

## SYLLABUS

1. Cardinal points of coaxial optical systems. Simple problems on combination of thin lenses, Eyepieces, Applanatic points.
2. Nature of light, Elementary ideas of electromagnetic wave and photon theories of light. Complex representation of waves and its application (to be used in theory of various phenomena).
3. Conditions for observing interference, Degree of coherence and visibility of fringes, Production of interference fringes and determination of wavelength, Michelson interferometer and its uses, Colour of thin films, Newton's Ring. Theory of Multiple Reflections, FP Etalon.
4. Temporal and spatial coherence. Michelson Stellar interferometer. Stimulated emission, Basic ideas about laser emission, Ruby and He-Ne laser as example.
5. Fresnel's theory of diffraction, Half-period elements. Diffraction from circular obstacle and aperture (Elementary theory), Zone Plate, Cornu's Spiral, Fresnel diffraction by straight edge and single slit.
6. Fraunhofer's diffraction by single slit and double slit. Theory of planck grating, width of principal maxima. Rayleigh's criterion of resolution, resolving power of prism, grating and FP etalon. Limit of resolution for telescope. Concave grating (elementary theory), and its mountings.
7. Unpolarised, polarised and partially polarised lights. Polarisation by reflection, Double refraction by uni-axial crystals, polarised, Huygen's theory of double refraction. Half and quarter waveplates. Production of elliptically polarised light.
8. Babinet Compensator, Analysis of elliptically polarised light by using a Nicol and a quarter wave plate, and by using Babinet compensator. Optically activity. Fresnel's theory of optical rotation, specific rotation, Biquartz and Laurent's half-shade polarimeters.

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### Chapter I

Q.1. Two lenses of focal lengths  $f_1$  and  $f_2$  separated by a distance  $d$ . Find out the focal length of the lens system. (2017)

Other Related System -

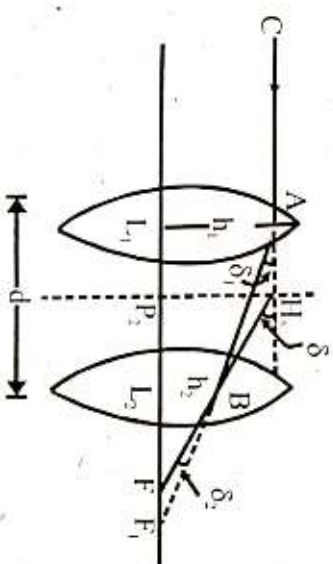
Q. What is an equivalent lens of a two lens system? Two lenses of focal lengths ' $f_1$ ' and ' $f_2$ ' are separated by a distance ' $d$ '. Find out the focal length of lens system. (2017)

Q. Find the expression for equivalent focal length of a system of two coaxial thin lenses having focal length  $f_1$  and  $f_2$  separated by ' $d$ ' distance apart. Hence find the position of principal points. (2018)

Ans. Equivalent Focal Length of Two Thin lenses:-

When two thin lenses are arranged coaxially, the image formed by the first lens system becomes the object for second lens system and the two systems act as a single optical system forming the final image from the original object.

Let us consider a simple optical system that consists of two thin lenses  $L_1$  and  $L_2$ , placed on a common axis and separated by a distance  $d$ , as shown in fig. The lenses are separated by a distance and have focal lengths  $f_1$  and  $f_2$ . We are interested to know how the combination works. We find that two lenses, separated by a finite distance, can be replaced by a single thin lens called an **equivalent lens**. The equivalent lens, when placed at a suitable fixed point, will produce an image of the same size as that produced by the combination of the two lenses. The focal length of equivalent lens is called **equivalent focal length**. We now derive an expression for the equivalent focal length,  $f$ , of the combination of two lenses.



Let a ray CA of monochromatic light parallel to the principal axis be incident on the first lens  $L_1$  at a height  $h_1$  above the axis. The ray CA is deviated through an angle  $\delta_1$  by the lens  $L_1$ . The incident ray CA, after refraction is directed toward  $F_1$ , which is the second principal focus of lens  $L_1$ . Then the deviation produced by the first lens is given by

$$\delta_1 = \frac{h_1}{f_1}$$

The emergent ray, AB, from the first lens is incident on the lens  $L_2$  at a height  $h_2$ . On refraction, the ray is deviated through an angle  $\delta_2$  by the lens  $L_2$  and meets the principal axis at F. Since the incident ray CA is parallel to the principal axis and after refraction through the optical system meets the axis at F, F must be the second principal focus of the combined lens system. The deviation produced by the second lens is given by

$$\delta_2 = \frac{h_2}{f_2}$$

If the incident ray CA is extended forward and the final emergent ray BF backward, they meet at a point  $H_2$ . It is clear that a single thin lens placed at P will produce the same deviation as that produced by the two lenses put together. The lens of focal length  $P_2F$  placed at  $P_2$  is termed as the equivalent lens, which can replace the two lenses  $L_1$  and  $L_2$ . The deviation produced by the equivalent lens is

**Focal Length of the Equivalent lens:-**

Deviation produced by the first lens  $L_1$  is  $\delta_1 = h_1 / f_1$   
Deviation produced by the second lens  $L_2$  is  $\delta_2 = h_2 / f_2$   
But  $\delta = \delta_1 + \delta_2$

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_2}{f_2} \quad \dots\dots(i)$$

The  $\Delta^{s} AL_1F_1$  and  $BL_2F_1$  are similar.

$$\frac{AL_1}{L_1F_1} = \frac{BL_2}{L_2F_1}$$

$$\frac{h_1}{f_1} = \frac{h_2}{f_2 - d} \quad \dots\dots(ii)$$

$$\text{or } h_2 = \frac{h_1(f_2 - d)}{f_1} \quad \dots\dots(iii)$$

Using equation (iii) into equation (i), we get

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1(f_2 - d)}{f_1 f_2} \Rightarrow \frac{1}{f} = \frac{1}{f_1} + \frac{f_2 - d}{f_1 f_2}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \quad \dots\dots(iv)$$

Therefore, the equivalent focal length is given by

$$f = \frac{f_1 f_2}{f_1 + f_2 - d} \quad \dots\dots(v)$$

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$$\text{or } f = \frac{-f_1 f_2}{\Delta} \quad \dots\dots(vi)$$

where  $\Delta = d - (f_1 + f_2)$  and is known as the **optical interval** between the two lenses. It is numerically equal to the distance between the second principal focus of the first lens and the first principle focus of the second lens.

From figure it may be seen that the rays converge at F and appear to have come from a plane  $EP_2$ . Plane  $EP_2$  is therefore, the position of a single lens, the effect of which is the same that of the two lenses,  $L_1$  and  $L_2$ . The distance  $L_2F$  is the equivalent focal length. The equivalent focal length is independent of the direction from which the light enters the system. The behaviour of the system cannot be reproduced in all respects by the equivalent thin lens placed in a fixed position. It will have to be placed in the position  $H_2P_2$  when light enters from left and in position  $H_1P_1$  when light enters from the right.

**Note:-**

(i) Equation (v) shows that if the distance  $d$  between the two convex lenses exceeds the sum of their focal lengths ( $f_1 + f_2$ ), the system becomes divergent, because of negative focal length.

(ii) If the medium between the two convex lenses is other than air, then the equation (v) for equivalent focal length would become

$$f = \frac{f_1 f_2}{f_1 + f_2 - d / \mu}$$

where  $\mu$  is the refractive index of the medium.

