x)dx = [f(b)g_1(b)-f(a)g_1(a)]-\int_a^b f'(x)g_1(x)dx$$

where $g'_1(x) = g(x)$ for every $x \in [a, b]$.

OR

If f is integrable over [a, b] then show that for each $\epsilon > 0$, there is a partition p_ϵ of [a, b], for which for each partition p of [a, b] which is a refinement of P_ϵ,

$$\left|\sum_{j=1}^{n} f(\xi_j)(x_j - x_j - 1) - \int_a^b f(x)dx\right| < \epsilon$$

for every choice of $\xi_j \in [x_{j-1}, x_j]$ where p is the partition

$P : a = x_0 < x_1 < \text{---------} < x_n = b$

4. (a) Test the convergence of the integral $\int_0^{\infty}\frac{\sin x}{x}dx$

OR

(a') Define absolute convergence of an integral $\int_a^{\infty} f(x)dx$. If a function f is bounded in [a, ∞) and $\int_a^{\infty} g(x)dx$ converges absolutely. Prove that $\int_a^{\infty} g(x)dx$ converges absolutely.

(b) Test the convergence of the integral $\int_a^{\infty}\frac{x^3}{\left(a^2+x^2\right)^2}dx$, where a > 0. State the test used.

OR

(b') Test the convergence of $\int_0^1\frac{dx}{\sqrt{[x(1-x)}}$.

5. (a) Prove that a subset A of a metric space x contains each of the boundary points of A if and only if each point of the complement A' of A in x is an interior point of A'.

OR

(a) Find all the interior points, boundary points, and the accumulation points of the set Q of all rational numbers in the metric space IR of all real numbers. What are the exterior points of the set Z of all integers in the real line R.

(b) If $\overline{A}$ denotes the union of A with the set of all accumulation points of A, prove that the complement $(A)'$ of $\overline{A}$ in X is equal to the set of all interior points of the complement A' of A in X.

OR

(b') Define a complete metric space and give an example of it. Prove that a subspace Y of a complete metric space X is complete if and only if Y is a closed subset of X.

MATHEMATICS (THIRD PAPER), 2011

Time Allowed : Three Hours] [Maximum Marks : 50

Note : Answer **all** questions. Marks are indicated against each part. Symbols have their usual meaning.

1. (a) Show that the work done by its tension T in a string, in a small extension from length ℓ to $\ell + \delta\ell$ is $-T\delta\ell$. 2

OR

Explain the general rule for deciding whether a given equilibrium position of a body is stable, if the force of gravity is the only external force acting on the body. 2

(b) A telegraph wire is made of a given material, and such a length ℓ is stretched between two posts, distance d apart and of the same height; as will produce the least possible tension at the posts. Show that $\ell = (d/\lambda)\sin h\lambda$, where $\lambda \tan h\,\lambda = \alpha$. 4

OR

A uniform string of weight W is suspended from two points at the same level and a weight W' is attached to its lowest point,. If α and β are now the inclinations to the horizontal of the tangents at the highest and lowest points, prove that $(\tan\alpha/\tan\beta) = 1 + (W/W')$. 4

(c) State and prove the principle of virtual work for a system of coplanar forces acting at different points of a rigid body. 4

OR

A smooth cone is placed with vertex downwards in a circular horizontal hole. Prove that the position of equilibrium with the axis vertical is unstable, if it is the only possible position of equilibrium. 4

2. (a) Establish the relation :

$\bar{F}(t) = m(t)\frac{d\bar{v}}{dt} + \lambda(\bar{u} - \bar{v})$, , for the motion of a particle of varying mass m (t) and velocity $\bar{v}$ under the force $\bar{F}(t)$, matter being emitted at a constant rate λ with velocity $\bar{u}$. 2

OR

Explain, what do you understand by "terminal velocity" of a body falling under gravity in a resisting medium. 2

(b) A particle, whose mass is m, is acted upon by a force $m\mu\,(x + a^4/x^3)$ towards the origin. If it starts from rest at a distance a, show that it will arrive at the origin in time $\pi/(4\sqrt{\mu})$. 4

OR

A particle moves along the axis of x starting from rest at $x = a$, for an interval t_1 from the beginning of the motion the acceleration is $-\mu x$, for a subsequent interval t_2 the acceleration is μx, and at the end of this interval the particle is at the origin; prove that $\tan\left(t_1\sqrt{\mu}\right)\tan h\left(t_2\sqrt{\mu}\right) = 1$. 4

(c) A heavy particle of mass m is attached to the end of an elastic string of natural length a, whose other end is fixed at 0. The particle is let fall from rest at 0. Show that part of the motion is simple harmonic, and that, if the greatest depth of the particle below 0 is $a\cot^2(\theta/2)$, $0 < \theta < \pi/2$, then the modulus of elasticity of the string is $\frac{1}{2}mg\tan^2\theta$. 4

OR

A light elastic string (modulus of elasticity λ) is stretched to double its length, and is tied to two fixed points distant 2a apart. A particle of mass m, tied to its middle point, is displaced in the line of the string through a distance equal to half its distance from the fixed points, and released. Find the time of a complete oscillation and the maximum velocity acquired in the subsequent motion. 4

3. (a) Prove that $\theta = vp/r^2$. 2

OR

Prove that the acceleration of a point moving in a curve with uniform speed is $\rho\psi^2$. 4

(b) A particle moves along a circle $r = 2a\cos\theta$ in such a way that its acceleration towards the origin is always zero. Show that its transversal acceleration varies as the fifth power of cosec θ. 4

OR

If the angular velocity $\sqrt{3}\omega$ of a particle about the origin is constant, and the rate of change of acceleration is directed wholly along the radius vector, prove that $r = a\cosh\omega t + b\sin h\,\omega t$, where, initially, the particle is at a distance a with velocity ωb along the radius vector. 4

(c) Using vector method, obtain radial and transversal accelerations of a particle moving in a plane curve. 4

OR

A point moves in a plane in such a way that its tangential and normal accelerations are always equal, and its velocity varies as $\exp\left(\tan^{-1}\frac{s}{c}\right)$, s being the length of the arc of the curve measured from a fixed point. Find the path. 4

4. (a) A particle is moving on a smooth curve under gravity and its velocity varies as the arcual distance from the highest point. Prove that the curve must be a cycloid. 2

OR

Define a cycloid and prove the relation : $s = \sqrt{(8ay)}$. 2

(b) A particle moves under gravity in a vertical circle, sliding down the convex side of a smooth circular arc. If its initial velocity is that due to a fall to the starting point from a height h above the centre, show that it will fly off the circle when at a height $\frac{2}{3}h$ above the centre. 4

OR

A heavy particle, hanging vertically from a fixed point by a light inextensible string of length l, is struck by a horizontal blow which imparts to it a velocity $2\sqrt{(g\ell)}$; prove that the string becomes slack when the particle has risen to a height $\frac{2}{3}\ell$ above that fixed point. 4

(c) A particle starts from rest at the cusp of a smooth cycloidal arc whose axis is vertical and vertex downwards; prove that, when it has fallen through half the distance measured along the arc to the vertex, two-thirds of the time of descent will have elapsed. 4

OR

Show that a cycloid can be a free path for a particle acted on at each point by a constant force parallel to the corresponding radius of the generating circle, this circle being placed at the vertex. 4

5. (a) State the Kepler's laws of planetary motion. 2

OR

Show that the motion of a particle under a central force takes place in a plane. 2

(b) Obtain the following differential equation : $\frac{h^2}{p^3}\frac{dp}{dr} = f$ to the orbit of a particle moving under central attractive acceleration f. 4

OR

Find the law of force to an internal point under which a particle will describe a circle. 4

(c) A particle moves with a central acceleration $\mu(r^5 - c^4r)$ towards the origin being projected from an apse at distance c with a velocity $\sqrt{\left(\frac{2\mu}{3}\right)}C^3$; show that its path is the curve $x^4+y^4=c^4$. 4

OR

A particle moves under a central attractive force varying inversely as the fifth power of the distance from the centre of force. It is projected from an apse at a distance a with a velocity equal to $\sqrt{5}$ times of that which would be acquired in falling from infinity. Show that the other apsidal distance is a/2. 4

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B.A./B.Sc. (Part-II) Exam., 2012

(Three Years Degree Course)

MATHEMATICS (FIRST PAPER)

(Linear Algebra)

Time Allowed : Three Hours] [Maximum Marks : 50

Note : Answer **all** questions. Marks allotted to each question are indicated against it.

1. (a) Define subspace generated by a subset of a vector space. Find the equations of the subspaces of $IR^3(IR)$ generated by

(i) $S_1 = \{(1, 1, 2), (2, 3, 5), (3, 5, 8)\}$

(ii) $S_2 = \{(2, 2, 4), (3, 3, 6)\}$

Also obtain the dimension of the linear sum of the subspaces generated by S_1 and S_2. 1+1½+1½+1

(b) If W is a subspace of a finite dimensional vector space V, prove that there exists, a subspace, W' of V such that $V = W \oplus W'$. Is W' unique? Justify your answer. 3½+1½

OR

(a) Define linearly dependent and linearly independent vectors of a vector space. Let {a, b, c} be a basis of IR^3 (IR) and $d = \alpha a + \beta b + \gamma c$, $\alpha, \beta, \gamma, \in IR$. Then prove that {a - d, b - d, c - d} will be a basis of $IR^3(IR)$ if and only if $\alpha + \beta + \gamma \neq 1$. 1+4

(b) If W is a subspace of a finite dimensional vector space V over a field, prove that :

$\dim V/W = \dim V - \dim W$, 4+1

and hence find the total number of cosets of W in V when W is a subspace of V over the field $Z/_3$ and dim V = 5, dim W=3.

2. (a) Let M_2 be the set of all 2×2 real matrices. If $T : M_2 \to M_2$ is linear transformation defined by $T(A) = A.B - B.A, \forall A \in M_2$ and $B = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$, then find (i) Ker T (ii) Im T and (iii) Trace of T. 1½+2+1½

(b) Define the transpose of a linear transformation and annihilator of a non-empty subset of a vector space, and prove that :

(i) The transpose of a linear transformation is a linear transformation.

(ii) The annihilator of a non-empty subset of a vector space is a subspace of the dual space of the vector space.

(iii) Kernel of the transpose of a linear transformation is the annihilator of the image of the linear transformation. 1+1+1+2

OR

(a) (i) Define a linear transformation. If $T : V_1 \to V_2$ is a linear transformation and W is a subspace of V_2, then prove that $T^{-1}(W)$ will be a subspace of V_1.

(ii) Let $T : V_1 \to V_2$ be a surjective, linear transformation. Suppose that $\dim V_1 = 12$, $\dim V_2 = 8$ and W be a subspace of V_2 such that $\dim W = 3$. Then find the dimension of $T^{-1}(W)$. 2+3

(b) Define dual space of a vector space and prove that the dimension of the dual space of a finite dimensional vector space is equal to the dimension of the vector space. 1+4

3. (a) Define row-space of a matrix. Find a basis of the row-space of the matrix :

$$A = \begin{bmatrix} 2 & 4 & 6 & 0 \\ 1 & 1 & 0 & -1 \\ -2 & -3 & -3 & 1 \\ -1 & -2 & -2 & 2 \\ 4 & 6 & 4 & -6 \\ 1 & 3 & 4 & -3 \end{bmatrix}$$

Hence obtain the rank of A. 1+3+1

(b) Define adjoint of a square matrix and prove that

(i) If A is an n-square non-singular matrix, then $\text{adj adj } A = (|A|)^{n-2} . A$

(ii) If A and B are non-singular matrices of the same order, then $\text{adj } (A.B) = \text{adj } B . \text{adj } A$ 1+2+2

OR

(a) Define trace of a square matrix. If A and B are m×n and n×m matrices, respectively, over a field, then prove that A.B and B.A have the same trace. Hence prove that similar matrices have the same trace. 1+3+1.

(b) Find a basis of the solution space of the following system of linear equations over the field of real numbers : 5

$2x + z - u + v = 0$, $x + z - u - v = 0$,

$6x + y + 4z + v = 0$

4. (a) Using Gram-Schmidt orthonormalization process orthonormalise the basis 5

$\{(1, 0, 3), (2, 2, 0), (3, 1, 2)\}$ of $IR^3(IR)$.

(b) (i) Prove that two eigen vectors of a Skew-Hermitian matrix corresponding to two distinct eigen values of the matrix are orthogonal.

(ii) If A and B are two square matrices of the same order such that A is Skew-Hermitian and A.B = B, then prove that B = 0. 3+2

OR

(a) If $S = \{x_1, x_2, \ldots x_n\}$ is an orthonormal subset of an inner product space $(V, < >)$, then for $x \in V$ prove that

$$\sum_{i=1}^{n} |< x, x_i >|^2 = \| x \|^2$$

if and only if S is a basis of V. 5

(b) Define eigen value of a square matrix. Let $A = [a_{ij}]$ be an n-square real matrix such that $|a_{ij}| \le m$, $\forall_i = 1, 2, \ldots, n$ and λ an eigen value of A. Then prove that

$|\lambda| \le mn$ 1+4

5. Find the matrix of the quadratic form-

$q(x_1, x_2, x_3) = 2x_1x_2 + 4x_1x_3 + 2x_2x_3$ on

$IR^3(IR)$ relative to the ordered basis

$\{(1, 1, 0), (1, 0, 1), (0, 0, 1)$ of $IR^3(IR)$. 10

OR

Reduce the following equation in standard form by suitable orthogonal change of variables

$4y^2 + 4z^2 + 4yz - 2x - 14y - 22z + 33 = 0$

and tell the nature of the surface represented by it. 8+2

MATHEMATICS (SECOND PAPER), 2012

Time Allowed : Three Hours] **[Maximum Marks :** 50

Note : Attempt **all** the **five** question. Each question carries 10 marks.

1. (a) If an infinite series of positive terms Σu_n is a Convergent series, then prove that $\lim_{n\to\infty} u_n = 0$

Deduce that the series $\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)^{1/n}$ diverges.

OR

If $\alpha > 0, \beta > 0$, then test the Convergence of the series :

$$1+\frac{\alpha+1}{\beta+1}+\frac{(\alpha+1)(\beta+1)}{(\beta+1)(2\beta+1)}+\frac{(\alpha+1)(2\alpha+1)(3\alpha+1)}{(\beta+1)(2\beta+1)(3\beta+1)}+\ldots\ldots$$

(b) Use Cauchy's Condensation test, to show that the series $\sum_{n=2}^{\infty}\frac{1}{n(\log n)^p}$ Converges for $p > 1$ and diverges if $p \le 1$

OR

Show that the series :

$$1^p+\left(\frac{1}{2}\right)^p+\left(\frac{1.3}{2.4}\right)^p+\left(\frac{1.3.5}{2.4.6}\right)^p+\ldots\ldots$$

Converges if $p > 2$ and diverges if $p \le 2$

2. (a) If $\sum_{n=1}^{\infty} u_n$ is an absolutely Convergent series, then prove that the series of positive terms and the series of negative terms of Σu_n are both Convergnt. Also prove that if $\sum_{n=1}^{\infty} u_n$ is conditionally convergent then the series of its positive terms and the series of its negative terms are both divergent.

OR

Prove that the series $x-\frac{x^3}{3}+\frac{x^5}{5}-\ldots\ldots$

Converges if and only if $x \in (-1, 1]$

(b) Let $\sum_{n=1}^{\infty} a_n$ be an absolutely Convergent series and $\sum_{n=1}^{\infty} b_n$ be a Convergent series Converging to zero then prove that their Cauchy product series will converge to zero.

OR

Test the Convergence of the series for $\sum_{n=1}^{\infty}\frac{\sin nx}{n}$ for positive values of x.

3. (a) If $f : [a, b] \to R$ is a continuous map on $[a, b]$ and $G(x) = \int_a^x f(u)du$, then prove

(i) $D_x G(x) = f(x)$ where $D_x = \frac{d}{dx}$ and also (ii) $F(b) - F(a) = \int_a^b f(x)dx$, where $f : [a, b] \to R$ is a primitive of f.

OR

State and prove the change of variable theorem of integration and apply it to integrate $\int_0^{\pi/2}(x^2 \sin x \cos x + x \sin^2 x)dx$

(b) If f is positive and monotinically decreasing in $[1, \infty]$. Show that the sequence $\{A_n\}$, where $A_n\left\{f(1)+f(2)+\ldots+f(n)-\int_1^n f(n)dx\right\}$ is convergent. Deduce the convergence of $\left\{1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}-\log n\right\}$.

OR

If $\int_a^b f(x)dx$ and $\int_a^b \phi(x)dx$ both exist and $\phi(x)$ keeps the same sign positive or negative throughout the interval of integration, then show that there exists a number μ lying between the infimum and supremum of f over [a, b] such that

$$\int_a^b f(x)\,\phi(x)dx = \mu \int_a^b \phi(x)dx$$

4. (a) If ϕ be bounded and monotonic in $[a,\infty]$ and tends to '0' as x tends to ∞. Also let $\int_a^x f(x)dx$ is bounded for $\forall x \geq a$, then prove that $\int_a^\infty f(x)\phi(x)dx$ is convergent.

OR

Show that $\int_a^\infty \sin x^2 dx$ is convergent

(b) If f and g are bounded and integrable function on $[a+\in, b]$ and 'a' is the only point of discontinuity such that $\lim_{x\to a} \frac{f(x)}{g(x)} = \ell$, where ℓ is non-zero finite, then prove that and $\int_a^b f(x)dx$ and $\int_a^b g(x)dx$ both either converge or diverge together.

OR

Show that $\int_0^{\pi/2} \sin x \log \sin x\, dx$ is convergent.

5. (a) Let X be a non-empty set and R is the set of real numbers. If a mapping $d : X \cdot X \to R$ is defined as $d(x,y) = \begin{cases} 0 & \text{when } x = y \\ 1 & \text{when } x \neq y \end{cases}$ for $\forall x, y \in X$, then show that (x,d) is a metric space. Give its name. Also show that the function $d : R \times R \to R$ defined by $d(x,y) = |x^2 - y^2|$, for $\forall x, y \in R$; is not a metric on R.

OR

Prove thet $\text{Int}\, A \cup \text{Int}\, B \subseteq \text{Int}\,(A \cup B)$ where Int A is the set of all interior points of set A. Give an example to show that equality does not hold in general.

(b) Define closure of a set in a metric space (X, d). If closure of a set A in X be denoted by $\overline{A}$, then show that

(i) $\overline{A} = \overline{B} \not\Rightarrow A = B$, where A and B are subsets of X.

(ii) $\overline{N} = N$, where N is set of natural number. Considered as a subset of the usual metric on IR.

OR

Prove that every closed subspace of a complete metric space is complete.

MATHEMATICS (THIRD PAPER), 2012

Time Allowed : Three Hours] [Maximum Marks : 50

Note : Answer **all** questions. Marks are indicated against each part. Symbols have their usual meaning.

1. (a) A hemisphere rests in equilibrium on a sphere of equal radius. Determine whether the equilibrium is stable or unstable when the flat surface of the hemisphere rests on the sphere. 2

OR

Show that for a common catenary $x = c \log\left(\frac{y+s}{c}\right)$. 2

(b) Show that the length of an endless chain which, will hang over a circular pulley of radius 'a' so as to be in contact with two third of the circumference of the pulley is- $a\left\{\frac{3}{\log(2+\sqrt{3})}+\frac{4\pi}{3}\right\}$ 4

OR

A heavy body rests on a fixed body. Determine the nature of equilibrium. 4

(c) State and prove the principal of virtual work for a system of coplanar forces acting on a particle. 4

OR

Two equal uniform rods, AB and AC, each of length 2b are freely joint as A and rest on a smooth vertical circle of radius 'a'. Show that, if 2θ be the angle between them, then- $a\cos\theta = b\sin^3\theta$. 4

2. (a) A particle is moving with S.H.M. of amplitude 'a' and periodic time T. Prove that $\int_0^T v^2 dt = \frac{2\pi^2 a^2}{T}$ 2

OR

A particle is projected upwards with velocity U under gravity (supposed constant) in a resisting medium whose resistance varies as the velocity. Prove that the greatest height received by the particle is- $\frac{VU}{g}+\frac{V^2}{g}\log\frac{V}{V+U}$, where V is the terminal velocity. 2

(b) A particle falls under gravity (supposed constant) in a resisting medium whose resistance varies as the square of the velocity. Find the motion if particle starts from rest. 4

OR

A spherical rain drop, falling freely, receives in each instant an increase of volume equal to λ times its surface at that instant. Find the velocity at the end of time t. 4

(c) A particle is projected vertically upwards from the earth surface with a velocity just sufficient to carry it to infinity. Prove that the time it takes in reaching a height h is $\frac{1}{3}\left(\frac{2a}{g}\right)^{1/2}\left\{\left(1+\frac{h}{a}\right)^{3/2}-1\right\}$. 4

where a is the earth's radius.

OR

One end of an elastic string is fastened at A and to the other end fastened at A and to the other end fastened a particle heavy enough to stretch the string to double its natural length a. Show that if the particle is dropped from A, it will descend a distance $(2+\sqrt{3})$ a before coming to rest. 4

(a) A particle is moving along a plane curve. Determine the radial and transverse components of rate of change of acceleration of the particle. 2

OR

Prove that the angular acceleration of the direction of motion of a point moving in a plane is $\frac{v}{\rho}\frac{dv}{ds}-\frac{v^2}{\rho^2}\frac{d\rho}{ds}$ 2

b) A point starts from origin in the direction of the initial line with velocity $\frac{f}{w}$ and moves with constant angular velocity w about the origin and with constant negative radial acceleration of magnitude f. Show that the rate of growth of the radial velocity is never positive. Also obtain the polar equation of the path. 4

OR

A particle moving in a plane describes the equiangular spiral $r = a \exp(\theta \cot \alpha)$. If the radius vector to the particle has a constant angular velocity, show that the resultant acceleration of the particle makes and angle 2α with the radius vector and is of magnitude $\frac{v^2}{r}$, where v is the speed.

(c) A particle, projected with a velocity u, acted on by a force which produces a constant acceleration f in the plane of the motion-inclined at a constant angle α with the direction of motion. Obtain the intrinsic equation of the curve described.

OR

A point moves in a plane curve so that its tangential acceleration is constant and the magnitudes of tangential velocity and normal acceleration are in a constant ratio. Show that the intrinsic equation of the path is of the form

$s = A\Psi^2 + B\Psi + C$ 4

4. (a) A particle is constrained to move along the arc of a smooth cycloid in a vertical plane, placed with the vertex downwards and axis verticle. Prove that the motion is simple harmonic and the periodic time is independent of amplitude. 2

OR

If motion of a particle starts from rest from cusp of a smooth cyclpid whose axis is vertical and vertex is downwards, then 2

$$\left(\frac{ds}{dt}\right)^2 = \frac{g}{4a}[(4a)^2 - s^2]$$

(b) A particle is projected from the lowest point of a smooth vertical circle with velocity v and moves on the inside of the circle. Find the condition under which the particle will describe the complete circle. 4

OR

A particle is projected from the lowest point inside a smooth sphere of radius 'a' with a velocity due to a height h above the centre. Find the point at which the particle will leave the sphere. 4

(c) A particle is projected with velocity V from the cusp of a smooth inverted cycloid down the arc. Show that the time of reaching the vertex is $2\left(\frac{a}{g}\right)^{1/2} \tan^{-1}\left(\frac{\sqrt{4ag}}{V}\right)$ 4

OR

If a particle slides down a smooth cycloid starting from a point whose arcual distance from the vertex is b, prove that its speed at any time is $\frac{2\pi b}{T}\sin\left(\frac{2\pi t}{T}\right)$, where T is the time for complete oscillation of the particle. 4

5. (a) For a central orbit, prove that 2

$$v^2 = h^2\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right]$$

OR

If in a central orbit the velocity at any points is inversely proportional to the distance of the point from the centre of force, show that the path is an equiangular spiral. 2

(b) Show that the only law for a central attraction for which the velocity in a circle at any distance is equal to the velocity acquiring in falling from infinity to the distance is that of inverse cube. 4

OR

The velocity at any point in a central orbit is $\left(\frac{1}{n}\right)^{th}$ of what it would be for a circular orbit at the same distance. Show that the central force varies as $\frac{1}{r^{2n^2+1}}$ and that the equation of the orbit is

$r^{n^2-1} = a^{n^2-1}\cos(n^2-1)\theta$ 2+2

(c) A particle moves under a central repulsive force mμ/(distance)^3, and is projected from an apse at a distance 'a' with velocity V. Show that the equation to the path is

$r\cos p\theta = a$, and that the angle θ described in times t is

$\frac{1}{p}\tan^{-1}\left(\frac{pV}{a}t\right)$, where

$$p^2 = \frac{\mu + a^2V^2}{a^2V^2}$$ 4

OR

A particle describes an ellipse under a force μ/(distance)2 towards the force. If it was projected with velocity V from a point distant r from the centre of force, show that its periodic time is

$$\frac{2\pi}{\sqrt{\mu}}\left[\frac{2}{r} - \frac{V^2}{\mu}\right]^{-3/2}$$ 4

B.A./B.Sc. (Part-II) Exam., 2013

MATHEMATICS (FIRST PAPER)

(Linear Algebra)

Note : Answer all questions. Marks allotted to each question are indicated against it.

1. (a) Show that the vector space $\mathbb{R}^3(\mathbb{R})$ is the linear sum of its subspaces $W_1 = \{(x,y,z): x,y,z \in \mathbb{R} \text{ and } x-y+z=0\}$ and $W_2 = \{(x,y,z): x,y,z \in \mathbb{R} \text{ and } \frac{x}{2} = \frac{y}{1} = \frac{z}{1}\}$. Is $\mathbb{R}^3$ the direct sum of W_1 and W_2? Justify your answer. 4 + 1

(b) Define maximal linearly independent subset of a vector space. If $S = \{a_1, a_2, \ldots, a_n\}$ is a subset of a vector space V over a field F. Then prove that the following statement are equivalent :

I : S is linearly independent and S generates V.

II : S is a maximal linearly independent subset of V. 1 + 4

(a) Let $S_1 = \{a_1, a_2, \ldots, a_n\}$ be a linearly independent subset of a vector space V over a field F and $a \in V$ but $a \notin L_{S_1}$.

Then show that the set $S_2 = \{a, a_1 + \lambda a, a_2 + \lambda a, \ldots, a_n + \lambda a\}$ where $\lambda \in F$ will be linearly independent. 5

(b) If W_1 and W_2 are two subspaces of a finite dimensional vector space V over a field F, then prove that

$\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$. 5

2. (a) Define kernel of a linear transformation . Find a linear transformation $T: \mathbb{R}^4 \to \mathbb{R}^3$ such that Ker T and $I_M T$ are respectively spanned by $\{(1,1,1,1),(1,0,0,1)\}$ and $\{(1,1,0),(1,0,1)\}$. 1+4

(b) Define transpose of a linear transformation and prove that the rank of the transpose of a linear transformation is equal to the rank of the linear transformation. 1+4

OR

(a) If a linear transformation $T: \mathbb{R}^4 \to \mathbb{R}^3$ is defined by $T\{(x,y,z,u)\} = (x-y+z+u, x+2z-u, x+y+3z-3u)\ \forall\ (x,y,z,4) \in \mathbb{R}^4$, then find (i) a basis of Ker T and (ii) a basis of $I_m T$. 2+3

(b) Define the annihilator of a non-empty subset of a vector space and prove that the sum of the dimensions of a sub-space of a finite dimensional vector space and its annihilator, is equal to the dimension of the vector space. 1+4

3. (a) Define trace of a square matrix . If A is a real matrix such that trace $(A.A^t) = 0$, then prove that $A = 0$. 1+2

(b) Define determinantal rank of a matrix. If A is an n-square matrix of rank n-1, than prove that the rank of the adjoint of A is 1. 1+3

(c) Can we express $(1,4,5) \in \mathbb{R}^3(\mathbb{R})$ as a linear combination of (1,1,2), (2,3,5) and (3,5,8) in $\mathbb{R}^3$? If yes, in how many ways ? 3

OR

(a) Define non-singular matrix. Prove that an n-square matrix is non- singular if an only if the rank of the matrix is n.

(b) If A and B are two matrices associated with the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ relative to ordered bases $X = \{(1,1,0),(0,1,1),(1,0,1)\}$ and $Y = \{(1,1,1),(1,2,3),(1,4,9)\}$ respectively, then find a matrix C such that $B = C.A.C^{-1}$. 3

(c) Find the value of k for which the system of linear equation $kx + y + z = 1$, $x + ky + z = k$, $x + y + kz = k^3$ over the field of real numbers does not have a solution. 4

4. (a) Define the orthogonal complements $S^{\perp}$ of a non -empty subset S of an inner product space. If $\{(1,1,2,0),(1,1,0,3)\}$ generates the subspace W of Euclidean space $\mathbb{R}^4$. then find an orthonormal basis of $W^{\perp}$ .1+4

(b) Define eigen values and eigen vectors of a linear operator on a vector space. Find eigen values and corresponding eigen-vectors of the linear operator T on a vector space $\mathbb{R}_2[x]$ of all real polynomial of degree at most 2, given by

$T(a_0+a_1x+a_2x^2) = (3a_0-2a_1)-(2a_0-3a_1)\ x\ +4a_2x^2\ \forall\ (a_0+a_1x\ +\ a_2x^2) \in \mathbb{R}_2[x]$. 1+4

OR

(a) (i) Define adjoint of a linear operator. Find the adjoint of the linear operator T_A on $M_n(F)$ (set of all n×n matrices over the field F) given by $T_A(X) = A.X\ \forall\ X \in M_n(F)$, where A is a fixed matrix in $M_n(F)$. 1+2

(ii) If T* is the adjoint of a linear operator T on the inner product space $(V, <\ >)$ over the field of complex numbers, then prove that rank of T* = rank of T. 2

(b) If S is a real skew-symmetric matrix such that $S^2 + I = 0$, then

(i) Prove that S is an orthogonal matrix of even order and -

(ii) Show that there are non-zero vectors X and Y which are orthogonal and satisfy the relations

$SX+Y = 0$, $SY = X$ and $X^tX = Y^tY$.

5.(a) Is matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ diagonalizable ? If yes, find a matrix P such that $P^{-1}.A.P$ is a diagonal matrix. 3+2

(b) Reduce the quadratic form $q(x, y, z) = x^2+5y^2-4z^2+2xy-4zx$ on $\mathbb{R}^3$ $(\mathbb{R})$ to canonical fom by $\alpha x'^2+\beta y'^2+\gamma z'^2$ by congruent reduction and find the transformation relation between (x,y,z) and (x',y',z') and tell the index and signature of the quadratic form. 3+1+1

OR

(a) Prove that every bilinear form on a real vector space can be uniquely expressed as the sum of a symmetric and skewsymmetric bilinear forms. 5

(b) Reduce the equation

$22x^2-12xy+17y^2-112x + 92y +178 = 0$

into standard from by suitable orthogonal standard form by suitable orthogonal change of variables and tell the nature of the conic section represented by it. 4+1

MATHEMATICS (SECOND PAPER), 2013

Note : Attempt all the five question. Each question carries 10 marks.

1. (a) Prove that a necessarey and sufficient condition for the convergence of the series $\sum_{n=0}^{\infty} u_n$ is that corresponding to an arbitrary chosen positive number there exists a positive interger N such that

$\left|\sum_{v=N+1}^{N+p} u_v < \epsilon\right|$ for all positive integral valuse of p.

OR

Prove that the infinite series :

$\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+.....+\frac{1}{n^p}+.....$ is convergent if $p>1$, and divergent if $p \le 1$.

(b) Prove that a series $\sum_{n=0}^{\infty} u_n$ of positive terms is convergent of divergent according as

$\lim_{n\to\infty} n\left(\frac{u_n}{u_n+1}-1\right) > 1$: or < 1

OR

Test the convergence of the infinite series

$$x+\frac{2^2x^2}{2!}+\frac{3^3x^3}{3!}+\frac{4^4x^4}{4!}+\frac{5^5x^5}{5!}+\ldots\ldots$$

2. (a) Define 'alternating series' and give an example . Prove that the series $\sum_{n=1}^{\infty}(-1)^{n-1}u_n$ is convergent $(u_n \geq 0 \forall_n)$ if

(i) $u_n \geq u_{n+1} \forall n \geq 1$ and (ii) $\lim_{n\to\infty} u_n = 0$

OR

State and prove Riemann's rearrangement Theorem for a conditionally convergent series.

(b) If the series $\sum_{n=0}^{\infty} a_n$ is convergent and $\{V_n\}_{n=0}^{\infty}$ is a monotonic sequence that converges to zero then show that the series $\sum_{n=0}^{\infty} a_n v_n$ is convergent.

Hence or otherwise, show that the series $\sum_{n=2}^{\infty}\left\{\frac{(n^4+1)^{1/4}-n}{\log n}\right\}$ is convergent.

OR

If one of two series $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ is absolutely convergent while the other is convergent and their sums are A and B respectively then show that Cauchy product series $\sum_{n=0}^{\infty} a_n$, where $c_n = a_0b_n + a_1b_{n-1} + \ldots\ldots\ldots + a_0b_n = \sum_{k=0}^{n} a_n a_k b_{n-k}, \forall n \geq 0$ is convergent and its sum is AB.

3. (a) State and prove Fundamental Theorem of Intergral Calculus.

OR

If $\int_a^b f(x)dx$ and $\int_a^b \phi(x)dx$ both exist and ϕ is monotonic in [a,b] then prove that there exists $\xi \in [a,b]$ such that

$$\int_a^b f(x)\phi(x)dx = \phi(a)\int_a^{\xi} f(x)dx + \phi(b)\int_{\xi}^b f(x)dx$$

(b) If a function $f:[a,b] \to \mathbb{R}$ is bounded and Riemann integrable in [a,b], then prove that $|f|$ is also bounded and Riemann integrable in [a,b]. Also, show that the cenverse of this is not necessarily true.

OR

If a function f is bounded in [a,b] and the set of its points of discontinuity in [a,b] has only a finite number of limit points then prove that f is Riemann integrable in [a,b].

Hence or otherwise, show that the function f defined by

$$f(x) = \frac{1}{2^n}, \text{when} \frac{1}{2^{n+1}} < x \leq \frac{1}{2^n}, n = 0,1,2,3,\ldots\ldots$$

is Riemann integrable in [0,1].

4. (a) Show that the improper integral $\int_0^1 x^{n-1}\log x\,dx$ converges if and only if $n > 0$.

OR

Discuss the convergence of the following integral

$$\int_0^{\infty} \frac{x^m(1+x^n)}{1+x^p}dx,$$ where m, n & p are positive real numbers.