**Q2. Discuss the Ramsden's Eyepiece briefly.**

**Other Related Questions:-**

**Q. Discuss the relative merits and demerits of Ramsden's eye piece.**

(2013)

**Ans. Ramsden Eyepiece:-**

Ramsden's eyepiece consists of two plano - convex lenses each of focal length  $f$  separated by a distance equal to  $(2/3)f$ . The lenses are kept with their curved surfaces facing each other, as shown in Figure, thereby reducing spherical aberration. The field lens is a little larger than the intermediate image and is placed close to this image to allow as much light as possible to pass through it. The eye lens has a smaller diameter but carries out the actual magnification

**Theory:-**

The objective forms the real inverted image I of a distant object. This serves as an object for the field lens, which gives rise to a virtual image  $I_1$ .  $I_1$  in turn serves as an object for the eye lens, which gives the final image at infinity, because  $I_1$  is made to lie at its principal focus.

**Equivalent focal length:-**

The equivalent focal length of the eyepiece can be found as follows. If

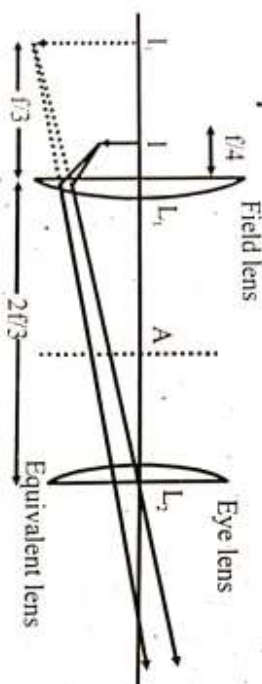
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F denotes the focal length of the equivalent lens, then

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\therefore \frac{1}{F} = \frac{1}{f} + \frac{1}{f^2} \times \frac{2}{3f} = \frac{2}{3f} + \frac{1}{f} = \frac{2+3}{3f} = \frac{5}{3f}$$

$$\therefore F = 3f/4$$



The equivalent lens of focal length  $3f/4$  must be placed behind the field lens at a distance

$$\alpha = \frac{F \times d}{f} = \frac{(3/4)f \times (2/3)f}{f} = \frac{f}{2}$$

Thus the equivalent lens lies between the field lens and the eye lens.  
**Position of the cross-wires:-**

The cross-wires should be placed at the position of I. Now, the position of I relative to field lens can be found as follows. If the final image is to be formed at infinity, the image I should lie in the focal plane of the equivalent lens. In other words, the distance AI should be equal to  $F = 3f/4$ . Since  $AL_1 = f/2$ ,  $L_1 = f/2$ . Therefore, the objective should produce the image at a distance of  $f/4$  in front of the field lens. A fine scale may be placed here if it is desired to measure the size of the image. Since the scale and image would be magnified equally, the measurement would be trustworthy.

**Merits and Demerits:-**

- The field of view of this eyepiece is fairly wide.
  - It is not entirely free from chromatic aberration since the distance between the two lenses is not equal to half the sum of their focal lengths. However, chromatic aberration is minimised by using an achromatic combination both for the field lens and the eye lens.
  - Spherical aberration is minimised by using two plano-convex lenses thereby spreading deviation over four surfaces.
- Ramsden's eyepiece is used partially in all instruments where measurement of the size of the final image are to be made. This eyepiece is sometimes preferred to an positive eyepiece because a real image is formed by the objective

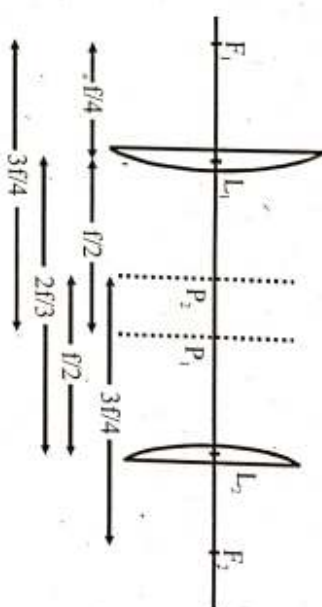
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in front of the field lens, which further acts as a real object for the eye lens.

**Cardinal points of Ramsden eyepiece:-**  
**Position of the principal planes:-**

The first principal plane of the equivalent lens lies at  $P_1$  behind the field lens at a distance  $\alpha$  given by the relation

$$\alpha = \frac{F \times d}{f} = \frac{(3/4)f \times (2/3)f}{f} = \frac{f}{2}$$



Thus, the equivalent lens is placed at  $P_1$  in between the field lens F and the eye lens E at a distance  $f/2$  behind the field lens, as shown in Fig. The point  $P_1$  where the principal plane cuts the axis is the first principal point.

The second principal plane is at a distance  $\beta$  from the eye lens  $L_2$ .

$$\beta = \frac{d \times F}{f_1} = \frac{-\frac{2}{3}f \times \frac{3}{4}f}{f} = -\frac{f}{2}$$

The second principal plane lies behind the eye lens  $L_2$  at a distance  $f/2$ . The point  $P_2$  where the principal plane cuts the axis is the second principal point.

**Nodal Points:-**

As the lens system is situated in air, the nodal points coincide with the principal points. Thus,  $P_1$  and  $P_2$  are the nodal points,  $N_1$  and  $N_2$  respectively.

**Position of Focal points:-**

The equivalent focal length of the eyepiece is  $3f/4$ . Hence the first focal point  $F_1$  lies at a distance  $3f/4$  from the first principal point  $P_1$ . That is at a distance of  $\left\{ \frac{3}{4}f - \frac{1}{2}f \right\} = \frac{1}{4}f$  to the left of the field lens  $L_1$ .

The second focal point  $F_2$  lies at a distance  $3f/4$  from the second principal point  $P_2$  i.e., at a distance  $f/4$  behind the eye lens  $L_2$  away from the field lens.

**Q.3. Explain briefly the Huygens eyepiece.**

**Other Related Questions:-**

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Q. Describe lense arrangement in Huygen's eyepiece explaining the considerations which lead to the used ratio of focal length. If  $F'$  is second focal length of the eye piece, find the focal lengths of lenses, their separation and positions of all cardinal points in terms  $F'$  and show these in diagram. Why is this eye-piece called negative eye-piece. (2015)

Q. Deduce the positions of the cardinal points of a Huygen's eyepiece and indicate them on a diagram. Also, find the position of the cross-wire. Why this eye piece can not be used to see your palm? (2013)

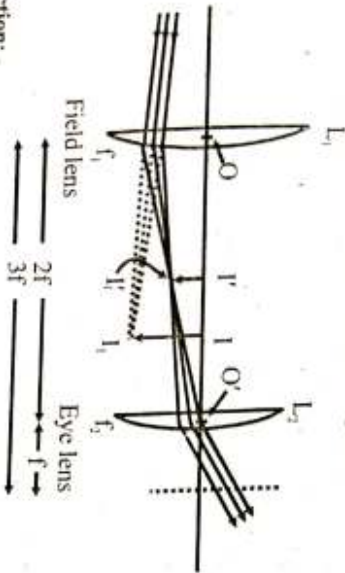
Q. Discuss the relative merits and demerits of Huygen's eye piece. (2013)

Q. Describe construction of Huygen's eye piece. Find position of cardinal points for a Huygen's eye piece of given focal length and show them in a diagram. Explain why this eyepiece cannot be used for measurement. (2011)

Q. The focal length of two lenses of Huygen's eye piece are  $3F$  and  $F$ . Find the positions of the principal point. (2014, 19)

Ans. Huygens Eyepiece:-

In the Huygens eyepiece a converging beam enters the field lens and forms a virtual image before the eye lens. The need for converging beam implies that this eyepiece does not act like a simple magnifier.



Construction:-

Huygen's eyepiece consists of two lenses having focal lengths in the ratio 3:1 and the distance between them is equal to the difference in their focal lengths. The focal lengths and the positions of the two lenses are such that each lens produces an equal deviation of the ray and the system is achromatic. This eyepiece is free from chromatic as well as spherical aberrations as it satisfies the two conditions simultaneously.