(b) If a real-valued function ϕ is bounded and monotonic in $[1,\infty]$ and $\int_1^{\infty} f(x)dx$ is convergent then prove that $\int_1^{\infty} f(x)\phi(x)dx$ is convergent.

OR

State and establish the-test for convergence of an improper integral $\int_a^{\infty} f(x)dx$

5. (a) Define 'metric space' and give an example. Let (X,d) be a metric space. Define σ on X×X by

$\sigma(x,y) = \min\{1.d(x,y)\} \forall (x,y) \in X \times X$

Show that σ is a metric on X .

Is $\sigma_1: X \times X \to \mathbb{R}$ defined by

$\sigma_1(x,y) = \max\{1,d(x,y)\}$ also a metric on X? Explain.

OR

Define 'open set' in a metric space. Prove that the intersection of a finite number of open sets is an open set in a metric space. Give and example to show that intersection of an infinite number of open sets need not be open.

(b) Define' closed set' in a metric space and give an example . Prove that in a metric space, a closed ball is a closed set. Show by an example that the converse of this is not necessarily true.

OR

Define 'Cauchy sequence' in a metric space and give an example. Prove that in a metric space, every Cauchy sequence is bounded. Show by an example that the converse of this is not necessarily true.

MATHEMATICS (THIRD PAPER), 2013

Note : Answer all questions. Marks are indicated against each part. Symbols have their usual meaning.

1. (a) Explain the general rule for deciding whether a given equilibrium position of a body stable or unstable in case the force of gravity is the only external force acting on the body. **OR** 2

State principle of virtual work for a system or coplanar forces acting at different points of a rigid body . 2

(b) A heavy string of uniform density and thickness is suspended from two given points in the same horizontal plane. A weight, on n^{th} that of the string, is attached to its lowest point. Show that if θ, ϕ are the inclinations to the vertical of the tangents at the highest and lowest points of the string, $\tan \phi = (1+n)\tan\theta$. 4

OR

The extremities of a heavy string of length 2ℓ and weight $2\ell w$, are attached to two small rings which can slide on a fixed horizontal wire. Each of these rings is acted on by horizontal force equal to ℓw . Show that the distance apart of the rings is $2\ell \log(1+\sqrt{2})$. 4

(c) Two uniform rods AB and AC, smoothly jointed at A, are in equilibrium in a vertical plane. B and C rest on a smooth horizontal plane and middle points of AB and AC are connected by a string. Show that the tension of the string is $\frac{W}{\tan B + \tan C}$ where W total weight of the rods and B and C are the inclinations to the horizontal of the rods AB and AC. **OR** 4

Show that in the position of stable equilibrium, the potential energy of the body is minimum. 4

2. (a) Define simple harmonic motion and explain the terms periodic time, frequency and amplitude of the motion. 2

OR

Explain the term "terminal velocity" of a particle falling under gravity in a resisting medium. 2

(b) A heavy particle is attached to an extensible string to a fixed point from which the particle is allowed to fall freely ; when the particle is in its lowest position the string is of twice its natural length. Prove that the modulus is four times the weight of the particle. 4

OR

A heavy particle is projected vertically upwards in a medium the resistance of which varies as the square of the velocity. It has a kinetic energy k in its upward path at a given point ; when it passes the same point on the way down, show that its loss of energy is $\frac{k^2}{k+k_1}$, where k_1 is the limit to which its energy approaches in its downwards course. 4

(c) A body of mass (m_1+m_2) is split into two parts of masses m_1 and m_2 by an internal explosion which generates kinetic energy E. Show that if after explosion the parts move in the same line as before, their relative speed is $\sqrt{\frac{2E(m_1+m_2)}{m_1 m_2}}$. 4

OR

Show that a particle executing S.H.M. requires one-sixth of its period to move from the position of maximum displacement to one in which the displacement is half the amplitude. 4

(a) A particle describes an equiangular spiral $r = ae^{m\theta}$ with a constant velocity . Find the components of the velocity along the radius vector and a perpendicular to it. **OR**

A particle is moving in a parabola with uniform angular velocity about the focus ; prove that its normal acceleration at any point is proportional to the radius of curvature of its path at that point. 2

(b) A points moves in a plane in such a manner that its tangential and normal accelerations are always equal, and its velocity varies as exp $\left(\tan^{-1}\frac{s}{c}\right)$.s being the length of the arc of the curve measured from a fixed point. Find the path. **OR** 4

If a rod, which always passes through the origin, rotates with uniform angular velocity ω' while one end describes the curve $r = a + be^{\theta}$, show that the radial acceleration of any point of the rod is the same at every instant and the radial velocity is the same at every point at a given instant. 4

(c) A point moves along the arc of a cycloid in such a manner that the tangent at it rotates with a constant angular velocity. Show that the acceleration of the moving point is constant in magnitude. 4

OR

A small bead slides with a constant speed v on a smooth wire bent in the form of a cardioid $r = a(1+\cos\theta)$.Find θ and prove that the radial acceleration is constant. 4

4. (a) Find the intrinsic equation $s = 4a \sin \psi$ of a cycloid. 2

OR

Prove that for a particle moving under gravity on the arc of a smooth cycloid, whose plane is vertical and vertex lowest ; the period is independent of the arc described. 2

(b) If a particle starts from rest at a given point of a cycloid with its axis vertical and vertex downwards, prove that it falls through 1/n of the vertical distance to the lowest point in the time $2\sqrt{\left(\frac{a}{g}\right)}\sin^{-1}\left(\frac{1}{\sqrt{n}}\right)$, where a is the radius of the generating circle. 4

OR

A heavy particle slides down a smooth cycloid starting from rest at the cusp, the axis being vertical and vertex downwards. Prove that the magnitude of the acceleration of the particle is equal to g at every point of its path, and that when it arrives at the vertex the pressure on the curve is equal to twice the weight of the particle. 4

(c) A particle is hanging from a fixed point O by meas of a light string of length a. There is a small smooth nail at O' in the same horizontal line with O at a distance b (<a) from O. Find the minimum velocity with which the particle should be projected from its lowest position in order that it may make a complete revolution round the nail without the string becoming slack. **OR** 4

A particle, attached to the end of a light inextensible string of length ℓ, whose upper end is tied to a fixed point O, describes a circle in a vertical plane. Determine the motion. 4

5. (a) Show that in every central orbit the sectorial area, swept out by the radius vector of the particle, increases uniformly per unit of time. 2

OR

State Kepler's laws of planetary motion. 2

(b) Find the differential equation to the path of a particle moving in a plane under a central attractive acceleration . 4

OR

Find the law of force towads the pole under which the curve $r^n = a^n \cos n\theta$ can be described. Discuss clearly the case when n = -2. 4

(c) A particle moves under a force $\mu\{3au^4 - 2(a^2 - b^2)u^5\}, a > b,$ and is projected from an apse at a distance (a + b) with velocity $\sqrt{\frac{\mu}{a+b}}$ .Show that it orbit is $r = a + b\cos\theta$. **OR** 4

A particle is projected from a point (a,0) with a velocity $\sqrt[2]{(\mu)}$ a at right angle to the radius vector under an acceleration to the pole of magnitude $\mu\left(r + \frac{a^4}{r^3}\right)$ .Show that the orbit is $r^2(2 + \cos\sqrt{(3)}\theta) = 3a^2$

B.A./B.Sc. (Part-II) Exam., 2014

(Three Years Degree Course)

MATHEMATICS (FIRST PAPER)

(Linear Algebra)

Time Allowed : Three Hours] [Maximum Marks : 50

Note : Answer all questions. Marks allotted to each question are indicated against it.

1. (a) Define basis of a vector space. Prove that a maxim ! linearly independent subset of a vector space is a basis of the vector space. 1+4

(b) If W is a subspace of a finite dimensional vector space V, prove that dim V/W = dim V-dim W.

OR

(a) Define linearly dependent and linearly independent vectors of a vector space over a field . Let $S = \{a_1, a_2, \ldots\ldots, a_n\}$ be a linearly independent subset of a vector space V over a field F and $a \in V$ but $a \notin L_s$. Then show that $\{a_1, a_2, \ldots\ldots, a_n, a\}$ will be linearly independent. 1+ 4

(b) If $W_1 = \{x,y,z) : x,y, z \in \mathbb{R}$ and $y+2z = 0\}$ and $W_2 = \{(x,y,z) : x, y, z \in \mathbb{R}$ and $x+y+z = 0\}$ are two subspaces of the vector space $\mathbb{R}^3(\mathbb{R})$ then find

(i) a basis of W_1 1+1+1½+ 1½

(ii) a basis of W_2

(iii) a basis of $W_1 \cap W_2$ and

(iv) a basis of $W_1 + W_2$.

2. (a) Let M_3 denote the vector space of 3×3 matrices over the field of real numbers. A map $T : M_3 \to M_3$ is defined by $T(A) = A - A^t \forall A \in M_3$. Then prove that T is linear transformation. Find a basis of Ker T and hence find the nullity of T and rank of T. $1_{1/2}+1_{1/2}+1+1$

(b) Prove or disprove the following :

(i) If $T_1 : \mathbb{R}^2 \to \mathbb{R}$ and $T_2 : \mathbb{R}^2 \to \mathbb{R}$ are two linear transformations such that Ker T_1= Ker T_2 and $I_m T_1 = I_m T_2$, then $T_1 = T_2$. $1_{1/2}$

(ii) If $T : V_1 \to V_2$ is a linear transformation such that $\rho(T) = 2$ and dim $V_2 = 5$, then the nullity of T^t will be 3. $1_{1/2}$

(iii) If W is a subspace of $\mathbb{R}^4$ ($\mathbb{R}$) spanned by subset $S = \{(2,3,-2,1), (4, 2,-1,3), (2,-5,4,3),(4,10,-7,1)\}$, then dimension of the annihilator of W will be 2. 2

OR

(a) Let M_2 denote the vector space of 2×2 matrices over the field of real numbers. A linear transformation $T : M_2 \to M_2$ is defined by $T(A) = AB - BA, \forall A \in M_2$ and $B = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$, then find $I_m T$, Ker T and trace of T. 2+2+1

(b) (i) Find dual basis to the ordered basis $X = \{(1,-2,3),(1,-1,1),(2,-4,7)\}$ of $\mathbb{R}^3$ ($\mathbb{R}$). 3

(ii) Prove that a non-zero linear functional on a vector space V is injective if and only if dim V = 1. 2

3. (a) If a linear transformation associated with a square matrix is non-singular, then prove that all the linear transformations associated with the matrix will be non singular. 5

(b) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}$$

and hence write A^{-1} as the product of elementary matrices. 5+2

OR

(a) Define row -rank and determinantial -rank of a matrix and prove that row-rank of a matrix is equal to the determinantial rank of the matrix. 1+1+3

(b) Define adjoint of a square matrix. If A is $n \times n$ ($n \geq 2$) matrix, then prove that adj adj $(|A|)^{n-2} A$. 1+4

4. (a) Using Gram -Schmidt orthonormalization process orthonormalize the ordered basis $\{(0,1,1), (1,0,1),(1,1,0)\}$ of $\mathbb{R}^3$ ($\mathbb{R}$).

(b) Define adjoint T* of a linear transformation T on an inner product space (V, < >) and prove that Ker (T*oT) = Ker T and hence prove that $\rho(T^* oT) = \rho(T) = \rho(ToT^*)$. 5

OR

(a) (i) Prove that eigen values of a Hermitian Matrix are all real . 3

(b) (ii) Prove that eigen vectors of a Hermitian matrix corresponding to distinct eigen values of the matrix are orthogonal. 2

b.(i) Prove that the determinant of a skew-symmetric matrix of odd order is zero. 2

(ii) If A is real skew -symmetric matrix such that $A^2 + I_n = 0$, then prove that A is of even order.

5. (a) (i) Prove that the matrix $A=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$ is not diagonalizable. 2

(ii) Prove that every real quadratic form q on a vector space V determines a unique symmetric bilinear form fs on V given by

$$f_s(x,y)=\frac{1}{2}[q(x+y)-q(x)-q(y)],\forall x,y\in V.$$ 3

(b) Reduce the equation $2y^2+2zx+4y+4=0$ to standard form by a suitable orthogonal change of variables and identify the conicoid represented by it. 4+1

OR

(a) Reduce the quadratic form

$q(x,y,z)=x^2-2yz+xz$ on $\mathbb{R}^3(\mathbb{R})$ to canonical form $\lambda_1x'^2+\lambda_2y'^2+\lambda_3z'^2$ by congruent reduction and find the transformation relation between (x,y,z) and (x',y',z'). Also find the index and signature of the qudratic form. 3+1+1

(b) If A is a real symmetric matrix, then prove that there exists a real symmetric matrix B such that $B^3=A$. 5

MATHEMATICS (SECOND PAPER), 2014

Time Allowed : Three Hours] **[Maximum Marks :** 50

Note : Attempt **all** the **five** question. Each question carries equal marks.

1. (a) Let $\sum_{n=1}^{\infty}u_n$ and $\sum_{n=1}^{\infty}v_n$ be two series of positive terms such that $\lim_{n\to\infty}\frac{U_n}{V_n}=\ell$. If ℓ is finite and non-zero then prove that the two series are either both convergent or both divergent. Test convergence of series $\sum_{n=1}^{\infty}\frac{2^n}{\sqrt{4^n+1}}$.

OR

Test convergence of series

$$x^2+\frac{2^2}{3.4}x^4+\frac{2^2.4^2}{3.4.5.6}x^6+\frac{2^2.4^2.6^2}{3.4.5.6.7.8}x^8+\ldots\ldots\ldots(x>0).$$

(b) State Cauchy's condensation test. Test convergence of series

$$\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-3}+\ldots\ldots$$

OR

Test convergence of series

$$x+x^{1+\frac{1}{2}}+x^{1+\frac{1}{2}+\frac{1}{3}}+\ldots\ldots\ldots(x>0).$$

2. (a) Prove that if a series $\sum_{n=1}^{\infty}u_n$ is absolutely convergent and series $\sum_{n=1}^{\infty}v_n$ is a rearrangement of $\sum_{n=1}^{\infty}u_n$ then $\sum_{n=1}^{\infty}v_n$ also absolutely convergent and has the same sum as $\sum_{n=1}^{\infty}u_n$.

OR

Define a conditionally convergent series giving an example. Prove that if a series $\sum_{n=1}^{\infty}u_n$ is conditionally convergent then it can be rearranged to converge to a real number α.

(b) Prove that if {bn} is positive monotonic decreasing sequence and series $\sum_{n=1}^{\infty} u_n$ is convergent then series $\sum_{n=1}^{\infty} u_n b_n$ is also convergent.

Test convergence of series :

$$\frac{1}{4^0.1}-\frac{1}{4.3}+\frac{1}{4^2.5}-\frac{1}{4^3.7}+\ldots\ldots\ldots$$

OR

Let be $\sum_{n=1}^{\infty} u_n$ be an absolutely convergent series and $\sum_{n=1}^{\infty} v_n$ be a convergent series converging to zero. Then prove that their Cauchy product series will converge to zero.

3. (a) Define a step function giving an example.

Evaluate

$$\int_{-2}^{2}[x^2+1]dx,$$

Where [y] denotes largers integer less then or equal to y.

OR

If $f:[a,b]\to \mathbb{R}$ is bounded function and r be a real number such that $a < r < b$, if $\int_a^r f(x)dx$ and $\int_r^b f(x)dx$ exist, then prove that $\int_a^b f(x)dx$ exists and $\int_a^b f(x)dx=\int_a^r f(x)dx+\int_r^b f(x)dx$.

(b) If $If:[a,b]\to \mathbb{R}$ is a bounded and integrable function,then prove that function $\lambda f:[a,b]\to \mathbb{R}$ is also integrable where λ is a fixed real number.

OR

Define primitive of a function and state and prove theorem on change of variable of integration.

4. (a) Discuss convergence of $\int_0^{\infty} e^{-x}\, x^{n-1}\, dx$, x is real .

OR

Show that $\int_0^{\infty}\frac{\sin x}{x}dx$ is not absolutely convergent.

(b) If f (x) is bounded and monotonic for $x \geq a$ and $\lim_{x\to\infty}\phi(x)=0$ and $\int_a^x f(x)dx$ is bounded for $x \geq$ then prove that $\int_a^{\infty} f(x)\phi(x)dx$ is convergent.

OR

If f, g are positive bounded and integrable functions on [a + ϵ ,b] and 'a' is the only point of discontinuity such that $\lim_{x\to a+0}\frac{f(x)}{g(x)}=\ell$ where ℓ is finite and non-zero then prove that $\int_a^b f(x)\,dx$ and $\int_a^b g(x)\,dx$ both either converge or diverge together.

5.(a) Let X be a non-empty set and d a metric function on X define function $d_1 : X\times X \to \mathbb{R}$ by $d_1(x,y)=\frac{d(x,y)}{1+d(x,y)}$ $\mathbb{R}$ is set of real numbers.

Show that d_1 is a metric function on X.

OR

Define open and closed sets in a metric space. Prove that in a metric space arbitrary union and finite intersection of open sets is open.

(b) Define closure of a set in a metric space . Let (X, d_1) and (Y, d_2) be two metric spaces, and $f : X \to Y$ be a function , then prove that f is continuous iff $f(\overline{A}) \subseteq \overline{f(A)}$, for all $A \subseteq X$.

OR

Let $(\mathbb{R},d)$ be a metric space, d being usual metric on $\mathbb{R}$ then prove that every Cauchy sequence in $\mathbb{R}$ is convergent in $\mathbb{R}$.

MATHEMATICS (THIRD PAPER), 2014

Time Allowed : Three Hours] [Maximum Marks : 50]

Note : Answer all questions. Marks are indicated against each part. Symbols have their usual meaning.

1. (a) In the case of common catenary prove that

$$T^2 - T_0^2 = w^2 s^2,$$

where T is the tension at any P, T_0 the tension at the lowest point C, w the weight per unit length and s the length of the arc CP of the catenary. 2

OR

Prove that the work done is stretching an elastic string is equal to the extension produced multiplied by the mean of the initial and final tensions. 2

(b) A uniform chain, of length 2ℓ and weight W, is suspended from two points, A and B, in the same horizontal line. A load P is now suspended from the middle point D of the chain and the depth of this point below AB is found to be h. Show that each terminal tensions is

$$\frac{1}{2}\left(P\frac{\ell}{h} + W\frac{h^2+\ell^2}{2h\ell}\right)$$

OR

Derive the Cartesian equation of the common catenary. 4

(c) A uniform rod, of length 2ℓ, is attached by smooth rings at both ends to a parabolic wire, fixed with its axis vertical and vertex downwards, and of latus rectum 4a. Show that the angle θ which the rod makes with the horizontal in a slanting position of equilibrium is given by $\cos^2\theta = \frac{2a}{\ell}$ and that, if these position exist, they are stable. 4

OR

A string, of length a, forms the shorter diagonal of a rhombus formed of four unifor rods, each of length b and weight W, which are hinged together . If one of the rods be supported in a horizontal position, prove that tension of the string is. $\frac{2W(2b^2-a^2)}{b\sqrt{4b^2-a^2}}$. 4

2. (a) A point in a straight line with S.H.M. has velocities v_1 and v_2 when its distances from the centre are x_1 and x_2. Show that the period of motion is . $2\pi\sqrt{\frac{x_1^2-x_2^2}{v_2^2-v_1^2}}$ 2

OR

A particle is falling under gravity in a resisting medium . If the law of resistance be mkv^2, find the terminal velocity. 2

(b) One end of and elastic string in fixed and to the other end is fastened a particle heavy enough to stretch the string to double its natural length a. The string is drawn vertically down till its is four times its natural length and then let go. Show that the particle returns to this point in time $\sqrt{\frac{a}{g}}\left(2\sqrt{3}+\frac{4\pi}{3}\right)$ 4

OR

A particle is projected vertically upwards from the earth's surface with a velocity just sufficient to carry it to infinity. Prove that the time it takes in reaching a hight h is $\frac{1}{3}\sqrt{\frac{2a}{g}}\left\{\left(1+\frac{h}{a}\right)^{3/2}-1\right\}$, where a is the earth's radius . 4

(c) A particle of unit mass is projected with velocity u at an inclination α above the horizon in a medium whose resistance is k times the velocity. Show that its direction will again make an angle α with the horizon after a time $\frac{1}{k}\log\left(1+\frac{2ku}{g}\sin\alpha\right)$

OR

A particle is projected vertically upwards with velocity V in a medium whose resistance varies as the square of the velocity. Prove that the velocity v when the particle has described distance x is given by $\frac{2gx}{k^2}=\log\frac{V^2+k^2}{v^2+k^2}$, where k is the terminal velocity. 4

3. (a) Prove that $\theta=\frac{vp}{r^2}$.

OR

Prove that if the tangential and normal accelerations of a particle describing a plane curve be constant throughout the motion, the angle ψ through which the direction of motion turns in time t is given by $\psi = A\log(1+Bt)$. 2

(b) If the radial the transverse velocities of a point are always proportional to each other and this holds for acceleration also, prove that its velocity will vary as some power the radius vector. 4

OR

Find the tangential and normal components of acceleration of a particle moving in the plane curve. 4

(c) A particle moves in a catenary ($s = c\tan\psi$), the direction of its acceleration at any point makes equal angles with the tangent and the normal to the path at the point. If the speed at the vertex where $\psi = 0$ be u, show that the velocity and acceleration at any other point are given by ue^{ψ} and $\frac{\sqrt{2}}{c}u^2e^{2\psi}\cos^2\psi$. 4

OR

A point starts from the origin in the direction of the initial line with velocity f/w, and moves with constant angular velocity w about the origin, and with constant negative radial acceleration-f. Show that the rate of growth of the radia velocity is never positive but tends to the limit zero, and prove that the equation of the path is $w^2r = f(1-e^{-\theta})$. 4

(a) What is a simple pendulum ? If the angle of swing is small, show that the motion is simple harmonic. 2

OR

A particle moves under gravity on the arc of a smooth cycloid whose plane is vertical and vertex lowest. Prove that the period is independent of the arc described. 2

(b) A particle moves under gravity in a vertical circle, sliding down the convex side of a smooth circular arc. If its initial velocity is that due to a fall to the starting point from a height h above the centre, show that it will fly off the circle when at a height $\frac{2}{3}$ above the centre. 4

OR

A particle is free to move on a smooth vertical circular wire of radius r. It is projected from the lowest point with velocity just sufficient to carry it to the highest point. Show that the reaction between the particle and the wire is zero after a time $\sqrt{\frac{r}{g}}\log(\sqrt{5}+\sqrt{6})$. 4

(c) Starting from cusp a particle slides down the arc of a smooth cycloid whose axis is vertical and vertex lowest ; prove that the time occupied in falling down the first half the vertical height is $\frac{\pi}{2}\sqrt{\frac{a}{g}}$ 4

OR

Two particles are let drop from the cusp of a cycloid down the curve of an interval of time t; Prove that they will meet in a time $2\pi\sqrt{\frac{a}{g}}+\frac{t}{2}$ 4

5. (a) A particle moves in a ellipse under a force which is always directed towards its focus. Find the law of force 2

OR

Prove that in every cental orbit the linear velocity varies inversely as the perpendicular from the centre upon the tangent to the path. 2

(b) Find the (p,r) equation of the path of a particle moving in a plane under a central attractive acceleration. 4

OR

A particle describes the curve $r^n \cos n\theta = a^n$ under a force P to the pole. Show that P is proportional to r^{2n-3}. 4

(c) A particle moves with a central acceleration which varies in versely as the cube of the distance; if it beprojected from an apse at a distance a from the origin with a velocity which is $\sqrt{2}$ times the velocity for a circle of radius a, show that the equation to its path is

$$r\cos\frac{\theta}{\sqrt{2}} = a$$ 4

OR

A particle moves with a central acceleration $\mu(r^5 - c^4 r)$ being projected from an apse at a distance c with a velocity $\sqrt{\frac{2\mu}{3}}c^2$; show that its path is the curve $x^4 + y^4 = c^4$ 4

B.A./B.Sc. (Part-II) Exam., 2015

(Three Years Degree Course)

MATHEMATICS (FIRST PAPER)

(Linear Algebra)

Time Allowed : Three Hours] [Maximum Marks : 50

Note : Answer **all** questions. Marks allotted to each question are indicated against it.

1. (a) Define a vector space. Let V be a vector space over F and θ be the zero vector of V, Prove that $1\frac{1}{2} + 1 + 1 + 1\frac{1}{2}$

(i) $a\theta = \theta \; \forall \;\; a \in F$,

(ii) $0\alpha = \theta \; \forall \;\; \alpha \in V$,

(iii) $a(-\alpha) = -(a\alpha) = (-a)\alpha$

$\forall a \in F$ and $\forall \alpha \in V$

(b)(i) Prove that the set of non-zero vectors $\alpha_1, \alpha_2, \ldots\ldots, \alpha_n$ of a vector space V over F is linearly dependent if some $\alpha_k, 2 \le k \le n$, is a linear combination of the vectors preceding it . $2\frac{1}{2}$

(ii) Show that the vectors (1, 2, 1), (2, 1, 0), (1, -1, 2) from a basis of $\mathbb{R}^3(\mathbb{R})$

OR

(a) Prove that corresponding to each sub-space W_1 of a finite dimensional vector space V(F), there exists a subspace W_2 such that V is the direct sum of W_1 and W_2. 5

(b) If W is a subspace of a vector space V(F), Prove that the set V/W of all cosets $W + \alpha$, where α is an arbitrary element of V, is a vector space over F for the vector addition and scalar multiplication compositions defined as follows :

$(W + \alpha) + (W + \beta) = W + (\alpha + \beta) \; \forall \alpha, \; \beta \in V$, and

$a(W + \alpha) = W + a\alpha, \; \forall \; a \in F$ and $\forall \alpha \in V$. 5

2. (a) Let V and W be vector spaces over the field F and let T be a linear transformation from V into W. Suppose that V is finite dimensional . Than Prove that . 5

rank (T) + nulity (T) = dim V.

(b) Prove that any two finite dimensional vector spaces V and W defined over the same field F are isomorphic if and only if they have the same dimension. 5

OR

(a) Define ranges space, null space of a linear transfromation $T : V \to W$. Show that range of T is a subspace of W and null space of T is a subspace of V. 1 + 2 + 2

(b) Find the dual basis of the basis
$B = \{(1,-1,3),(0,1,-1), (0,3,-2)\}$ for $\mathbb{R}^3$ $(\mathbb{R})$ 5

3. (a) Reduce the matrix

$$a = \begin{bmatrix} 1 & 4 & 3 & 2 \\ 1 & 2 & 3 & 4 \\ 2 & 6 & 7 & 5 \end{bmatrix}$$

to normal form and find its rank . 5

(b) Let $V = \mathbb{R}^3$ and $T : V \to V$ be a linear transformation defined by
$T(a,b,c) = (a + c, -2a + b, -a + 2b + c)$,
Find the matrix of T with respect to the basis.
$\{(1,0,1),(-,1,1,1),(0,1,1)\}$ 5

OR

(a) Show that the equations
$x + y + z = 6$, $x+2y+3z = 14$, $x + 4y + 7z = 30$ are consistent and solve them. 5

(b) Define adjoint of a square matrix and prove that
adj (AB) = (adjB) (adj A). 1+4

4. (a) Define inner product on a vector space V(F), Let V(C) be the vector space of all continuous complex-valued functions on the unit interval $0 \le t \le 1$ If $f(t), g(t) \in V$, let us define

$$(f(t), g(t)) = \int_0^1 f(t)\overline{g(t)}\, dt.$$

Show that (f(t), g(t)) is an inner product on V(c). 1+4

(b) If α, β are vectors in an inner product space V, prove that 2 + 3

(i) $\|\alpha + \beta\| \le \|\alpha\| + \|\beta\|$

(ii) $\|\alpha + \beta\|^2 + \|\alpha - \beta\|^2 = 2(\|\alpha\|^2 + \|\beta\|^2)$

OR

(a) State and prove Bessel's inequality for an inner product space 1+4

(b) Define eigenvalues and eigenvectors of a square matrix A. Prove that the eigenvectors corresponding to distinct eigenvalues of a matrix are linearly independent. 1 + 4

(a) Prove that an n × n matrix is diagonalizable if and only if it possesse n linearly independent eigenvectors. 5

(b) Which of the following quadratic forms are congruent with each other? Why ?

(i) $x^2 + y^2 + z^2 + xy$

(ii) $xy + yz + zx$,

(iii) $x^2+2y^2+ 2z^2+3xy$. $1\frac{1}{2}+1\frac{1}{2}+1\frac{1}{2}+\frac{1}{2}$

OR

(a) Show that the matrix $\begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonalizable . Also find the transforming matrix and diagonal matrix. 5

(b) Show that a real quadratic form $ax^2+bxy+cy^2$ in two variables x,y is positive definite if and only is $a > 0$ and $b^2 - 4ac < 0$

MATHEMATICS (SECOND PAPER), 2015

Time Allowed : Three Hours] **[Maximum Marks** : 50.

Note : Attempt **all the five** questions. Each question carries equal marks. Symbols used have their usual meaning.

1. (a) Let $\sum_{n=1}^{\infty} a_n$ be a convergent series of positive real numbers. Test the convergence of $\sum_{n=1}^{\infty} a_n^2$, $\sum_{n=1}^{\infty} \frac{1}{1+a_n}$ and $\sum_{n=1}^{\infty} \frac{a_n}{1+a_n}$

OR

State and prove a necessary and sufficient condition for a series of positive real numbers to be convergent.

(b) State Raabe's test for the convergence of a series of positive real numbers. Test the convergence of $\sum_{n=1}^{\infty} \frac{(nx)^n}{n!}$ for $x > 0$.

OR

State Cauchy's integral test for the convergence of a series of positive real numbers. Test the convrgence of $\sum_{n=2}^{\infty} \frac{1}{n\ell og\,(\ell ogn)}$

2. (a) State leibniz's test for convergence of an alternating series. Prove $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ that is a conditionally convergent series.

OR

Define an absolutely convergent series. If $\{a_n\}$ is a sequence of real numbers such that $\{n^2 a_n\}$ is convergent, prove that the series $\sum_{n=1}^{\infty} a_n$ is convergent.

(b) Prove that $\sum_{k=1}^{\infty} \sin kx$ has bonded partial sums. for each real number x.

Determine all real numbers x for which $\sum_{n=1}^{\infty} \left(1+\frac{1}{2}+\ldots\ldots\ldots+\frac{1}{n}\right)\frac{\sin nx}{n}$, convergent.

OR

Define Cauchy product of two series of real numbers. Show by an example that Cauchy product of two convergent series need not be convergent.

3. (a) When do we say that a bounded function $f : [a,b] \to IR$ is Riemann integrable ?

Prove that the function $f : [0, 2] \to \mathbb{R}$ defined by

$f(x) = \begin{cases} 1 & \text{if } 0 \le x \le 1, \\ 2 & \text{if } 1 < x \le 2, \end{cases}$ is Riemann intergrable and find its integral.

OR

Define a step function and its integral. If f and g are Riemann integrable functions on [a,b] such that $f(x) \le g(x)$ for all x in [a,b], prove that

$$\int_a^b f(x)dx \le \int_a^b g(x)dx.$$

(b) State and prove second mean value theorem for integrals.

OR

Let $f : [a,b] \to \mathbb{R}$ be a Riemann integrable function. If F is an antidervative of f on [a,b], prove that

$$\int_a^b f(x)dx = F(b) - F(a).$$

4. (a) Test the convergence of intergrals $\int_1^{\infty} x^2 e^{-3x} dx$ and $\int_0^{\pi/2} \frac{\cos x}{x} dx$.

OR

Determine real values of α and β for which intergral

$\int_0^{\frac{\pi}{2}} \sin^{\alpha} x \cos^{\beta} x \, dx$, is convergent.

(b) State Cauchy's criterion for convergence of improper intergrals of first kind. Test the convergence of $\int_0^{\infty} e^{-x^2} dx$.

OR

State and prove Dirichlet's test for convergence of improper integrals.

5. (a) Define a metric space, with an example. If d is a metric on a set S. prove that $|d(x,y) - d(z,y)| \le d(x,z)$ for all x,y and z in S.

OR

Prove that any finite set in a metric space is a closed set. Is {1} an open set in IR, with usual metric ? Support your answer.

(b) Define a closure point of set in a metric space. Prove that a point p is a closure point of a set A in a metric space if and only if there is a sequence in A converging to p.

OR

Define a complete metric space. Prove that any discrete metric space is complete. Is subspace of a complete metric space is complete ? Support your answer.

MATHEMATICS (THIRD PAPER), 2015

Time Allowed : Three Hours] [Maximum Marks : 50

Note : Answer **all** questions. Marks are indicated against each part. Symbols have their usual meaning.

1.(a)(i) Show that the work done by the tension T in a string in a small extension from length ℓ to $\ell+\delta\ell$ is $(-T\delta\ell)$.

(ii) A hemisphere rests in equilibrium on a fixed sphere of equal radius. Show that the equilibrium is unstable when the curved, and stable when the flat, surface of the hemisphere rests on the sphere. 2+4

OR

(a')(i) Explain the concept of stable and unstable equilibrium position with the help of a example.

(ii) A square of side 2a is placed with its plane vertical, between two smooth pegs which are in a horizontal line and at a distance d. Show that it will be in equilibrium when the inclination of one of its edges to the horizon is either $\pi / 4$ or $\frac{1}{2}\sin^{-1}(a^2 - d^2)/d^2]$ 2+4

(b) Establish the cartesian equation of a common catenary. 4

OR

(b') A uniform chain of length ℓ, is suspended from two points A,B in the same horizontal line. If the tension at A is twice that at the lowest point, show that the span AB is $\frac{\ell}{\sqrt{3}}\ell og\left(2+\sqrt{3}\right)$. 4

OR

2. (a) One end of a elastic string whose modulus of elasticity is λ and whose natural length is a, is tied to a fixed point on a smooth horizontal table, and the other is tied to a mass m lying on the table. The particle is pulled to a distance where the extension of the string becomes b, and then let go ; describe the motion and find the period of one complete oscillation. 4

OR

(a') Discuss the simple harmonic motion of a particle. Also, describe how such a motion can be represented geometrically. 4

(b) A machine-gun of mass Mo stands on a horizontal plane and contains the ammunition of total mass m, which is fired horizontally at a uniform rate with constant velocity u relative to the gun.The coefficient of sliding friction between the gun and the plan is μ. Explain the method employed to deal with changing mass problems and show that if the total ammunition is fired in time T, the velocity of the gun is then u log (1+m/M0)-μgT. 6

OR

(b') A particle moves in a straight line under the action of an attractive force varying inversely as the square of the distance from a fixed point. Discuss the motion. 6

3. (a) Find the tangential and normal components of (i) velocity, and (ii) rate of change of acceleration, of a particle moving in a plane. 6

OR

(a') (i) Prove that the acceleration of a point moving along a curve in a plane with uniform speed is $\rho(\psi)^2$.