1. The lens combination acts as an achromatic system if  $D = \frac{f_1 + f_2}{2}$  where  $D$  is the distance between the lenses and  $f_1$  and  $f_2$  are the focal lengths of the two lenses.

2. The lenses produce equal deviation of the incident ray when the distance between the two lenses is equal to  $(f_1 - f_2)$ .

The field and the eye lenses used are plano-convex and are placed with their convex surfaces towards the incident ray. In this way, the total deviation

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due to the combination is divided into four parts which makes the combination to have minimum spherical aberration.

If spherical and chromatic aberration are to be minimised simultaneously, the following condition is to be satisfied.

$$D = \frac{f_1 + f_2}{2} \quad \text{Chromatic aberration}$$

$$D = f_1 - f_2 \quad \text{Spherical aberration}$$

Combining the two conditions, we obtain

$$\frac{f_1 + f_2}{2} = f_2 - f_1$$

$$\text{or } f_2 = 3f_1$$

$$\therefore D = 3f_1 - f_1 = 2f_1$$

To satisfy the conditions for minimum chromatic and spherical aberrations, the focal length of the field lens should be three times the focal length of the eye lens and the distance between them should be equal to twice the focal length of the eye lens. Huygen's eyepiece is constructed on this principle.

Theory:-

The objective forms an image, which serves as a virtual object for the field lens. The field lens forms real inverted image  $I'$ . If this image is situated at the principal focus of the eye lens, then the final image is at infinity.

Equivalent Focal Length:-

The equivalent focal length of the eyepiece can be found as follows. If  $F$  is the equivalent focal length of the eyepiece, then it is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{3f} - \frac{2f}{f \cdot 3f} = \frac{2}{3f}$$

$$F = 3f/2$$

The equivalent lens lies behind field lens at a distance of

$$x = \frac{d \times F}{f} = \frac{2f \times 3f/2}{f} = 3f$$

In other words, the equivalent lens is at a distance of  $3f$ .  $2f - 2f = f$  behind the eye lens.

Position of Cross - wires:-

It is noted above that the principal focus of the equivalent lens lies  $f/2$  ahead of the eye lens. It is here that the image due to objective must be formed in order that the final image is at infinity. The rays coming from the objective are, however, intercepted by the field lens, thus displacing the image to position

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1. It is here that the cross-wires should be placed.

We further noted above that the image  $I_1$  is at a distance of  $f/2$  from the eye lens or  $3f/2$  from the field lens. Hence, for field lens  $u = -3f/2$ . Then

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$-\frac{2}{3f} + \frac{1}{v} = \frac{1}{f}$$

$$\text{or } v = f$$

In other words,  $I_1$  lies midway between the two lenses and so fixes the position of the cross-wires or scale, if used. Since the image formed by the objective lies behind the field lens (instead of in the front) this eyepiece is sometimes referred to as a negative eyepiece. It may be noted that the image of the scale is formed by the eye lens whereas the final image is produced by both the lenses. Hence the image and the scale would not be magnified equally and so the measurements will not be reliable. The image of the cross-wire or scale would have all the defects of an image formed by a single lens. Hence in instruments using Huygen's eyepieces, scale is not used except when magnification is low.

**Merits and demerits:-**

- (i) The Huygen's eyepiece is fully free from chromatic aberration because the distance between the lenses is equal to half the sum of their focal lengths.
- (ii) Spherical aberration is also minimum because the distance between the two lenses is equal to the difference of their focal lengths.
- (iii) The field of view of this eyepiece is smaller than that of Ramsden's eyepiece.

**Cardinal Points of Huygens Eye piece:-**

**Position of Principal Points:-**

The equivalent lens should be placed at a distance  $\alpha$  from the field lens, which is given by

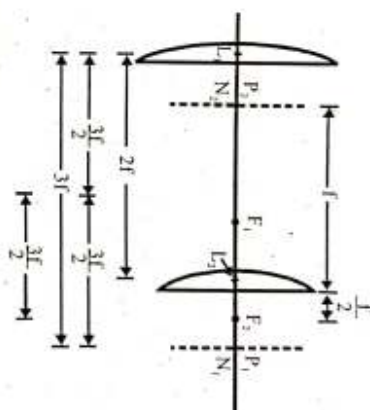
$$\alpha = \frac{d \times F}{f} = \frac{2f \times 3f/2}{f} = 3f$$

Since the distance  $d$  between the field lens  $L_1$  and the eye lens  $L_2$  is  $2f$ , the position of equivalent lens is  $3f$ , i.e., it should be placed away from the eye lens, as shown by dotted line at  $P_1$  in Fig.

The first principal point  $P_1$  lies at a distance  $\alpha = 3f$  from the field lens. The second principal point  $P_2$  lies at a distance  $\beta$  from the eye lens towards the field lens and is given by

$$\beta = -\frac{d \times F}{f} = -\frac{2f \times \frac{3}{2}f}{3f} = -f$$

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**Position of the Focal Points:-**

The first focal point  $F_1$  lies at a distance  $3f/2$  from the first principal point  $P_1$ , i.e., at a distance  $\left\{ \frac{3f}{2} - f \right\} = \frac{f}{2}$  from the eye lens on the side of the field lens.

The second focal point lies at a distance  $3f/2$  from the second principal point  $P_2$ , i.e., at a distance  $\left\{ \frac{3f}{2} - f \right\} = \frac{f}{2}$  from the eye lens away from the field lens.

**Q.4. Define cardinal points of a co-axial two lens system. Show that the distance between two principal points two principal points and two nodal points are equal for a given coaxial two lens system always. (2016)**

**Other Related Questions:-**

**Q. Define principal points and nodal points of two lens system. Prove that the distance between two principal points and two nodal points of a given lens system are always equal. (2014)**

**Ans. Cardinal Points:-**

In the case of refraction through a thin lens, the thickness of the lens has been neglected in calculating the various formulae. It is then immaterial from which point of the lens the distances are measured. But we cannot apply the above approximation for an optical system consisting of a combination of lenses. One way of calculating the position and size of the image formed by an optical system is to consider refraction at each surface of a lens successively, but the method proves to be more tedious. In 1841, Gauss showed that any number of coaxial lenses could be treated as a single unit, without the necessity of treating the single surfaces of lenses separately. 'The lens makers' formula can be applied to the system provided the distances are measured from two hypothetical parallel planes, fixed with reference to the refracting system. The points of intersection of these planes with the axis are called the principal

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points or Gauss points. In fact there are six points in all, which characterize an optical system. They are

- (i) two focal points,
- (ii) two principal points and
- (iii) two nodal points.

These six points are known as cardinal points of an optical system. The planes passing through these points and which are perpendicular to the principal axis are known as cardinal planes. The cardinal points and cardinal planes are intrinsic properties of a particular optical system and determine the image forming properties of the system. If these are known, one can find the image of any object without making a detailed study of the passage of the rays through the system. It is not necessary to consider the refraction of the rays at the various surfaces.

#### Principle Points and Principle Planes:-

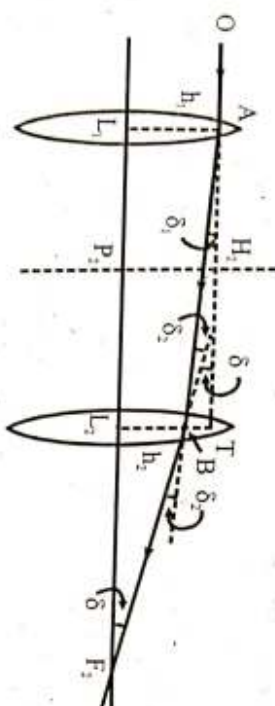


Figure (a)

Consider an optical system having its principal focal points  $F_1$  and  $F_2$ . A ray  $OA$  travelling parallel to the principal axis and incident at  $A$  is brought to a focus at  $F_1$  in the image space of the optical system as shown in figure (a). The actual ray is refracted at each surface of the optical system and follows the path  $OABF_1$ . If we extend the incident ray  $OA$  forward and the emergent ray  $BF_1$  backward, they meet each other within the optical system at  $H_1$ . Now, we can describe the refraction of the incident ray  $OA$  in terms of a single refraction at a plane passing through  $H_1$ . A plane drawn through the point  $H_1$  and perpendicular to the axis may be regarded as the surface at which refraction takes place.