(ii) Define the angular velocity w of a particle moving in a plane. Also, establish a relationship between w and the velocity v of the paticle. 2 + 4

(b) A particle is acted on by a force parallel to the axis of y whose acceleration (always towards the axis of x) is μ / y^2, and when y = a, it is projected parallel to the axis of x with constant velocity $\sqrt{(2\mu / a)}$. Find the equation of the path described by the particle. 4

OR

(b') A particle is acted on by a force parallel to the axis of y whose acceleration is λy, and is initially projected with a constant velocity $a\sqrt{\lambda}$ parallel to the axis of x from a point where y = a; prove that it will describe a catenary. 4

4. (a) A heavy particle, hanging vertically from a fixed point by a light inextensible string of length ℓ, is struck by a horizontal blow which imparts to it a velocity u; discuss the motion of a particle. 6

OR

(a') A heavy particle is attached to a fixed point by a fine inextensible string of length ℓ; the particle is projected horizontally from the lowest point with velocity $\sqrt{\left[\ell g\left(2+\frac{3\sqrt{3}}{2}\right)\right]}$. Determine the point, where the particle after leaving the circular path cut it again. 6

(b) If a particle starts from rest at a given point of a cycloid with its axis verticle and vertex downwards, prove that it falls through $\frac{1}{n}$ of the vertical distance to the lowest point in the time. $\sqrt{(a/g)}\sin^{-1}(1/\sqrt{n})$, where a is the radius of the generating circle. 4

OR

(b') A particle oscillates in a complete cycliod from cusp to cusp. Prove that the velocity v at any point P is equal to the resolved part of the velocity V at the vertex along the tangent at P. 4

5.(a) (i) Explain the term "Velocity for a circle of radius r = a".

(ii) Find the law of force to an internal point under which a praticle will describe a circle. 2 +4

OR

(a')(i) Explain the term" velocity from infinity to the point r = a". 2 +4

(ii) Find the law of force towards the pole under which the curve $r^n = a^n \cos n\theta$ can be described.

(b) A particle move under central attractive acceleration [= μ/(distance)^2] being projected with velocity V from a point distant R. Show that its path is a rectangular hyperbola if the angle of projection is

$$\sin^{-1}\left[\frac{\mu}{V\sqrt{\left(V^2 - \frac{2\mu}{R}\right)}}\right] \quad 4$$

OR

(b') A particle describe an ellipse under a force μ/(distance)^2 towards the focus; if it was projected with velocity V from a point distant R from the centre of force, show that its periodic time is :

$$\frac{2\pi}{\sqrt{\mu}}\left[\frac{2}{R} - \frac{V^2}{\mu}\right]^{-\frac{3}{2}} \quad 4$$

B.Sc. (Part-II) Examination, 2016

MATHEMATICS

First Paper

(Linear Algebra)

Time : 3.00 Hours] [M.M. 50

Note : Answer all questions. All questions carry equal marks.

1. (a) Let S be a generating set containing n elements of a vector space V over a field F. Prove that any linearly independent subset T of V contains at most n elements.

OR

Determine the subspace of $\mathbb{R}^3$ generated by $S = \{(x,y,z) \in \mathbb{R}^3 : y = x^2\}$ and give a basis of this subspace.

(b) Let V be a finite dimensional vector space over a field F. Let w_1 and w_2 be subspaces of V. Prove that

$$\dim(w_1 + w_2) = \dim w_1 + \dim w_2 - \dim(w_1 \cap w_2)$$

OR

Let $w_1 = \{(x_1,x_2,x_3,x_4) \in \mathbb{R}^4 : x_1 - 2x_2 + x_3 = 0\}$ and $w_2 = \{(\lambda, 2\lambda, -3\lambda, 4\lambda) : \lambda \in \mathbb{R}\}$. Prove that w_1 and w_2 are subspaces of $\mathbb{R}^4$ amd $\mathbb{R}^4$ is a direct sum of w_1 and w_2.

2. (a) Let V_1, V_2 and V_3 be finite dimensional vector spaces over a field F. Let $T_1 : V_1 \to V_2$ and $T_2 : V_2 \to V_3$ be linear transformations. Prove that $T_2 o T_1 : V_1 \to V_3$ is a linear transformation and rank $(T_2 o T_1) \leq \min\{\text{rank}(T_1), \text{rank}(T_2)\}$.

OR

State and prove Fundamental Theorem of vector space homomorphisms.

(b) Let V_1 and V_2 be finite dimensional vector spaces over a field and $T: V_1 \to V_2$ be a linear transformation. Define the transpose T^t of T. Prove that rank (T^t) = rank (T).

OR

Let V be a finite dimensional vector space over a field F. Let V** denote the dual of the dual space V* of V. Prove that for each $v \in V$, the map $fv: V^* \to F$ defined by $fv(\varphi) = \varphi(v)(\varphi \in V^*)$, is a linear transformation and the map

$\theta: V \to V^{**}$ defined by

$\theta(v) = fv$ is an isomorphism.

3. (a) Let $A = \begin{pmatrix} 2 & 0 & -1 \\ 1 & 1 & -1 \end{pmatrix} \in M_{2\times 3}(IR)$ and $T: IR^3 \to IR^2$ be the linear transformation associated to A with respect to the standard ordered basis of IR^3 and IR^2. Determine a basis of Ker T.

OR

Let F be a field and V be a vector space of dimension n over F. Let B be an ordered basis of V and $A \in Mn(F)$. Prove that A is non-singular if and only if the linear transformation $T: v \to V$ associated to A with respect to B, is non-singular.

(b) Let A be an n-square real matrix such that $A^2 = A$. Prove that there exists a non-singular matrix $P \in Mn(IR)$ such that $P^{-1}AP$ is in normal form.

OR

Determine whether following homogeneous system has a nontrivial solution over IR:

$3x + 5y - 4z = 0,$

$-3x - 2y + 4z = 0,$

$6x + y - 8z = 0.$

If yes, describe the solution space.

4. (a) Use Gram-Schmidt process to orthonormalize the basis $\{(1, 1, 1), (0, 1, -1), (2, 1, 1,)\}$ of IR^3.

OR

Let (V,<,>) be a finite dimensional inner product space. Prove that an orthonormal set $\{v_1, v_2, ..., v_n\}$ is an orthonormal basis of v if an only if for each

$$v \in v, \|v\|^2 = \sum_{i=1}^{n} |\langle v, v_i \rangle|^2$$

(b) Let V be a finite dimensional complex inner product space. Prove that a linear transformation T on V is unitary if and only if for any orthonormal basis $\{v_1, v_2, ..., v_n\}$ of V, $\{T(v_1), T(v_2), ..., T(v_n)\}$ is orthonormal basis of V.

OR

Let T be a linear transformation on a finite dimensional vectore space V over a field F. Let $S = \{v_1, v_2, ..., v_r\}$ be a set of eigenvectors corresponding to distinct eigenvalues of T. Prove that S is linearly independent.

5. (a) Let $A \in Mn(C)$ be a Hermitian matrix. Prove that there exists an n-equare unitary matrix U such that U^*AU is a diagonal matrix.

OR

Prove that $f((x_1, y_1, z_1), (x_2, y_2, z_2)) = x_1y_2$ $2x_2y_1 + 3x_1y_2 + x_2y_3 + 4x_3y_3$ defines a bilinear form on IR^3. Find the matrix of this form with respect to the ordered basis $\{(1, 1, 0), (1, -1, 0), (3, 1, 1)\}$ of IR^3

(b) Reduce the quadratic form

$q((x,y)) = 2x^2 + 2xy + 2y^2 + 2x - 2y - 4$ to normal form. Also find the matrix of the transformation and rank of q.

OR

Classify the quadrics in IR^3 represented by $3x^2 + 2y^2 + 3z^2 - z + 2x - 2 = 0$.

MATHEMATICS
Second Paper, 2016
(Analysis)

Time : 3.00 Hours] [M.M. 50

Note : Attempt all the questions. Each question carries equal marks. Symbols have their usual meaning.

1. (a) Prove that an infinite series of positive real numbers is convergent if and only if the sequence of its partial sums is bounded above.

OR

Let $\sum_{n=1}^{\infty} a_n$ be a divergent series of positive terms and $\{s_n\}_{n=1}^{\infty}$ denote the sequence of its partial sums. Test the convergence of the series $\sum_{n=1}^{\infty} \frac{a_n}{s_n}$.

(b) State and prove Cauchy's condensation test for the convergence of an infinite series of positive terms.

OR

Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be series of positive terms

satisfying $\frac{a_n + 1}{a_n} \le \frac{b_n + 1}{b_n}$, for all $n \in N$. If $\sum_{n=1}^{\infty} b_n$ is convergent, then prove that $\sum_{n=1}^{\infty} a_n$ is also convergent.

Test the convergence of $\sum_{n=1}^{\infty} \frac{\sqrt{n} + 1}{n^5 + n^2}$.

2. (a) Define absolutely convergent series and conditionally convergent series. Prove that every absolutely convergent series is covnergent. Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3n - 2}$ is conditionally convergent.

OR

Let $\sum_{n=1}^{\infty} a_n$ be a conditionally convergent series of real numbers. Let $l \in IR$. Prove that there is a rearrangement $\sum_{n=1}^{\infty} b_n$ of $\sum_{n=1}^{\infty} a_n$ such that $\sum_{n=1}^{\infty} b_n = l$.

(b) Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of real numbers whose partial sums form a bounded sequence. Let $\{b_n\}_{n=1}^{\infty}$ be a monotonic decreasing sequence of nonnegative real numbers which converges to 0. Prove that the series $\sum_{n=1}^{\infty} a_n b_n$ is convergent.

OR

State Merten's theorem. Find Cauchy product $\sum_{n=0}^{\infty} c_n$ of the series $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$, where $a_n = b_n = r^n$, for all n and $|r| < 1$. Show that $\sum_{n=0}^{\infty} c_n$ is absolutely convergent and find its sum.

3. (a) Define a partition of an interval [a, b], Let $h : [a, b] \to IR$ be a step function associated with a partition P of [a, b]. Define $\int_a^b h(x)dx$ and show that value of $\int_a^b h(x)dx$ does not change if P is replaced by its refinement.

OR

Let f and g be bounded real valued functions defined on [a, b]. Prove that

$$\overline{\int_a^b}(f+g)(x)dx \le \overline{\int_a^b} f(x)dx + \overline{\int_a^b} g(x)dx$$

(b) If a function f is Riemann integrable over [a, b], then show that $|f|$ is also Riemann integrable over [a, b] and

$$\left|\int_a^b f(x)dx\right| \le \int_a^b |f(x)|\,dx.$$

OR

State and prove a necessary and sufficient condition for Riemann integrability of a bounded function $f : [a, b] \to IR$.

4. (a) Let f and g be positive and Riemann integrable functions on [a, x] for all $x > a$ such that $\lim_{x\to\infty} \frac{f(x)}{g(x)} = l, 0 < l < \infty$.

Show that $\int_a^\infty f(x)dx$ is convergent if and only if $\int_a^\infty g(x)dx$ is convergent.

OR

Prove that the improper integreal $\int_a^\infty f(x)dx$ is convergent if and only if for any $\varepsilon > 0$ there exists $\delta > a$ a such that $\left|\int_a^b f(x)dx\right| \le \varepsilon$, wherever $t_1, t_2 > \delta$

(b) Determine values of m and n, for which $\int_0^1 x^{m-1}(1-x)^{n-1}dx$ is convergent.

OR

Examine the convergence of

(i) $\int_0^1 \frac{x^{p-1}}{1+x}dx$ and

(ii) $\int_1^\infty x^{n-1}e^{-x}dx, n \in N$

5. (a) Let (X, d) be a metric space. If $\rho(x, y) = \frac{d(x,y)}{1+d(x,y)}, x, y \in X$, then prove that (X, ρ) is a bounded metric space.

OR

Prove that the union of an arbitrary family of oper subsets in a metric space is an open subset. Is

intersection of an arbitrary family of open subsets also open? Support your answer.

(b) Let (X, d) be a metric space. Define closure cl(A) of A, where $A \subseteq X$. Let A and B be subsets of X. Show that $c|(A \cup B) = c|(A) \cup d(B)$. Show that $cl(A \cap B) = cl(A) \cap cl(B)$. does not hold in general.

OR

Let $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ be convergent sequences in a metric space (X, d) such that $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$, where $a, b \in X$. Prove that $\lim_{n\to\infty} d(a_n, b_n) = (a, b)$.

MATHEMATICS
Third Paper, 2016
(Mechanics)

Time : 3.00 Hours] [M.M. 50

Note : Answer all questions. Marks are indicated against each part. Symbols have their usual meaning.

1. (a) Establish the intrinsic equation of a common catenary. Also prove that for a common catenary, $x = c \log(\sec\psi + \tan\psi)$. 3+2

(b) Show that the length of an endless chain which will hang over a circular pulley of radius a, so as to be in contact with two-thirds of the circumference of the pulley is a $\left\{\frac{3}{\log(2+\sqrt{3})} + \frac{4\pi}{3}\right\}$.

OR

(a') State and prove the principle of virtual work for a system of coplanar forces acting at different points of a rigid body.

(b') A uniform rod of length *l* is attached by smooth rings at both ends to a parabolic wire, fixed with its axis vertical and vertex downwards, and the latus rectum 4a; show that the angle θ which the rod makes with the horizontal in a slanting position of equilibrium, is given by $\cos^2\theta = \frac{2a}{\ell}$, and that if this position exists, it is stable.

2. (a) Define a simple harmonic motion. Is it periodic? A point in a straight line with S.H.M. has velocities v_1 and v_2 when its distances from the centre are x_1 and x_2. Show that the period of motion is $2\pi\sqrt{\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}}$

1½+½+3

(b) Two masses M and M' are attached to the lower end of an elastic string whose upper end is fixed and are hung at rest. M' falls off; show that the distance of M from the upper and of the string at time t is $a + b + c\cos t\sqrt{\left(\frac{g}{b}\right)}$, where a is the unstretched length of the string, b and c the distances by which it would be xtended when supporting M and M' respectively.

OR

(a') If h be the height due to the velocity v at the earth's surface, supposing its attraction constant and H the corresponding height when the variation of gravity is taken into account, prove that $\frac{1}{h} - \frac{1}{H} = \frac{1}{r}$, where r is the earth's radius. 5

(b') If a particle is projected with velocity V along a smooth horizontal plane in a medium whose resistance per unit mass is μ times the cube of the velocity, show that the distance it has described in time t is $\frac{1}{\mu V}\{\sqrt{(1+2\mu V^2 t)}-1\}$ and that its velocity, then is $\frac{V}{\sqrt{(1+2\mu V^2 t)}}$. 5

3. (a) Find the radial and transverse components of velocity and acceleration of a particle moving in a plane curve. 5

(b) A point P describes, with a constant angular velocity of OP, an equiangular spiral of which O is the pole. Find its acceleration and show that its direction makes the same angle with the tangent at P as the radius vector OP makes with the tangent.

OR

(a') Prove that if the tangential and normal acceleration of a particle describing a plane curve be constant throughout the motion, the angle ψ through which the direction of motion turns in time t is given by $\psi = A \log(1+Bt)$. 5

(b') If the radial and transverse velocities of a point are always proportional to each other and this holds for acceleration also, prove that its velocity will vary as some power of the radius vector.

4. (a) A particle is projected from the lowest point with velocity u and moves along the inside of a smooth vertical circle of radius r. Prove that-

(i) When $u^2 \le 2gr$, the particle oscillates on each side of the lowest point,

(ii) When $u^2 > 2gr$ but $< 5gr$, the particle leaves the circle and describes a parabola,

(iii) When $u^2 > 5gr$, the particle makes complete revolutions.

(b) A particle is projected with velocity V from the cusp of a smooth vertical cycloid down the arc. Show that the time of reaching the vertex is

$$2\sqrt{\frac{a}{g}}\tan^{-1}\left(\frac{\sqrt{4ag}}{V}\right)$$ 4

OR

(a') A particle is free to move on a smooth vertical circular wire of radius r. It is projected from the lowest point with velocity just sufficient to carry it to the highest point. Show that the reaction between the particle and the wire is zero after a time

$$\sqrt{\frac{r}{g}}\log(\sqrt{5}+\sqrt{6}).$$ 5

(b') A heavy particle slides down a smooth cycloid starting from rest at the cusp, the axis being vertical and vertex downwards. Prove that the magnitude of the acceleration of the particle is equal to g at every point of its path.

5. (a) For a central orbit prove that

$$v^2 = h^2\left\{u^2+\left(\frac{du}{d\theta}\right)^2\right\},$$

where $u = \frac{1}{r}$ and v is the velocity at any point of the orbit.

(b) A particle moving under a constant force from the centre is projected in a direction perpendicular to the radius vector with the velocity acquired in falling to the point of projection from the centre. Show that its path is

$$\left(\frac{a}{r}\right)^3 = \cos^2\left(\frac{3\theta}{2}\right).$$

OR

(a') Derive the differential equation to the path of a particle moving under a central attractive force.

(b') A particle is projected at right angles at a distance a under a central force $\frac{\mu}{r^n}$ with the velocity from infinity. Show that the orbit is

$$r^{\frac{n-3}{2}} = a^{\frac{n-3}{2}} \cos\frac{n-3}{2}\theta.$$

●

B.A./B.Sc. (Part II) Examination, 2017

MATHEMATICS (FIRST PAPER)

(Linear Algebra)

Time : Three hours] [Maximum Marks : 50

Note : Answer **all** questions. All questions carry equal marks. Symbols used have their usual meaning.

1. (a) Let V be a finitely generated vector space over a field F. Prove that V contains a basis over F.

OR

(a') Prove that $W = \{(x_1, x_2, x_3, x_4, x_5) \in \mathbb{R}^5 : x_2 + x_3 + x_4 = 0\}$ is a subspace of $\mathbb{R}^5$. Determine $\dim_{\mathbb{R}} W$ and a subspace U of $\mathbb{R}^5$ such that $\mathbb{R}^5 = W \oplus U$.

(b) Let $V = \mathbb{Z}_3[x]$ denote the vector space (over $\mathbb{Z}_3$) of all polynomials over $\mathbb{Z}_3$ in indeterminate x.

Let $S = \{x^n + x^{n+2} : n \in \mathbb{N} \cup \{0\}\}$ and $W = \langle S \rangle$. Prove that $\dim_{\mathbb{Z}_3} V/W = 2$.

OR

(b') Let W_1 and W_2 be subspaces of a finite dimensional vector space V over a field F. Prove that. :

$$\dim_F(W_1 + W_2) = \dim_F W_1 + \dim_F W_2 - \dim_F(W_1 \cap W_2)$$

2. (a) Define the kernel of a linear transformation between two vector spaces over the same field. Give an example of two distinct linear transformations from $\mathbb{R}^4$ to $\mathbb{R}^4$ which hae the same Kernel and image.

(a') Define an isomorphism between two vector spaces over the same field. Prove that two finite dimensional vector spaces over a field F are isomorphic if and only if they have same dimension.

(b) Define the annihilator of a non-empty subset of a vector space over a field. Determine the annihilator of the subset $S=\{(x,y,z)\in \mathbb{R}^3 : xy=1\}$ of the real vector space $\mathbb{R}^3$.

OR

(b') Let V be a non-zero finite dimensional vector space over a field F. Le T: V→ V be a linear transformation such that $T^n=0$ but $T^{n-1}\neq 0$ fo some $n\in N, n>1$. Prove that there exists non-zero $V\in V$ such tha $\{v, T(v), T^2(v), ..., T^{n-1}(v)\}$ is linearly independent over F.

3. (a) Let F be a field and $A, B\in Mn(F), A\neq B$. Prove that A and B ar similar if and only if they are matrices of the same linear transformatio $T: F^n \to F^n$ with respect to different choice of ordered bases of F^n.

OR

(a') Define an elementary matrix. Write

$$A=\begin{pmatrix}1 & 1 & 1\\ 0 & -2 & 3\\ 0 & 0 & 3\end{pmatrix}\in M_3(\mathbb{R})$$

as a product of elementary matrices.

(b) Define row reduced Echelon form of a matrix over a field. Let

$$A=\begin{pmatrix}1 & 2 & -1\\ 2 & 1 & 0\\ 3 & 0 & 1\end{pmatrix}\in M_3(\mathbb{R})$$

Find a non-singular matrix $P\in M_3(\mathbb{R})$ such that P.A is in row reduced Echelon form.

OR

(b') Find the values of a for which the following system of linear equations over $\mathbb{R}$ is consistent :

$x+3y-7z=1, 2x+7y+az=3, x+ay-7z=0$

4. (a) Let V be a finite dimensional inner product space over C and $T: V \to V$ be a linear transformation. Define T*, the adjoint of T and prove that rank (T*)=rank (T).

OR

(a') Let W be the subspace of standard real inner product space $\mathbb{R}^4$ spanned by $\{(1,2,1,-1),(2,1,1,-1)\}$.

Let $v=(3,2,1,1)\in \mathbb{R}^4$. Determine the orthogonal projection of v on W and the perpendicular length of v from W.

(b) Let $A\in M_n(\mathbb{R})$ and $B=A^t.A$.

Prove that all eigenvalues of B arer non-negative and rank (B)=rank (A).

OR

(b') Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation associated to

$$A=\begin{pmatrix}1 & 2 & 1\\ 0 & -1 & -1\\ 0 & 0 & 2\end{pmatrix}\in M_3(\mathbb{R})$$

with respect to the standard ordered basis of $\mathbb{R}^3$. Determine all one dimensional T-invariant subspaces of $\mathbb{R}^3$.

5. (a) Define a quadratic form on a finite dimensional vector space over a field. Prove that a quadratic form q on $\mathbb{R}^n$ is uniquely associated with a symmetric bilinear form on $\mathbb{R}^n$ is uniquely associated with a symmetric bilinear form on $\mathbb{R}^n$.

OR

(a') State Sylvester's law of inertia. Use it to determine the number of distinct congruence classes of n×n real symmetric matrices.

(b) Classify the curve in $\mathbb{R}^2$ represented by

$$f(x,y) = 2x^2 - 4xy - y^2 + 4 = 0.$$

OR

(b') Let f be the bilinear form on $\mathbb{R}^3$ defined by

$$f((x_1,x_2,x_3),(y_1,y_2,y_3)) = 2x_1y_1 - 3x_2y_2 + 4x_3y_3 + x_1y_2 + 2x_2y_1$$

Find

(i) Symmetric and skew symmetric parts of f;

(ii) The matrix of f with respect to the ordered basic $\{(1,1,0),(1,-1,0),(1,1,1)\}$ of $\mathbb{R}^3$.

MATHEMATICS (SECOND PAPER)-2017

(ANALYSIS)

Time : Three hours] [Maximum Marks : 50

Note : Attempt **all** questions. Each question carries equal marks. Symbol have their usual meaning.

1. (a) State and prove 'D' Alembert's Ratio Test' for the Convergence of a series $\sum_{n=1}^{\infty} U_n$ of positive terms.

OR

Prove that the series $\sum_{n=1}^{\infty} U_n$ of positive terms is convergent or divergent according as

$$\lim_{n\to\infty}\left\{n\left(\frac{U_n}{U_{n+1}}-1\right)-1\right\}\log n > 1 \text{ or } < 1.$$

If for this series,

$$\lim_{n\to\infty}\left\{n\left(\frac{U_n}{U_{n+1}}-1\right)-1\right\}\log n = 1.$$

then what is the nature of $\sum_{n=1}^{\infty} U_n$? Justify the answer.

(b) Test the convergence of the following series :

$$x + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \frac{4^4x^4}{4!} + \ldots, \text{ for } x > 0.$$

Test the convergence of the following series :

$$1 + \frac{3}{7}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + \frac{3.6.9.12}{7.10.13.16}x^4 + \ldots, x > 0.$$

(a) If the series $\sum_{n=1}^{\infty} a_n$ of real numbers is convergent and $\{V_n\}_{n=1}^{\infty}$ is monotonic sequence that converges to zero, then prove that $\sum_{n=1}^{\infty} a_n V_n$ is convergent. Also show that $\sum_{n=1}^{\infty} \frac{1}{n} \sin n\theta$ is convergent for all real values of θ.

OR

Prove that by a suitable rearrangement of the terms of a conditionally convergent series, it can be made to converge to any given number α.

(b) If $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ are absolutely convergent and their sums are A and B respectively, then prove that the Cauchy's product series $\sum_{n=0}^{\infty} C_n$ where $C_n = \sum_{k=0}^{n} a_k b_{n-k} \forall n \geq 0$, is absolutely convergent and its sum is AB.

OR

Assuming the expansion

$$\tan^{-1}x = x - \frac{x^3}{3} + \frac{X^5}{5} - \frac{X^7}{7} + \ldots = \sum_{N=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}, -1 < x \le 1,$$

Prove that

$$\left(\tan^{-1}x\right)^2 = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+2}}{n+1}\left(1 + \frac{1}{3} + \frac{1}{5} + \ldots + \frac{1}{2n+1}\right)$$

3. (a) State and prove a necessary and sufficient condition for the Riemann integrability of a function $f:[a,b] \to \mathbb{R}$.

OR

If $f:[a,b] \to \mathbb{R}$ is monotonic in [a, b] then prove that it is Riemann integrable over [a, b].

(b) State and prove 'Bonnett's form of Second Mean Value Theorem' of Integral Calculus.

OR

A function f is defined in [0,1] as follows :

$f(x) = \frac{1}{3^{r-1}}$, where $\frac{1}{3^r} < x \le \frac{1}{3^{r-1}}$, for $r = 1,2,3,\ldots$ and $f(0) = 0$.

Show that $\int_0^1 f(x)dx$ exists and is equal to $\frac{3}{4}$.

4. (a) Let $0 < a < b < \infty$. Show that the improper integral $\int_a^b \frac{dx}{(x-a)^n}$ converges if and only if n<1.

Hence or otherwise, examine the convergence of $\int_0^1 \frac{dx}{x^{1/3}(1+x^2)}$

OR

Show that $\int_0^\infty x^n e^{-x}\,dx$ is convergent if and only if n>-1.

(b) State and prove 'Dirichlet's Theorem' for the convergence of the improper integral $\int_a^\infty f(x)\phi(x)dx$.

OR

Show that $\int_0^\infty \sin x^3\,dx$ is convergent.

5. (a) Let l' be the set of all absolutely summable sequences of real numbers i.e.

$$\ell' = \left\{x = (x_n)_{n=1}^{\infty} \in \mathbb{R}^N : \sum_{n=1}^{\infty}|x_n| < \infty\right\}, N$$

being the set of all natural numbers. For

$x = (x_n)_{n=1}^{\infty}$ and $y = (y_n)_{n=1}^{\infty} \in \ell'$,

let $d_1(x,y) = \sum_{n=1}^{\infty}|(x_n - y_n)|$. Show that d_1 is a metric on l'.

OR

Let (x, d) be a metric space. Prove that -

(i) The null space ϕ and the whole space x are closed in x;

(ii) Union of a finite number of closed subsets in X is a closed set in X; and

(iii) Intersection of any collection of closed subsets in X is a closed set in X.

(b) Prove that every convergent sequence is a Cauchy sequence in a metric space. Is the converse of this true ? Justify the answer.

OR

Let (X, d) be a metric space and A be a subset of X. Define the closure $\overline{A}$ ofA. Show that-

(i) $\overline{X} = X$

(ii) $A \subseteq B \subseteq X \Rightarrow \overline{A} \subseteq \overline{B}$

(iii) $\overline{A \cap B} \subseteq \overline{A} \cap \overline{B}$

Show by an example that in (iii), inclusion may be proper.

MATHEMATICS (THIRD PAPER)-2017
(MECHANICS)

Time : Three hours] [Maximum Marks : 50

Note : Answer all questions. Symbols have their usual meaning. Each question carries equal marks.

1. (a) Explain the terms "sag", "span" and "parameter" of a common catenary. If PN be the ordinate of any point P of a catenary and NL the perpendicular from N to the tangent at P, then show that NL=c and PL=s.

OR

(a') Define a common catenary and prove that for such a catenary, $s = c \sinh\left(\frac{x}{c}\right)$

(b) A hemisphere rests in equilibrium on a fixed sphere of equal radius; show that the equilibrium is unstable when the curved surface, and stable when the flat surface, of the hemisphere rests on the sphere.

OR

(b') Two equal uniform rods, AB and AC, each of length 2b are freely jointed at A and rest on a smooth vertical circle of radius a. If 2θ be the angle between the rods. Then prove that $b \sin^3\theta = a \cos\theta$.

2. (a) Using vector method, obtain tangential and normal accelerations of a particle moving in a plane curve.

OR

(a') A particle moves in a catenary (intrinsic equation $s = c \tan\psi$), and the direction of its acceleration at any point makes equal angles with the tangent and the normal to the path at that point. If the speed at the vertex be u, then show that the acceleration at any other point equals $\frac{\sqrt{2}}{c} u^2 e^{2\psi} \cos^2\psi$.

(b) Prove that the parabola $y^2 = 4ax$ can be described under a constant force parallel to the axis of y, and a force proportional to y parallel to the axis of x.

OR

(b') Find the curve described by a moving particle having a constant acceleration in direction inclined at a constant angle to the tangent.

3. (a) Give a geometrical representation of simple harmonic motion.

One end of an elastic string of length *l* and modulus of eleasticity λ is tied to a fixed point on a smooth horizontal table, and the other is tied to a mass m lying on the table. The particle is pulled to a distance where the extension of the string becomes l_1 and then let go. Discuss the motion. Also, find the period of one complete oscillation.

OR

Explain the term "terminal velocity" of a particle moving in a resisting medium and find an expression for it if the resistance varies as the velocity.

A heavy particle is projected vertically upwards in a medium the resistance of which varies as the square of the velocity. It has a kinetic energy k in its upward path at a given point; when it passes the same point on the way down, show that its loss of energy is $\frac{k^2}{(k+k_1)}$, where k_1 is the limit to which its energy approaches in its downwards course.

4. (a) A heavy particle slides under gravity down the inside of a smooth

circular tube of radius b held in a vertical plane. If it starts from the highest point with the velocity $\sqrt{(2bg)}$, then find the pressure on the curve at the point where, in the subsequent motion, the verticle component of the acceleration is a maximum.

OR

(a') A particle is hanging from a fixed point 0 by means of a light in extensible string of length a. There is a small smooth nail at 0_1 in the same horizontal line with 0 at a distance b (< a) from 0. Find the minimum velocity with which the particle should be projected from its lowest position so that it may make a completly revolution round the nail without the string becoming slack.

(b) A particle falls down a cyclid under its own weight. If it starts from a point on the cycloid whose height above the vertex is h, with a velocity due to a fall through h, then show that it will attain the same height as it started from, first in time $\left[s\pi/\left\{2\sqrt{2gh}\right\}\right]$, where s denotes the arcual distance between the vertex and the point of start.

OR

(b') A particle is allowed to fall down the arc of a smooth cycloid $s = 4a\sin\psi$, whose axis is verticle and vertex upwards. It starts with a negligible velocity from the highest point. Find the height of the point above the base of the cycloid where the particle leaves the curve.

5. (a) Establish the (p,r) equation to the path of a particle moving in a plane with centrial attractive attraction f.

OR

(a') Prove that a particle moving under a central attractive force varying inversely as the square of the distance from the centre of the force always describes a conic type of orbit.

) If a particle moves with a central acceleration $\mu\left(r^5 - a^4 r\right)$ towards the origin and is projected from an apse at a distance a with a velocity $(2\mu/3)^{1/2} a^3$, then prove that its path is the curve $x^4 + y^4 = a^4$.

OR

') A particle moves under a central attractive force varying inversely as the fifth power of the distance from the centre of the force. It is projected from an apse at a distance b with a velocity equal to $\sqrt{5}$ times of that which would be acquired in falling from infinity. Find the other apsidal distance.

B.A./BSc. (Part-II) Examination, 2018

Three-Year Degree Course

MATHEMATICS (Paper : First)

(Linear Algebra)

Time : Three Hours] [Maximum Marks : 50

Note : Answer all questions. All questions carry equal marks. Symbols used have their usual meaning.

1. (a) Prove that the union of two subspaces of a vector space is a subspace of the vector space if and only if one of them is contained in the other. Give an example to show that the union of two subspaces of a vector space may not be a subspace of the vector space.

OR

(a') Define a subspace of a vector space. If W is a subspace of IR^3 generated by $\{(1, 1, -2), (0, -1, 1), (3, -2, -1)\}$ then find the dimension of another sub-space W^1 of IR^3 such that $IR^3 = W \oplus W'$

(b) Let W be a subspace of a vector space V over a field F. If V is finit dimensional, then prove that

$$\dim (V/W) = \dim V - \dim W$$

OR

(b') Let W be the subspace of $V = M_2(IR)$ consisting of the diagonal matrices in $M_2(IR)$. Find a basis of V/W.

2. (a) State and prove the fundamental theorem of vector space homomorphisms.

OR

(a') Define rank and nullity of a linear transformation, Let T be a linear operator on a finite dimensional vector space V such that rank (T^2)=rank (T). Prove that range (T) $\cap$ Kernel (T)=0.

(b) Define the transpose of a linear trasformation. Prove that the rank of the transpose of a linear transformation T between finite dimensional vector spaces over the same field is equal to the rank of T.

OR

(b') Let V_1 and V_2 be vector spaces over the same field F and $T : V_1 \to V_2$ be a subjective linear transformation. Suppose that W is a subspace of V_2 of dimension 2. If $\dim V_1 = 7$ and $\dim V_2 = 5$. then find $\dim T^{-1}(W)$.

3. (a) Find the inverse of the matrix :

$$A=\begin{bmatrix}1 & 1 & 3\\ 2 & 3 & 4\\ 3 & 4 & 6\end{bmatrix}$$

and write A^{-1} as the product of elementary matrices.

OR

(a') Reduce the matrix :

$$A=\begin{bmatrix}2 & 3 & -1 & -1\\ 1 & -1 & -2 & -4\\ 3 & 1 & 3 & -2\\ 6 & 3 & 0 & -7\end{bmatrix}$$

to normal form and find its rank.

(b) Define the adjoint of a square matrix. If A and B are non-singular matrices of same order over the same field F. Then prove that adj(AB) = (adj) B (adj A).

OR

(b') Let the rank of the coefficient matrix of a non-homogeneous system of m linear equations in n variables be p. If the system is consistent, the show that the number of linearly independent solutions of the system is n-p+1.