This plane is called the principal plane of the optical system. Thus, the four consecutive deviations of the light rays caused by the four surfaces of the optical system are equivalent to a single refraction at  $H_1$ , taking place at the principal plane. We now define the principal plane of an optical system as the loci where we assume refraction to occur without reference as to where the refraction actually occurs.

$H_2$  is the principal plane in the image space and is called the **second principal plane**. The point  $P_2$  at which the second principal plane intersects the axis, is called the **second principal point**.

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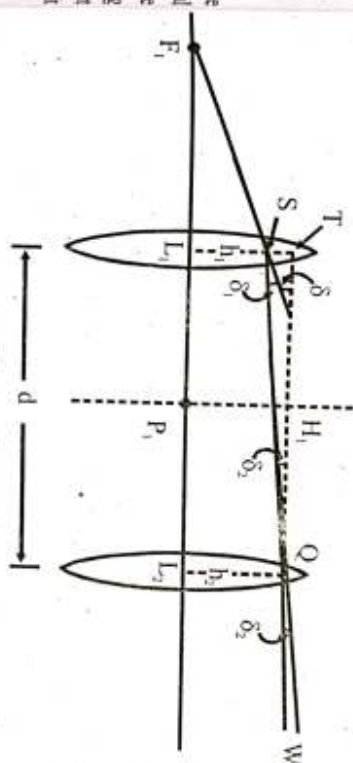


Figure (b)



Figure (c)

By adopting similar procedure, as shown in Fig., we can locate the principal plane  $H_2$  and principal point  $P_2$  in the **object space**. Consider the ray  $F_1S$  passing through the first principal focus  $F_1$ , such that after refraction it emerges along  $QW$  parallel to the axis at the same height as that of the ray  $OA$  in figure (a). The rays  $F_1S$  and  $QW$  when produced intersect at  $H_2$ . A plane perpendicular to the axis and passing through  $H_2$  is called the **first principal plane**. The point of intersection  $P_2$  of the first principal plane with the axis is called the **first principal point**.

It is seen from figure that two incident rays are directed towards  $H_1$  and after refraction seem to come from  $H_2$ . Therefore,  $H_2$  is the image of  $H_1$ . Thus  $H_1$  and  $H_2$  are the conjugate points and the planes  $H_1P_1$  and  $H_2P_2$  are a pair of conjugate planes. It is also seen that

$$H_2P_2 = H_1P_1$$

Hence, the lateral magnification of the planes is +1. Thus, the first and second principal planes are planes of unit magnification and are therefore called unit planes and the points  $P_1$  and  $P_2$  are called unit points.

**Note:-** The principal planes are conceptual planes and do not have physical existence within the optical system.

#### Some Remarkable Features of Principal Planes:-

- (1) Even a complex optical system has only two principal planes.
- (2) Between  $H_1$  and  $H_2$  all rays are parallel to the principal axis.
- (3) The location of the principal planes is characteristic of a given optical system.

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Their positions do not change with the object and image distance used.

- (4) The principal planes are conjugate to each other. An object in the first principal plane is imaged in the second principal plane with unit magnification. Any ray directed towards a point on the first principal plane emerge from the lens as if it originated at the corresponding point (at the same distance above or below the axis) on the second principal plane. Hence, the name unit plane.
- (5) The principal points  $H_1$  and  $H_2$  provide a set of references from which several system parameters are measured.

#### Focal Points And Focal Planes:-

If a parallel beam of light travelling from the left to the right (in object space) is incident on the optical system, the beam comes together at a point,  $F_1$  on the other side (in image space) of the optical system. The beam passes through point  $F_2$  whatever may be its path inside the system. The point  $F_2$  is called the second focal point of the optical system. A beam of light passing the point  $F_1$  on the axis of the object side is rendered parallel to the axis after emergence through the optical system. The point  $F_1$  is called the first focal point.

#### We can now define the focal points as follows:-

The first focal point is a point on the principal axis of optical system such that a beam of light passing through it is rendered parallel to the principal axis after refraction through the optical system.

The second focal point is a point on the principal axis of the optical system such that a beam of light travelling parallel to the principal axis of the optical system, after refraction through the system, passes through it.

The planes passing through the principal focal points  $F_1$  and  $F_2$  and perpendicular to the axis are called first focal plane and second focal plane respectively. The main property of the focal planes is that the rays start from a point in the focal plane in the object space correspond to a set of conjugate parallel rays in the image space. Similarly, a set of parallel rays in the object space corresponds to a set of rays intersecting at a point in the focal plane in the image space.

The distance of the first focal point from the first principal point, i.e.,  $P_1$  is called the first focal length,  $f_1$ , of the optical system and the distance of the second focal point from the second principal point,  $P_2$ , is called the second focal length  $f_2$ .  $f_1$  and  $f_2$  are also known as the focal lengths in object space and image space respectively.

When the medium is same on the two sides of the optical system  $f_1$  (numerically) =  $f_2$ .

#### Nodal Points And Nodal Planes:-

**Nodal points** are points on the principal axis of the optical system where light rays, without refraction, intersect the optic axis. In a thin lens, the nodal points are the same as the principal points.

Light passing through the centre of a thin lens does not deviate. In an optical system the centre separates into two nodal points. The planes passing

through the nodal points and perpendicular to the principal axis are called the **nodal planes**. Whereas the principal planes are planes where all refraction are assumed to occur. The nodal planes are planes where refraction does not take place. Figure (d) represents an optical system with the help of its cardinal planes. It is seen from the figure (d) that ray of light,  $AN_1$ , directed towards one of the nodal points,  $N_1$ , after refraction through the optical system, along  $N_1N_2$ , emerges out from the second nodal point  $N_2$ , in a direction,  $N_2R$ , parallel to the incident ray. The distances of the nodal points are measured from the focal points.

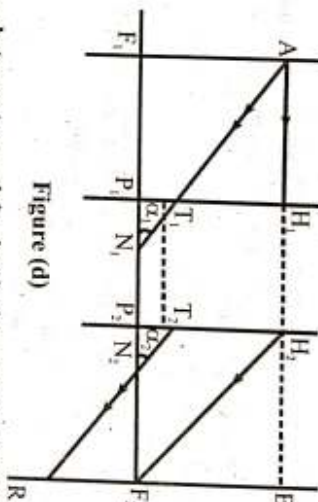


Figure (d)

The distance between two nodal points is always equal to the distance between two principal points:-

Referring in figure (d), we see that in the right angled  $\Delta T_1 P_1 N_1$  and  $\Delta T_2 P_2 N_2$

$$T_1 P_1 = T_2 P_2$$

$$\angle T_1 N_1 P_1 = \angle T_2 N_2 P_2 = \alpha$$

Therefore, the two  $\Delta$ 's are congruent.

$$\therefore P_1 N_1 = P_2 N_2$$

Adding  $N_1 P_2$  to both the sides, we get

$$\therefore P_1 N_1 + N_1 P_2 = P_2 N_2 + N_1 P_2$$

$$\therefore P_1 P_2 = N_1 N_2$$

Thus, the distance between the principal points  $N_1$  and  $N_2$  is equal to the distance between the principal points  $P_1$  and  $P_2$ .

**Q.5. Ramsden's eyepiece is a positive eye piece whereas Huygen's eye piece is negative eye piece.** (2012)

**Ans.** Ramsden's eyepiece is a positive eyepiece. The image formed by the objective lies in front of the field lens. Therefore, cross-wire can be used.

Huygen's eyepiece is a negative eyepiece. The image formed by the objective lies in between the two lenses. Therefore, cross-wires cannot be used.

**Q.6. Compare performance of Huygen's and Ramsden's eyepieces.** (2011, 18)

**Other Related Questions:-**

**Q. Write short note on comparison of Huygen's and Ramsden's eye pieces.**

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(2009)

Ans. Comparison of Ramsden eyepiece with Huygens eyepiece:-

| S.No. | Ramsden Eyepiece                                                                                                                                        | Huygen's Eyepiece                                                                                                                                  |
|-------|---------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------|
| 1.    | Ramsden's eyepiece is a positive eyepiece. The image formed by the objective lies in front of the field lens. Therefore, cross wires can be used.       | Huygen's eyepiece is a negative eyepiece. The image formed by the objective lies in between the two lenses. Therefore, cross wires cannot be used. |
| 2.    | The condition for minimum spherical aberration is not satisfied. But by spreading the deviations over four surfaces, spherical aberration is minimized. | The condition for minimum spherical aberration is satisfied.                                                                                       |
| 3.    | It does not satisfy the condition for achromatism but can be made achromatic by using an achromatic doublet as the eye lens.                            | It satisfies the condition for achromatism.                                                                                                        |
| 4.    | It is achromatic for only two chosen colors.                                                                                                            | It is achromatic for all colors.                                                                                                                   |
| 5.    | The other types of aberrations are better eliminated. Coma is absent and distortion is 5% higher.                                                       | Other aberrations like pincushion distortion are not eliminated.                                                                                   |
| 6.    | The eye clearance is 5% higher.                                                                                                                         | The eye clearance is too small and less comfortable.                                                                                               |
| 7.    | It is used for quantitative purposes in microscopes and telescopes.                                                                                     | It is used for qualitative purposes in microscopes and telescopes.                                                                                 |
| 8.    | Its power is positive.                                                                                                                                  | Its power is positive.                                                                                                                             |
| 9.    | The two principal planes are crossed.                                                                                                                   | The two principal planes are crossed.                                                                                                              |
| 10.   | It can be used as a simple microscope because the first principal plane lies to the left of the field lens and the focal plane is real.                 | It cannot be used as simple microscope because the first focal plane lies to the right of the field lens and the focal plane is virtual.           |
| 11.   | The nodal points coincide with the principal points.                                                                                                    | The nodal points coincide with the principal points.                                                                                               |

Q.7.

Derive Newton's formula for a lens system.

Ans.

Newton's Formula:-

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(2016)

Referring to the Fig., it is seen that  $\Delta^{ve} A_1 B_1 F_1$  and  $F_1 K_1 P_1$  are similar.

$$\frac{K_1 P_1}{AB} = \frac{P_1 F_1}{B F_1} \quad \text{But, } K_1 P_1 = A_1 B_1$$

$$\frac{A_1 B_1}{AB} = \frac{f_1}{x_1} \quad \dots\dots\dots(i)$$

Further,  $\Delta^{ve} A_1 B_1 F_2$  and  $H_2 P_2 F_2$  are similar.

$$\frac{A_1 B_1}{H_2 P_2} = \frac{B_1 F_2}{P_2 F_2} \quad \text{But, } H_2 P_2 = AB$$

$$\frac{A_1 B_1}{AB} = \frac{x_2}{f_2} \quad \dots\dots\dots(ii)$$

From equation (i) and (ii), we get

$$\frac{b_2}{b_1} = \frac{f_1}{x_1} = \frac{x_2}{f_2} \quad \text{or} \quad x_1 x_2 = f_1 f_2$$



Figure

This is the Newton's formula. The distance of the image and the object have been measured from their respective foci. But it is sometimes convenient to measure the conjugate distance from the principal points.

Q.8. How two coherent sources are obtained in Fresnel's biprism experiment? Describe briefly this method to determine wave length of a monochromatic light. Also derive the formula of fringe width. (2012)

Other Important Question-

Q. How do you obtain coherent sources in Fresnel biprism experiment? Explain with the help of diagram.

Ans. Fresnel Biprism:-

Fresnel used a biprism to show interference phenomenon. The biprism consists of two prisms of very small refracting angles joined base to base. In practice, a thin glass plate is taken and one of its faces is ground and polished till a prism figure (1(a)) is formed with an obtuse angle of about  $179^\circ$  and two

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side angles of the order of  $30^\circ$ .

When a light ray is incident on an ordinary prism, the ray is bent through an angle called the angle of deviation. As a result, the ray emerging out of the prism appears to have emanated from a source  $S'$  located at a small distance above the real source, as shown in figure 1 (b). We say that the prism produces a virtual image of the source. A biprism, in the same way, creates two virtual sources  $S_1$  and  $S_2$ , as seen in figure 1 (c). These two virtual sources are images of the same source  $S$  produced by refraction and are hence coherent.

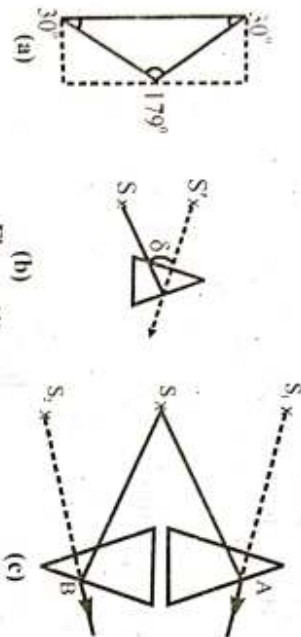


Figure (1)

#### Experimental Arrangement:-

The biprism is mounted suitably on an optical bench. An optical bench consists of two horizontal long rods, which are kept strictly parallel to each other and at the same level. The rods carry uprights on which the optical components are positioned. A monochromatic light source such as sodium vapour lamp illuminates a vertical slit  $S$ . Therefore, the slit  $S$  acts as a narrow linear monochromatic light source. The biprism is placed in such a way that its refracting edge is parallel to the length of the slit  $S$ . A single cylindrical wavefront impinges on both prisms. The top portion of wavefront is refracted downward and appears to have emanated from the virtual image  $S_1$ . The lower segment falling on the lower part of the biprism, is refracted upward and appears to have emanated from the virtual source  $S_2$ . The virtual sources  $S_1$  and  $S_2$  are coherent (see Fig. 2), and hence the light waves are in a position to interfere in the region beyond the biprism. If a screen is held there, interference fringes are seen. In order to observe fringes, a micrometer eyepiece is used.

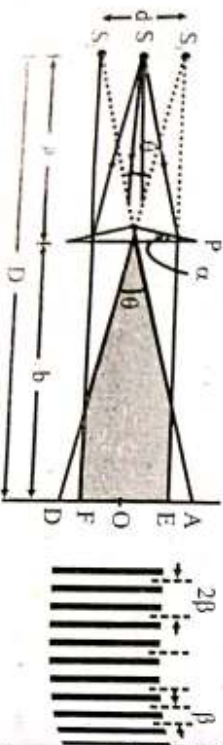


Figure (2)

(b)

#### Theory:-

The theory of the interference and fringe formation in case of Fresnel biprism is the same as the double-slit. As the point  $O$  is equidistant from  $S_1$  and  $S_2$ , the central bright fringe of maximum intensity occurs there. On both sides of  $O$ , alternate bright and dark fringes, as shown in figure. 2(b), are produced. The width of the dark or bright fringe is given by:

$$\beta = \frac{\lambda D}{d} \quad \dots\dots(i)$$

where  $D (= a + b)$  is the distance of the sources from the eyepiece

#### Determination of wavelength of light:-

The wavelength of the light can be determined using the eq.(i). For using the relation, the values of  $\beta$ ,  $D$  and  $d$  are to be measured. These measurement are done as follows:

#### Adjustments:-

A narrow adjustable slit  $S$ , the biprism, and a micrometer eyepiece are mounted on the uprights and are adjusted to be at the same height and in a straight line. The slit is made vertical and parallel to the refracting edge of the biprism by rotating it in its own plane. It is illuminated with the light from the monochromatic source. The biprism is moved along the optical bench till, on looking through it along the axis of the optical bench, two equally bright vertical slit images are seen. Then the eyepiece is moved till the fringes appear in the focal plane of the eyepiece.