4. (a) State and prove the Bessel's inequality for an inner product space.

(a') Use Gram-Schmidt process to orthonormalize the basis $\{(1,0,1),(1,0,-1),(0,3,4)\}$ of IR^3 with the standard inner product.

(b) Let W be a suspace of a finite dimensional inner product space V. Prove that $V=W\oplus W^{\perp}$.

OR

(b') Define eigen values and eigen vectors of a square matrix. Prove that eigen values of a Hermitlan matrix are real.

5. (a) Let n be a Positive integer and let V be the vector space of all square matrices of order n over the field of real numbers. Show that the equation f(A,B)=n trace (AB)-trace(A) trace (B) defines a bilinear form f on V. Is f symmetric ? Support your answer.

OR

(a') If $A=\begin{bmatrix}2 & 1 & 1\\ 1 & 2 & 1\\ 1 & 1 & 2\end{bmatrix}$, then find an orthogonal matrix P such that P^{-1}AP is a diagonal matrix.

(b) Prove that every quadratic form q on a real vector space V determines a unique symmetric bilinear form f_s on V given by :

$$f_S(x,y)=\frac{1}{2}\left[q(x+y)-q(x)-q(y)\right]\ \forall x,y\in V.$$

(b') Reduce the equation:
$x^2-z^2-4xy+4yz+6=0$ to standard form by suitable orthogonal change of variables and identify the surface represented by it.

MATHEMATICS (Paper : Second)- 2018
(Analysis)

Time : Three Hours] [Maximum Marks : 50

Note : Attempt all questions. All questions carry equal marks. Symbols used have their usual meaning.

1. (a) Prove that the series

$$\sum_{n=1}^{\infty}\frac{1}{n^p}$$

is convergent for $p>1$ and divergent for $p\le 1$.

Let $\sum_{n=1}^{\infty}u_n$ be a series of positive terms and $\lim_{n\to\infty} n\log\frac{u_n}{u_{n+1}}=l$

Prove that the series $\sum_{n=1}^{\infty}u_n$

is convergent or divergent according as $l>1$ or $l<1$ respectively.

(b) Test the convergence of the following series :

$$1+\frac{2^2}{3^2}+\frac{2^2}{3^2},\frac{4^2}{5^2}+\frac{2^2}{3^2},\frac{4^2}{5^2}+\frac{6^2}{7^2}+\ldots\ldots\ldots$$

OR

Test the convergence of the following series :

$$\sum_{n=1}^{\infty}\left(\sqrt{n^4+1}-\sqrt{n^4-1}\right)$$

2. (a) State and prove Leibnitz's test for convergence of an alternating series

OR

Prove that if the terms of an absolutely convergent series are rearranged, the series remains convergent and its sum is unaltered.

(b) Show that the sum of the series

$$1-\frac{1}{2}-\frac{1}{4}+\frac{1}{3}-\frac{1}{6}-\frac{1}{8}+\frac{1}{5}-\frac{1}{5}-\frac{1}{10}-\frac{1}{12}+\ldots\ldots\ldots$$

is half the sum of the series

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\frac{1}{7}-\frac{1}{8}+\ldots\ldots\ldots$$

OR

Prove that $\sum_{k=1}^{\infty}\sin kx$

has bounded partial sum, for each $x \in \mathbb{R}$. Determine all real numbers x for which $\sum_{n=1}^{\infty}\left(1+\frac{1}{2}+\ldots\ldots\ldots+\frac{1}{n}\right)\frac{\sin nx}{n}$, is convergent.

3. (a) Let f and g be bounded real valued functions defined on [a,b]. Prove that $\int_a^b (f+g)(x)\,dx \le \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$.

OR

Prove that a bounded function $f:[a,b]\to\mathbb{R}$ which has only a finite number of points of discontinuity in [a,b] is Riemann integrable in [a,b].

(b) State and prove the fundamental theorem of integral calculus.

OR

If $\int_a^b f(x)\,dx$ and $\int_a^b \phi(x)\,dx$ both exist and $\phi(x)$ keeps the same sign throughout the interval of integration, prove that there exists a number μ lying between the bounds of f such that

$$\int_a^b f(x)\phi(x)\,dx = \mu\int_a^b \phi(x)\,dx$$

4. (a) Test the convergence of the integral $\int_0^{\pi}\frac{\cos x}{1+x^2}dx$

OR

Let ϕ be bounded and monotonic in $[a,\infty]$ and let $\int_a^r f(x)\,dx$ be convergent. Then prove that $\int_a^x f(x)\phi(x)\,dx$ is convergent.

(b) Show that $\int_a^{\pi}\frac{\sin x}{x}dx$ is convergent.

OR

Show that the integral $\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$ is convergent if m>0, n>0.

5. (a) Define a metric space. Let (X,d) be a metric space and let M be a positive real number. Let $d^*(x,y)=\frac{Md(x,y)}{1+d(x,y)}\forall x,y\in x$

Prove that (X, d*) is also a metric space.

OR

Let (X, d) be a metric space and let A be any subset of X. Let $\overset{0}{A}$ denote the interior of A. Prove that

(i) $\overset{0}{A}$ is an open set.

(ii) $\overset{0}{A}$ is the largest open set contained in A.

(iii) A is open if and only if $\overset{0}{A}$ =A.

(b) Prove that a subset of a metric space is closed if and only if its complement is open.

OR

Define a convergent sequence in a metric space, prove that a convergent sequence in a metric space is bounded and its limit is unique.

MATHEMATICS (Paper : Third)-2018
(Mechanics)

Time : Three Hours] [Maximum Marks : 50

Note : Answer all questions. Each question carries equal marks. Symbols have their usual meaning.

1. (a) State the principle of virtual work and enumerate the forces that may be omitted in forming the equation of virtual work giving the reasons for their omission.

OR

A smooth sphere of radius r and weight w rests in a horizontal circular hole of radius a. A string is wrapped once round the sphedre about the hole and then pulled tight. What must be the tension in the string so that it may just raise the sphere ?

(b) A heavy string of unifor density and thickness is suspended from two given points in the same horizontal plane. A weight, one nth that of the string, is attached to its lowest point; show that if θ, ϕ be the inclinations to the verticle of the tangents at the highest and the lowest points of the string, $\tan\phi = (1+n)\tan\theta$

OR

Two rods, each of length 2a, have their ends united at an angle α, and are placed in a verticle plane on a sphere of radius r. Prove that the equilibrium is stable or unstable according as $\sin\alpha > or < \frac{2r}{a}$

2 (a) Using vector method, obtain radial and transverse components of acceleration of a particle moving in a plane curve.

OR

A point moves on a curve with constant tangential acceleration so that the magnitudes of a tangential velocity and the normal acceleration are in a constant ratio; find the intrinsic equation of the curve.

(b) If a rod, which always passes through the origin, rotates with uniform angular velocity ω, white one end describes the curve $r = a + be^{\theta}$, show that the radial acceleration of any point of the rod is the same at every instant and the radial velocity is the same at every point at a given instant of time.

OR

A point describes a path with an acceleration which is always directed towards a fixed point O, and varies directly as the distance from it Initially, it is projected from a point A with a velocity V in a direction at right angles to OA. find the path of the particle.

3. (a) A particle, whose mass is m, is acted upon by a force $m\mu\left(x + \frac{a^4}{x^3}\right)$ towards the origin. If it starts from rest at a distance a, show that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\pi}}$.

OR

If h be the height due to the velocity v at the earth's surface supposing its attraction constant and H the corresponding height when the variation of gravity is taken into account, prove that $\frac{1}{h} - \frac{1}{H} = \frac{1}{r}$, where r is the earth's radius.

(b) A body is attached to one end of an inextensible string, and the other end moves in a verticle line with simple harmonic motion of amplitude a, making n complete oscillations per second. Show that the string will not remain tight during the motion unless $n^2 < \left[g/\left(4a\pi^2\right)\right]$

OR

A particle moving in a straight line is subject to a resistance which produces the retardation kv^2, where v is the velocity and k is a constant. Show that v and to (the time) are given in terms of s (the distance) by the equations $V = \frac{u}{1+ksu}, t = \frac{s}{u} + \frac{1}{2}ks^2$, where u is the initial velocity.

4. (a) A heavy particle, hanging vertically from a fixed point by a light inextensible string of length l, is struck by a horizontal blow which imparts to it a velocity $2\sqrt{(gl)}$; prove that the string becomes slack when the particle has risen to a height $\frac{2}{3}$ above the fixed point.

OR

A heavy particle is placed at the highest point of a smooth verticle circle of radius a, and is allowed to slide down starting with a negligible velocity. If the particle leaves the circular path, find the latus rectum of the parabola subsequently described.

(b) Show that a cycloid can be a free path for a particle acted on at each point by a constant force parallel to the corresponding radius of the generating circle, this circle being placed at the vertex.

OR

A particle is allowed to fall down the arc of a smooth cycloid, whose axis is vertical and vertex upwards. It starts with a negligible velocity from the highest point. Prove that it will leave the curve, which it has fallen through half the verticale height of the cycloid.

5. (a) Obtain the law of central attraction for which the velocity in a circle at any distance is equal to the velocity in falling from infinity to that distance.

OR

A particle, subjected to a force producing acceleration $\mu\frac{(r+2a)}{r^5}$ towards the pole, is projected from the point (a,0) with a velocity equal to the velocity from infinity at an angle $\cot^{-1}2$ with the initial line. Show that the equation to the path is $r = a(1 + 2\sin\theta)$

(b) Obtain the differential equation to the path of a particle moving in a plane under a central attractive acceleration f. Also, explain the terms 'apse' and 'apse lines'.

OR

A particle describes an ellipse under a force $\frac{\mu}{(\text{distance})^2}$ towards the focus. If it was projected with velocity V from a point distant r from the centre of force, show that its periodic time is :

$$\frac{2\pi}{\sqrt{\mu}}\left[\frac{2}{r}-\frac{V^2}{\mu}\right]^{-3/2}$$

B.Sc. (Part-II) Examination, 2019

Three-year Degree Course

MATHEMATICS (FIRST PAPER)

(Linear Algebra)

Time : Three Hours] **[Maximum Marks : 50**

Note : Attempt all the questions. All questions carry equal marks, Symbols used have their usual meaning.

1. (a) Show that the vector space $\mathbb{R}^3$ over $\mathbb{R}$ is the sum of its subspaces $W_1 = \{(x, y, z) \in \mathbb{R}^3 : x + 2y + z = 0\}$ and $W_2 = \left\{(x, y, z) \in \mathbb{R}^3 : \frac{x}{-1} = \frac{y}{1} = \frac{z}{1}\right\}$. Is this sum a direct sum ? Justify your answer.

OR

Let V be a vector space over a field F and $S = \{v_1, v_2, \ldots\ldots v_n\}$ be a subset of V with each $v_i \neq 0 (1 \leq i \leq n)$. Prove that S is linearly dependent over F if and only if $\exists k (2 \leq k \leq n)$ such that is a linear combination of $v_1, v_2, \ldots\ldots v_{k-1}$.

(b) Determine the dimension of the subspace W of $\mathbb{R}^4$ generated by $S = (1, 1, 2, 0), (1, -1, 2, 0), (0, 1, 2, 1)$

OR

For the subspace $W = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1 + x_2 = 0, x_3 + x_4 = 0\}$ of the vector space $\mathbb{R}^4$ over $\mathbb{R}$, give a basis of the quotient space $\mathbb{R}^4 / W$.

2. (a) Define the Kernel of a linear transformation. Let V_1 and V_2 be vector spaces over the same field and $T : V_1 \to V_2$ be a linear transformation. Let W be a subspace of V_1. Prove that $T^{-1}(R(W)) = KerT + W$.

OR

Let V_1, V_2 and V_3 be finite dimensional vector spaces over the same field. $T_1 : V_1 \to V_2$, $T_2 : V_2 \to V_3$ be linear transformations. Prove that $T_2 \circ T_1 : V_1 \to V_3$ is a linear transformation and $rank(T_2 \circ T_1) \leq \min\{rank(T_1), rank(T_2)\}$.

(b) Define the dual space of a vector space. Find the dual basis of the dual space of $\mathbb{R}^3$ relative to the ordered basis $S = \{(1, 1, 0), (1, 2, 0), (1, 1, 1)\}$ of $\mathbb{R}^3$.

OR

Define the annihilator of a non-empty subset of a vector space. If V is a finite dimensional vector space over a field F and W is a subspace of V, then prove that :
dim W + dim W° = dim V,
where W° is the annihilator of W.

3. (a) Define the row-rank and the column-rank of a matrix. Prove that for any field F and $A \in M_{m\times n}(F)$, (row-rank)(A) = (column-rank)(A).

OR

Define an elementary matrix. Write

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & -2 & 1 \\ 0 & 0 & 2 \end{pmatrix} \in M_3(\mathbb{R})$$ as a product of elementary matrices.

(b) Define the row reduced Echelon form of a matrix. Let

$$A = \begin{pmatrix} 1 & -1 & 2 & 0 \\ 2 & 3 & -1 & 1 \\ -1 & 2 & 0 & 2 \end{pmatrix} \in M_{3\times 4}(\mathbb{R})$$. Find a non-singular matrix $P \in M_3(\mathbb{R})$ such that PA is in row reduced Echelon form.

OR

Define adj(A), the adjoint of a matrix $A \in M_n(F)$, where F is a field and prove that A. adj (A) = det (A) I_n = adj(A). A.

4. (a) Let S = $\{(1, 1, 0), (1, 3, 0), (1, 1, 1)\} \subseteq \mathbb{R}^3$. Use Gram-Schmidt process to orthonormalize S.

OR

Let V be a finite dimensional inner product space and $T \in V^*$. Prove that there exists a unique $y \in V$ such that $T(x) = \langle x, y \rangle \forall x \in V$.

(b) Let F be a field and $A \in M_n(F)$. Define the trace of A. If $B \in M_n(F)$ is similar to A, then prove that A and B have same trace. Is the converse true ? Justify your answer.

OR

Let $A \in M_n(\mathbb{C})$. Prove that if A is Hermitian, then each eigen value of A is real. Is the converse true ? Justify your answer.

5. (a) Let V be a vector space of dimension $n \in \mathbb{N}$ over a field F. Let $T : V \to V$ be a linear transformation. If T has n distinct eigen values, then prove that T is diagonalizable. Is the converse true ? Justify your answer.

OR

Let $A \in M_n(\mathbb{R})$ be a symmetric matrix. Prove that there exists an orthogonal matrix $P \in M_n(\mathbb{R})$ such that $P^t AP$ is a diagonal matrix.

(b) Prove that every bilinear form on a vector space over $\mathbb{R}$ can be uniquely expressed as the sum of a symmetric and a skew symmetric bilinear forms.

OR

Reduce the equation $2y^2 + 2x + 4y + 4 = 0$ to standard form by a suitable orthogonal change of variable and identify the surface represented by it.

MATHEMATICS (Second Paper) - 2019

(Analysis)

Time : Three Hours] **[Maximum Marks : 50**

Note : Attempt all questions. Each question carries equal marks. Symbols have their usual meaning.

1. (a) Let $\sum_{n=1}^{\infty} a_n$ be a convergent series of positive real numbers. Show that $\sum_{n=1}^{\infty} \frac{\sqrt{a_n}}{n^p}$ is convergent if $p > \frac{1}{2}$. Give a counter example for $p = \frac{1}{2}$.

OR

Prove that a series of positive real numbers is convergent if and only if the sequence of its partial sums is bounded above.

(b) State and prove the root test for convergence of series of positive real numbers.

OR

Test the convergence of following series :

$$1^p + \left(\frac{1^p}{2}\right) + \left(\frac{1.3}{2.4}\right)^p + \left(\frac{1.3.5}{2.4.6}\right)^p + \ldots$$

2. (a) Define an absolutely convergent series. Prove that every absolutely convergent series is convergent. Is the converse true ? Support your answer.

OR

Let $\{a_n\}$ be a decreasing sequence of positive real numbers converging to 0 and let $b_n = \frac{a_1 + a_2 + \ldots + a_n}{n}$ for all n. Show that $\sum_{n=1}^{\infty}(-1)^n b_n$ is convergent.

Is it absolutely convergent ? Support your answer.

(b) State and prove Dirichlet's test for convergence of series of real numbers.

OR

Define the Cauchy product of two series. If $\sum_{n=0}^{\infty} a_n$ is absolutely convergent and $\{r_n\}_{n\geq 0}$ is a sequence converging to 0, show that $\lim_{n\to\infty} \gamma_n = 0$, where $\gamma_n = \sum_{k=0}^{n} a_k r_{n-k}$ for all non-negative integers n.

3. (a) Prove that every step function $f:[a, b]\to\mathbb{R}$ is bounded and Riemann integrable on $[a, b]$. Is the converse true ? Support your answer.

OR

Let $f:[a, b]\to\mathbb{R}$ be a function such that f is Riemann integrable on $[c, b]$ for all c with $a < c < b$. Prove that f is Riemann integrable on $[a, b]$.

(b) State the second mean value theorem of Riemann integration. Let $f:[a, b]\to\mathbb{R}$ be a continuous function such that $\int_a^b f(x)dx = 0$ and $f(x)\geq 0$ for all x in $[a, b]$. Show that f is identically zero on $[a, b]$.