#### (i) Determination of fringe width $\beta$ :-

When the fringes are observed in the field view of the eyepiece, the vertical cross-wire is made to coincide with the centre of one of the bright fringes. The position of the eyepiece is read on the scale, say  $X_N$ . The micrometer screw of the eyepiece is moved slowly and the number of the bright fringes  $N$  that pass across the cross-wire is counted. The position of the cross-wire is again read, say  $X_N$ . The fringe width is then given by  $\beta = \frac{X_N - X_0}{N}$ .

#### (ii) Determination of 'd':-

(a) A convex lens of short focal length is placed between the slit and the eyepiece without disturbing their positions. The lens is moved back and forth near the biprism till a sharp pair of image of the slit is obtained in the field view of the eyepiece. The distance between the images is measured. Let it be denoted by  $d$ . If  $u$  is the distance of the slit and  $v$  that of the eyepiece from the lens figure 3(a) then the magnification is

$$\frac{v}{u} = \frac{d}{d} \quad \dots\dots(ii)$$

The lens is then moved to a position nearer to the eyepiece, where again a pair

images of the slit is seen. The distance between the two sharp images is again measured. Let it be  $d_2$ . Again magnification is given by

$$\frac{u}{v} = \frac{d_1}{d_2} \quad \dots\dots(iii)$$

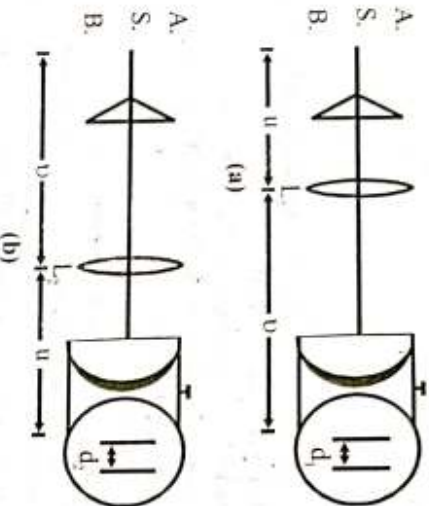


**Figure 3 - Measurement of the distance between the two virtual sources**  
Note that the magnification in one position is the reciprocal of the magnification in the other position.

Multiplying the equations (ii) and (iii), we obtain

$$\frac{d_1 d_2}{d^2} = 1$$

$$d = \sqrt{d_1 d_2} \quad \dots\dots(iv)$$

Using the values of  $\beta$ ,  $d$  and  $D$  in the eq.(i), the wavelength  $\lambda$  can be computed.

(b) Alternatively, the value of  $d$  can be determined as follows. The deviation produced in the path of a ray by a thin prism is given by

$$\delta = (\mu - 1) \alpha$$

where  $\alpha$  is the refracting angle of the prism. From the figure (2), it is seen that  $\delta = \theta/2$ . Since  $d$  is very small, we can also write  $d = a\theta$ .

$$\frac{\theta}{2} = \frac{d}{2a} = (\mu - 1) \alpha$$

$$\therefore d = 2a(\mu - 1) \alpha$$

**Q.9.** What is an eye piece? Name different types of eye pieces. Explain why an eye-piece should consist of two lenses. What are advantages of an eye-piece over a single lens?

Other Related Questions -

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**Q. Write short note on Advantages of a Two-lens eye piece. (2010)**  
**Ans. Eye-piece:-**

It is a device used to magnify an image formed by a lens or lenses preceding it in any optical instrument. For example, the objective of a microscope or telescope forms a real image of an object under examination. The function of the eye-piece is to enlarge this image so that the final image is free from various aberrations as seen by the eye.

**Why the eye piece consists of two lenses:-**

If a single convex lens is used as an eye-piece, it suffers from chromatic and spherical aberrations. It also suffers from an entirely different cause.



When eye is placed close to the eye lens E and the diameter of the eye lens is large as compared to the diameter of the pupil of the eye, the rays passing through the outer portions of the eye lens do not enter the eye. This decreases the intensity of the image and also makes the field of view limited. To overcome this difficulty, the eye lens E chosen is of small aperture and an additional lens F called the field-lens is introduced between the objective O and the eye-lens. The image formed by the objective is made to coincide with the principal plane of the field lens F. The combination of these two lenses is called a compound eye-piece.

**Advantages:-**

The field-lens F which is of a relatively large aperture as compared to the eye-lens turns the rays of light passing through its outer portions towards the principal axis which are thus made to pass through the middle of the eye-lens E.



Thus, all the rays from different parts of the wide area enter the eye of the observer and the field of view is increased in this manner. The rays from outer part could not have entered the eye in the absence of the field lens.

**Q.10. Write short note on Aplanatic points. (2015, 17, 19)**

Other Related Questions -

**Q. What are aplanatic points? Find position of aplanatic points in refraction**

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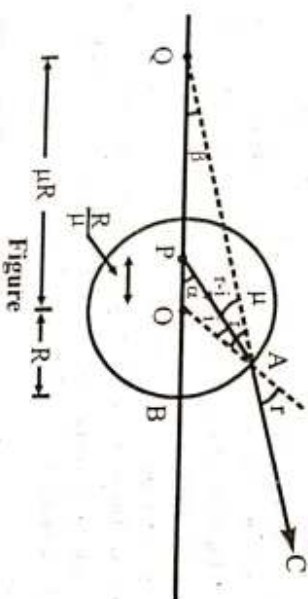
at a spherical surface separating two transparent media. Discuss application to oil immersion objectives and meniscus lenses.

Ans. Aplanatic Points: -

A spherical lens, which is free from the defects of spherical aberration and coma is called an **aplanatic lens**. A pair of conjugate points free from spherical aberration and coma is called aplanatic points.



Fig. illustrates the property of an aplanatic lens. Let O be the centre of curvature of the lens of refractive index  $\mu$  and radius of curvature R. P is a point on the axis of the lens such that  $PO = R/\mu$ . It can be shown that all rays passing through the point P appear to diverge through the point Q irrespective of the slope angle made by the incident rays. PA is the incident ray and AC is the refracted ray. The ray AC appears to diverge from the point Q which is the image of P. Let  $i$  and  $r$  be the angles of incidence and refraction and  $\alpha$  and  $\beta$  the slope angles made by the incident and refracted rays.



$$\text{Then, } \frac{\sin i}{\sin r} = \frac{1}{\mu}$$

.....(i)

In the  $\triangle APO$ ,

$$\frac{\sin i}{\sin \alpha} = \frac{PO}{R} = \frac{R}{\mu R} = \frac{1}{\mu}$$

.....(ii)

From (i) and (ii)

$$\frac{\sin i}{\sin r} = \frac{\sin i}{\sin \alpha}$$

$$\therefore \angle \alpha = \angle r$$

In the  $\triangle APO$ ,

.....(iii)

$$\angle \alpha = \beta + (r - i)$$

Substituting

.....(iv)

In the  $\triangle AOQ$   $\angle \alpha = \angle r$  in equation (iv), we get

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$$\frac{\sin r}{\sin \beta} = \frac{\sin i}{\sin \alpha} = \mu = \frac{OQ}{OA} = \frac{OQ}{R}$$

$$OQ = \mu R$$

Thus, if the distance of the object point P is  $R/\mu$  from the centre of curvature, then distances of the image point Q is  $\mu R$  irrespective of the slope angles  $\alpha$  and  $\beta$ . The object and image distance of the conjugate points that satisfy the Abbe condition can be seen as

$$BP = R + \frac{R}{\mu} \text{ and } BQ = R + \mu R \quad \text{.....(v)}$$

The object and image points which are at distances given by eq.(v) from the pole of the spherical surface are free spherical aberration as well as coma. Such points are known as aplanatic points.