OR

Let $f:[a, b]\to\mathbb{R}$ be a continuous function. Prove that there exists a differentiable function $g:[a, b]\to\mathbb{R}$ such that $g' = f$ on $[a, b]$. Does every Riemann integrable function $f:[a, b]\to\mathbb{R}$ has a primitive on $[a, b]$? Support your answer.

4. (a) Define the convergence of improper integrals of second kind and test the convergence of $\int_1^2 \frac{\sqrt{x}}{\log x}dx$.

OR

State and prove the limit comparison test for improper integral with bounded integrands.

(b) Discuss the convergence and divergence of the integral $\int_0^1 x^m \operatorname{cosec}^n x\, dx$.

OR

State and prove Abel's test for convergence of improper integrals.

5. (a) Define a closed set in a metric space. Prove that intersection of any family of closed sets in a metric space, is a closed set.

OR

Define a limit point of a set in a metric space. Prove that the set of limit points of a finite set in a metric space is empty and deduce that every finite set is closed.

(b) Prove that every convergent sequence in a metric space is a Cauchy sequence. Determine all Cauchy sequences in a discrete metric space.

OR

Define a complete metric space and give an example. Prove that any closed subspace of a complete metric space is complete.

MATHEMATICS (Third Paper) - 2019
(Mechanics)

Time : Three Hours] **[Maximum Marks : 50**

Note : Answer **all** questions. Each question carries equal marks. Symbols have their usual meaning.

1. (a) State and prove the principle of virtual work for a system of coplanar forces acting at different points of a rigid body.

OR

The middle points of the opposite sides of a jointed quadrilateral are connected by light rods of lengths l and l^1. If T and T^1 be the tensions in these rods, prove that

$$\frac{T}{l}+\frac{T^1}{l^1}=0$$

(b) Show that the length of an endless chain which will hang over a circular pulley of radius a so as to be in contact with two-thirds of the circumference of the pulley is

$$a\left\{\frac{3}{\log(2+\sqrt{3})}+\frac{4\pi}{3}\right\}.$$

OR

A uniform rod of length l is attached by smooth rings at both ends to a parabolic wire, fixed with its axis vertical and vertex downwards, and of latus rectum $4a$; show that the angle θ which the rod makes with the horizontal in a slanting position of equilibrium is given by $\cos^2\theta = 2a/l$, and that if this position exists it is stable.

2. (a) A particle is moving in a plane curve. Find components of its acceleration along the tangent and the normal to the curve at any instant.

OR

A point moves along the arc of a cycloid in such a manner that the tangent at it rotates with a constant angular velocity. Show that the acceleration of the moving point is constant in magnitude.

(b) A boat which is rowed with constant velocity U starts from a point A on the bank of a river which flows with a constant velocity V; and it point always towards a point B on the other bank exactly opposite to A. Find the equation of the path of the boat. If $V = U$, show that the path is a parabola whose focus is B.

OR

If the radial and transverse velocities of a point are always proportional to each other and this holds for acceleration also, prove that its velocity will vary as some power of the radius vector.

3. (a) If in a simple harmonic motion u, v, w be the velocities at distances a, b, c from a fixed point on the straight line which is not the centre of force, show that period T is given by the equation

$$\frac{4\pi^2}{T^2}(b-c)(c-a)(a-b)=\begin{vmatrix} u^2 & v^2 & w^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}.$$

OR

A particle, whose mass is m, is acted upon by a force $m\mu\left(x+\frac{a^4}{x^3}\right)$ towards the origin. If it starts from rest at a distance a, show that it will arrive at the origin in time $\pi/\left(4\sqrt{\mu}\right)$.

(b) A particle falls under gravity (supposed constant) in a resisting medium whose resistance varies as the square of the velocity. Find the motion if the particle starts from rest.

OR

A spherical raindrop, falling freely, receives in each instant an increase of volume equal to λ times its surface at that instant. Find the velocity at the end of time t, and the distance fallen through in that time.

4. (a) A particle is free to move on a smooth vertical circular wire of radius r. It is projected from the lowest point with velocity just sufficient to carry it to the highest point. Show that the reaction between the particle and the wire is zero after a time.

$$\sqrt{\frac{r}{g}}\log\left(\sqrt{5}+\sqrt{6}\right).$$

OR

A particle moves under gravity in a vertical circle, sliding down the convex side of a smooth circular arc. If its initial velocity is that due to a fall to the starting point from a height h above the centre, show that it will fly off the circle when at a height $\frac{2}{3}h$ above the centre.

(b) A heavy particle slides down a smooth cycloid starting from rest at the cusp, the axis being vertical and vertex downwards. Prove that the magnitude of the acceleration of the particle is equal to g at every point of its path.

OR

A particle is moving on a smooth curve under gravity and its velocity varies as the distance (measured along the arc) from the highest point. Prove that the curve must be a cycloid.

5. (a) Prove that in every central orbit, the sectorial area traced out by the radius vector to the centre of force increases uniformly per unit of time, and the linear velocity various inversely as the perpendicular from the centre upon the tangent to the path.

OR

A particle moves in an ellipse under a force which is always directed towards its focus; find the law of force, and the velocity at any point of its path.

(b) A particle moves under a central repulsive force, $\left\{=\frac{m\mu}{(\text{distance})^3}\right\}$, and is projected from an apse at a distance a with velocity V. Show that the equation to the path is $r\cos p\theta = a$, and that the angle θ described in time t is

$$\frac{1}{p}\tan^{-1}\left(\frac{pV}{a}t\right), \text{ where } p2=\frac{a^2V^2+\mu}{a^2V^2}.$$

OR

A particle moves in a curve under a central attraction so that its velocity at any point is equal to that in a circle at the same distance and under the same attraction; show that the law of force is that of the inverse cube and that the path is an equiangular spiral.

B.A./B.Sc. (Part-II) Examination-2022-23

MATHEMATICS

(FIRST PAPER) (Linear Algebra)

Time Allowed : Three hours] *[Maximum Marks : 50*

Note : Attempt all questions. Marks allotted to each question are indicated against it.

1.(a) Define a maximal linearly independent subset of a vector space over a field. If $S=\{a\ a_2.....a_n\}$ is a subset of a vector space V over a field F, then prove that the following statements are equivalent:

I : S is linearly independent and S generates V.

II : S is a maximal linearly independent subset of V. [1+2+2=5]

OR

(a') Define a basis of a vector space. Let $W_1=\{(x,y,0): x,y\in R\}$ be a subspace of $R^3(R)$ and a subspace W_2 of $R^3(R)$ be generated by $\{(1,2,3),(1,-1,1)\}$. Then find a basis of $W_1\cap W_2$. [1+4=5]

(b')Find a basis of the subspace.

$W_1=\{(x_1,x_2,x_3,x_4,x_5)\in R^5 : x_1+x_2+x_3+x_4+x_5=0\}$ of the vector space $R^5(R)$. Also find a subspace W_2 of $R^5(R)$ such that $R^5=W_1\oplus W_2$.

[2 ½ + 2 ½ = 5]

OR

(b') (i) Find the equation of the coset $W+(\alpha,\beta,\gamma)$ Where $W=\{(x,y,0): x,y\in R\}$ is a subspace $R^3(R)$.

(ii) If W is a subspace of a finite dimensional vector space V over the field of residue-classes modulo 5 such that dim V = 5, dim W= 2, find the total number of cosets of W in V. [2]

2.(a) Let V_1 and V_2 be two finite dimensional vector spaces over the same field F and $T: V_1\to V_2$ be a linear transformation. Prove that T is an isomorphism if and only if given an ordered basis $S=\{a_1,a_2,.....a_n\}$ of V_1 $T(S)=\{T(a_1),T(a_2),.....,T(a_n)\}$ is an ordered basis of V_2. [5]

OR

(a') Define the kernel of a linear transformation between two vector spaces over the same field. Give an example of two distinct linear transformations from R^4 to R^4 which have the same kernel and image. [1+4=5]

(b) Define the annihilator of a non-empty subset of a vector space. If W is a subspace of R^4 (R) spanned by {1, 2, -3, 4), (0, 1, 4, -1)}, find a basis of the annihilator of W. [1+4=5]

(b') Define the transpose of a linear transformation and prove that the rank of the transpose of a linear transformation is equal to the rank of the linear transformation. [1+4=5]

3.(a) (i) Suppose that A and B are idempotent matrices of the same order such that $I-(A+B)$ is invertible. Then prove that A and B have the same rank. [2]

(ii) Define trace of a square matrix. If A and B are $m \times n$ and $n \times m$ matrices respectively over a field, then prove that $A.B$ And $B.A$ have the same trace. [1+2=3]

OR

(a') Let W be the subspace of Mn (R) consisting of skew-symmetric matrices. Determine a basis of W. [5]

(b) Let V be the real vector space of all the polynomials in x of degree at most 3. Let D denotes the differential operator on V. If A and B are the matrices of D relative to the ordered bases $X = \{1, 1+x, 1+x^2, 1+x^3\}$ and $Y = \{1, x, x^2, x^3\}$ respectively, then find the matrix C such that $A = C.B.C^{-1}$ [5]

OR

(b') Find the value of k for which the system of linear equations.

$kx + y + z = 1, x + ky + z = k, x + y + kz = k^2$

over the field of real numbers does not have a solution. [5]

4.(a) State Cauchy-Schwarz's inequality in an inner product space. Using this inequality find the maximum value of $x+2y+3z$ on $x^2+2y^2+3z^2=1$ [1+4=5]

(b) Let A be a diagonalizable real matrix of order $n \times n$ having 0 and 1 as its only eigenvalues. If the null space of A has dimension r, prove that:

(i) The characteristic polynomial of A is $\lambda^r (\lambda-1)^{n-r}$ [2]

(ii) The trace and determinant of A are $n-r$ and 0, respectively. [2]

(iii) the rank A is $n-r$.

(b) Define adjoint T^* of a linear transformation T on an inner product space $(V, \langle\ \rangle)$. Find the adjoint of the linear transformation $T: \mathbb{C}^3 \to \mathbb{C}^3$ given by $T[(x,y,z)] = (x + iy-z, ix-y+z, x+y+iz)$ $\forall (x,y,z) \in \mathbb{C}^3$. [1+4=5]

OR

(b') Let W be the subspace of the standard real inner product space R^4 spanned by {(1, 2, 1, -1), (2,1,1,-1)}.

Let $x = (3,2,1,1) \in R^4$. Determine the orthogonal projection of x on W. [5]

5.(a) Prove that the union of two subspaces of a vector space is a subspace of the vector space if and only if one is contained in the other. [5]

OR

(a') If a map $T: R^3 \to R^3$ is defined by

$T[(x,y,z)] = (y+z, z, 0)\ \forall (x,y,z) \in R^3$

such that $T^2(v) \neq (0,0,0); v \in R^3$, then prove that:

(i) T is a linear transformation.

(ii) $S = \{v, T(v), T^2(v)\}$ is a basis of R^3.

Also obtain the matrix of T relative to S. [1 1/2 + 1 1/2 + 2 = 5]

(b) Using Gram-Schmidt orthonormalization process orthonormalize the ordered basis. $\{(0,1,1),(1,0,1),(1,1,0)\}$ of R^3 (R) [5]

OR

(b') (i) Prove that eigenvalues of a Hermitian matrix are all real. [3]

(ii) Prove that eigenvectors of a Hermitian matrix corresponding to distinct eigenvalues of the matrix are orthogonal.

MATHEMATICS (SECOND PAPER), 2022-23
(*Analysis*)

Time : Three hours] **[Maximum Marks : 50**

Note : Attempt all questions. Each question carries equal marks. Symbols have their usual meaning.

1.(a) State and prove Raabe's test for the convergence of a series of positive terms.

OR

State Logarithmic test and test the convergence of the series.

$$1 + \frac{x}{1!} + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \ldots\ldots, x > 0.$$

(b) Test the convergence of series $\sum_{n=1}^{\infty} a_n$ where.

(i) $a_n = \frac{n^2-1}{n^2+1} x^n, x > 0$

(ii) $a_n = (n^3+1)^{1/3} - n$

OR

State and prove Cauchy's root test for the convergence of a series of positive terms.

2.(a) Define an absolutely convergent and conditional convergent series. Show that the series.

$$x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\cdots$$

converges absolutely for all values of x.

OR

Let $\sum_{n=1}^{\infty} b_n$ be a re-arrangement of a series $\sum_{n=1}^{\infty} a_n$ of positive terms which converges to ℓ. Then prove that $\sum_{n=1}^{\infty} b_n$ also converges to ℓ.

(b) State Abel's test for convergence of a series. Test the convergence of the series. $\sum_{n=2}^{\infty} \frac{\left(n^3+1\right)^{1/3} - n}{\log n}$

OR

Show that $\sum_{n=1}^{\infty} (-1)^n \frac{(n+2)}{2^n+5} \cos nx$ is convergent for all real values of x.

3.(a) Prove that if f and g are two bounded and Riemann integrable functions on [a,b], then (f+g) is also Riemann integrable on [a, b]. and

$$\int_a^b f + g\,(x)\,dx = \int_a^b f\,(x)\,dx + \int_a^b g\,(x)\,dx$$

OR

Prove that if f is bounded and Riemann integrable on [a, b] then $|f|$ is also bounded and Riemann integrable on [a, b] and

$$\left|\int_a^b f\,(x)\,dx\right| \le \int_a^b |f\,(x)|\,dx$$

(b) Let function f be bounded and Riemann integrable on [a,b] if φ is an antiderivative of f on [a,b] then prove that

$$\int_a^b f\,(x)\,dx = \varphi\,(b) - \varphi(a)$$

Let f be Riemann integrable on [a, b] and m and M be real numbers such that $m \le f(x) \le M$ for all x in [a, b]. Prove that.

$$m\,(b-a) \le \int_a^b f\,(x)\,dx \le M\,(b-a)$$

4 (a) Test the convergence of the following integrals.

(i) $\int_0^{\infty} \frac{x}{(1+x)^3}\,dx$ (ii) $\int_0^{\pi/4} \frac{1}{\sqrt{\tan x}}\,dx$

OR

Let φ be bounded and monotonic in $[a, \varphi[$ and $\varphi(x)$ tends to 0 as $x \to \infty$ and $\int_a^t f\,(x)\,dx$ be bounded for all $t \ge a$. Prove that $\int_a^{\infty} f\,(x)\,\varphi\,(x)\,dx$ converges.

(b) State comparison test for the convergence of improper integrals. Show that $\int_a^b \frac{dx}{(x+a)^n}$ converges if and only if $n<1$.

OR

Test the convergence of the integral $\int_0^1 \frac{dx}{\sqrt{x(1-x)}}$

5.(a) Show that the following series is convergent.

$$\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-3}+\ldots\ldots\ldots$$

OR

Test the convergence of the series.

$$\frac{1}{1p}-\frac{1}{2p}+\frac{1}{3p}-\frac{1}{4p}+\ldots\ldots \text{for } p>0$$

(b) Compute $\int_{-1}^{1} f(x)\,dx$, where $f(x)=|x|$

OR

Show that $\int_0^{\infty}\frac{x\tan^{-1}x}{(1+x^4)^{1/3}}dx$ is divergent.

MATHEMATICS (THIRD PAPER), 2022-23
(*Mechanics*)

Time : Three hours] **[Maximum Marks : 50**

Note : Answer all questions selecting any two parts from each question. Each question carries equal marks. Symbols have their usual meaning.

1.(a) What do you understand by a common catenary? Explain. For a common catenary, prove that $s = c\sinh\frac{x}{c}$

(b) If a chain is suspended from two points B, C on the same level, and the depth of the middle point below BC is $\frac{l}{n}$, where $2l$ is the length of the chain, show that the horizontal span BC is equal to

$$l\left(n-\frac{1}{n}\right)\log\frac{n+1}{n-1}$$

(c) A string of length l is attached to a fixed point A, and the other end B is pulled with a force Wa inclined at an angle α to the horizon. W being the weight per unit length of the string. Show that the vertical distance of A above B is.

$$\sqrt{(l^2+a^2+2la\sin\alpha)}-a$$

2.(a) A point moves in a straight line towards a centre of force μ /(distance)^3 starting from rest at a distance a from the centre of force; show that the time of reaching a point distant b from the centre of force is $\left[\left\{a\sqrt{\left(a^2-b^2\right)}\right\}/\sqrt{\mu}\right]$ and that is velocity then is $\left[\left\{\sqrt{\mu}\sqrt{\left(a^2-b^2\right)}\right\}/ab\right]$

(b) A heavy particle of mass m is attached to the end of an elastic string of natural length l, whose other end is fixed at O. The particle is let fall from rest at O. Show that the part of the motion is simple harmonic, and that, if the greatest depth of the particle below O is $l\cot^2\left(\frac{\theta}{2}\right)\left(0<\theta<\frac{\pi}{2}\right)$, then the modulus of elasticity is $\frac{1}{2}mg\tan^2\theta$.

(c) If h be the height due to the velocity v at the earth's surface supposing its attraction constant and H the corresponding height when the variation of gravity is taken into account, prove that.

$\frac{1}{h}-\frac{1}{H}=\frac{1}{r}$, where r is the earth's radius.

3.(a) A particle moves along a circle $r = 2a\cos\theta$ in such a way that its acceleration towards the origin is always zero. Show that its transversal acceleration varies as the fifth power of cosec θ.

(b) A curve is described by a particle having a constant acceleration inclined at a constant angle to the tangent; find the curve.

(c) A body rotates with uniform angular acceleration α. If ω is the angular velocity when the body has turned through an angle θ from rest, show that $\omega^2 = 2\alpha\theta$.