An aplanatic lens is mostly used as the front lens of a high power microscope objective called the oil immersion objectives. As it is not possible to place an object inside a solid spherical lens, the lens is ground a little and the object to be examined is embedded in between a drop of oil and the lens surface. The oil chosen is such that it has the same refractive index as that of the lens.

#### Oil Immersion: -

In light microscopy, oil immersion is a technique used to increase the resolving power of a microscope. This is achieved by immersing both the objective lens and the specimen in a transparent oil of high refractive index, thereby increasing the numerical aperture of the objective lens.

Immersion oils are transparent oils that have specific optical and viscosity characteristics necessary for use in microscopy. Typical oils used have an index of refraction around 1.515. An oil immersion objective is an objective lens specially designed to be used in this way. Many condensers also give optimal resolution when the condenser lens is immersed in oil.

#### Meniscus Lenses: -

Meniscus lens is a lens which has two spherical curved surfaces, convex on one side and concave on the other side.

It is thicker at the center than at the edges. The lens provides a smaller beam diameter in order to reduce the beam waste and spherical aberration. When a meniscus lens is combined with another lens, the focal length is shortened and the numerical aperture of the system is increased. This reduces the image distortion and increases image resolution.

#### Numericals

Q.11. An Huygens' eye piece has an eye-lens of 4 cm. focal length. Find the value of its equivalent focal length. (2013, 16)

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**Sol.** Let  $f$  is the focal length of eye-lens and the focal length of field lens = 3  
Hence focal length of the field lens =  $3 \times 4 = 12$  cm  
and the equivalent focal length

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} = \frac{f_2 + f_1 - d}{f_1 f_2} \quad (d = 2f = 2 \times 4 = 8)$$

or  $F = \frac{f_1 f_2}{f_1 + f_2 - d}$

Putting all values, we get

$$F = \frac{12 \times 4}{12 + 4 - 8} = 6 \text{ cm}$$

**Q.12.** Two lenses of powers 2.5 dioptre and -5.0 dioptre are kept coaxially separated by a distance 30.0 cm. Find the positions of principal points and locate them on a diagram.

**Other Related Questions-**

**Q.** Two lenses of powers +2.5D and -5 D are kept coaxially separated by 30cm. Find the position of cardinal points.

(2012, 19)

**Sol.** The focal length of the first lens,  $f_1 = \frac{1}{P} = \frac{1}{2.5D} = \frac{1m}{2.5} = 40 \text{ cm}$

The focal length of the second lens,  $f_2 = \frac{1}{P} = \frac{1}{-5D} = -\frac{1m}{5} = -20 \text{ cm}$

The equivalent focal length of the coaxial system is given by

$$f = \frac{f_1 f_2}{f_1 + f_2 - d}$$

$$\therefore f = \frac{40 \text{ cm} \times (-20)}{40 \text{ cm} - 20 \text{ cm} - 30 \text{ cm}} = +80 \text{ cm}$$

$$\alpha = + \frac{df}{f_2} = \frac{(30 \text{ cm}) \times (80 \text{ cm})}{-20 \text{ cm}} = -120 \text{ cm}$$

The first principal point  $P_1$  is at a distance 120 cm to the left of the first lens.

$$\beta = - \frac{df}{f_1} = - \frac{30 \text{ cm} \times 80 \text{ cm}}{40 \text{ cm}} = -60 \text{ cm}$$

The second principal point  $P_2$  is at a distance of 60 cm to the left of the second lens.

**First Focal Point:-**  
The distance of  $F_1$  from the lens  $L_1$  is

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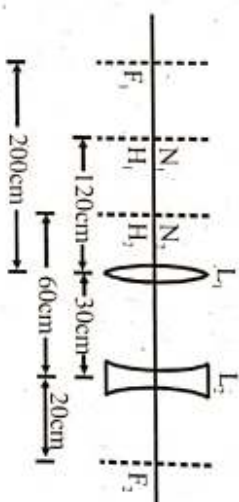
$$L_1 F_1 = -f \left[ 1 - \frac{d}{f_2} \right] = -80 \text{ cm} \left[ 1 - \frac{30 \text{ cm}}{(-20 \text{ cm})} \right] = -200 \text{ cm}$$

**Second Focal Point:-**

The distance of  $F_2$  from the lens  $L_2$  is

$$L_2 F_2 = +f \left[ 1 - \frac{d}{f_1} \right] = 80 \text{ cm} \left[ 1 - \frac{30 \text{ cm}}{40 \text{ cm}} \right] = +20 \text{ cm}$$

The first focal point  $F_1$  is to the left of the first lens and is at a distance of 200 cm from it. The second focal point  $F_2$  is to the right of the second lens and is at a distance of 20 cm from it. The cardinal points are plotted in figure. **Nodal Points** - As the system is situated in the air, the nodal points are same as the principal points.



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**Chapter 2****Q.1. Discuss fringes of constant inclination and optical path difference.**

(2017)

**Ans.** The fringes of constant inclination obtained with thick plates are called Haidinger's fringes, though they were first systematically studied by Mascari by Lummer. Since the light in a given fringe refers to a constant value of  $\theta$ , the telescope is normal to the surface, the fringes are circular rings in the focal plane of the objective. When fringes of constant inclination are viewed with naked eye, it is desirable to focus the eye for infinity but, owing to the small range of angles included by the eye pupil, there is great depth of focus and the fringes may still be seen when the eye is focused near the film, provided it is very thin.

**Q.2. Discuss localized and nonlocalized fringes.**  
**Other Related Questions -**

(2009, 17)

**Q. Explain localized and non localized fringes.**

(2010, 12, 16)

**Q. What do you mean by localized fringes.**

(2015)

**Q. What do you mean by unlocalized fringes.**

(2015)

**Ans.** Fringe formation and localization in holographic interferometry, speckle interferometry, and Michelson interferometry is very important to understand the nature of interfering waves and the perturbation involved in one of the waves. The fringes of optical interferometry can be classified into two parts and they are (i) Localized and (ii) Non localized. The localized fringes are obtained in Michelson interferometer, irrespective of coherence of sources, if the two mirrors are tilted with respect to each other. On the other hand if they are parallel, then fringes never localize at any plane. The non localized fringes will be formed, if the interfering beams from two arms of the interferometer have path length variation within the coherence length of source. Unlike fringe formations in classical interferometric systems, localization of fringes in holographic interferometry, speckle interferometry depends on the reflected or transmitted object waves as the other interfering waves could be plane laser beam. In such cases the fringes localize only at planes where the phases remain constant and the plane is normal to the direction of interference. This condition can not be satisfied for most of the interferometric experiments. This condition may introduce error in measuring the cause of interference. For example if, we want to measure the tilt of an object or any other motion of an object in holographic interferometry, it will not be possible to accurately compute the amount of motion without the fringes being localized at a particular plane. The fringe formation medium, polarization etc.

**Chapter 3****Q.1. Explain Newton's rings are circular but air-wedge fringes are straight.**

(2013, 17)

**Ans. Newton's Rings are Circular but Air - wedge Fringes are Straight: -**

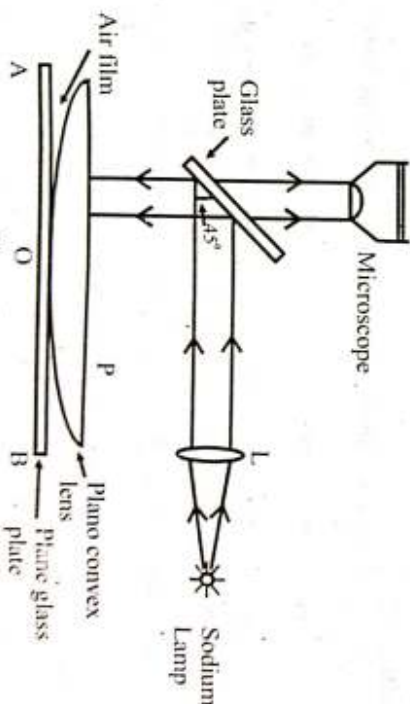
In both Newton's rings and Air-wedge fringe arrangements, each fringe is the locus of points of equal thickness of the film. In Newton's rings arrangement, the locus of points of equal thickness of air film lie on a circle with the point of contact of the plano-convex lens and glass plate as centre. Hence the fringes are circular and concentric. In case of wedge-shaped air film, the loci of points of equal thickness are straight lines parallel to the edge of the wedge. Hence, the fringes are straight and parallel.

**Q.2. Write short notes on Newton's ring.**  
**Ans. Newton's Ring: -**

(2012)

Newton's rings are an example of fringes of equal thickness. Newton's rings are formed when a plano - convex lens  $P$  of a large radius of curvature placed on a sheet of plane glass  $AB$  is illuminated from the top with monochromatic light (figure (1)). The combination forms a thin circular air film of variable thickness in all directions around the point of contact of the lens and the glass plate. The locus of all points corresponding to specific thickness of air film falls on a circle whose centre is at  $O$ . Consequently, interference fringes are observed in the form of a series of concentric rings with their centre at  $O$ . Newton originally observed these concentric circular fringes and hence they are called **Newton's rings: -**

The experimental arrangement for observing Newton's rings is shown in figure (1).

**Figure 1**

Monochromatic light from an extended source S is rendered parallel by

a lens L. It is incident on a glass plate inclined at  $45^\circ$  to the horizontal, and is reflected normally down onto a plano-convex lens placed on a flat glass plate. Part of the light incident on the system is reflected from the glass-to-air boundary, say from point D (fig. 2). The remainder of the light is transmitted through the air film. It is again reflected from the air-to-glass boundary, say from point J. The two rays reflected from the top and bottom of the air film are derived through division of amplitude from the same incident ray CD and are therefore coherent. The rays 1 and 2 are close to each other and interfere to produce darkness or brightness. The condition of brightness or darkness depends on the path difference between the two reflected light rays, which in turn depends on the thickness of the air film at the point of incidence.

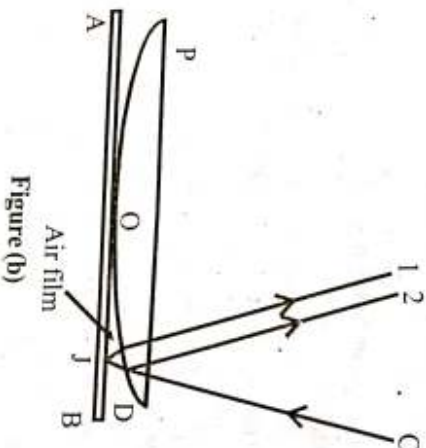


Figure (b)

Q.3. What are the essential condition to obtain stationary interference in the super position of two light beams. (2017)

Other Related Questions:-

Q. State the conditions which are necessary to observe the stationary interference pattern. (2017)

Q. Two light beams superpose to generate stationary interference pattern. Discuss. (2014)

(i) The conditions which are absolutely essential for obtaining such a pattern-pattern, i.e. one with high visibility and appreciable fringe width. (2011)

Q. State and explain the conditions for good contrast of the interference pattern. (2011)

Q. State and explain the conditions which are necessary to observe the stationary interference pattern. (2013)

Ans. Conditions for obtaining Steady Interference pattern:- (2019)

(i) The two sources of light must be coherent. That is these two sources should emit light waves with a constant phase difference. In such case, coherent sources are obtained from single source. (2019)

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- (ii) The amplitudes of waves must be equal. i.e. the brightness of the two sources should be same.
- (iii) The sources of light should be narrow.
- (iv) The two sources of light must be of monochromatic.
- (v) The distance between the sources and screen must be reasonably large.
- (vi) The distance between coherent sources must be small.
- (vii) The sources should emit light waves in nearly same direction.
- (viii) The two interfering waves must be in the same state of polarization.

Q.4. The dark region of interference pattern means destruction of light energy. Is it against the principle of conservation of energy. (2016)

Ans. Actually when we think for a while it seems to be violation of law of conservation of energy but there is not actually. (2016)

Reason:-

Interference is defined as the superposition of two coherent waves and redistribution of energy. Hence the same energy loss at dark point is transferred at bright point that's why bright point is brighter than direct sum of the intensities of two waves too.

Hence in conclusion we can say that yes there seems violation but actually it is not. Because the average energy of waves after superposition is equal to the sum of their energy.

Mathematical Proof:-

Let amplitude of two waves is  $a_1$  and  $a_2$  then their intensities.

$$I_1 = ka_1^2 \text{ and } I_2 = ka_2^2$$

Intensity after superposition at any point

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta \quad (\delta \rightarrow \text{Phase difference})$$

$$\text{Now, } I_{av} = \frac{\int_0^{2\pi} I d\delta}{\int_0^{2\pi} d\delta} = \frac{\int_0^{2\pi} (I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta) d\delta}{\int_0^{2\pi} d\delta}$$

$$= \frac{\int_0^{2\pi} (ka_1^2 + ka_2^2 + 2ka_1 a_2 \cos \delta) d\delta}{\int_0^{2\pi} d\delta}$$

$$= \frac{(ka_1^2 \delta + ka_2^2 \delta) \Big|_0^{2\pi}}{(\delta) \Big|_0^{2\pi}} = ka_1^2 + ka_2^2 \Rightarrow I_{av} = I_1 + I_2$$

Hence energy is conserved.

Q.5. When the centre of Newton's ring will be bright? (2016)

Ans. Newton's Rings with bright center due to reflected light:-

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Ordinarily the center of a Newton's ring is dark. At the center, the geometrical path difference is zero but the effective path difference between the two interfering rays at the point of contact is  $\lambda/2$  which is the condition for minimum intensity. This path difference ( $\lambda/2$ ) is due to a phase change  $\pi$  which occurs in the interfering ray reflected from the lower denser surface of the film (that is, from glass plate) developed between the plano-convex lens and glass plate. Hence the center of a Newton's ring is dark. To obtain a bright center, a liquid of refractive index  $\mu$  is introduced between a lens of refractive index  $\mu_1$  and a plate of refractive index  $\mu_2$  such that  $\mu_1 > \mu > \mu_2$ . Under this condition the reflection of both the interfering rays at the point of contact becomes zero which is the condition of maximum intensity. Hence the center of Newton's rings appears bright. This situation was achieved by Young. In his experiment, he poured a few drops of sassafras oil ( $\mu = 1.57$ ) between a crown glass ( $\mu = 1.50$ ) lens and a flint glass ( $\mu = 1.65$ ) plate and obtained a bright center of the rings system.

**Q.6. What do you mean by fringes of equal inclination.**

(2009, 10, 12, 14, 15)

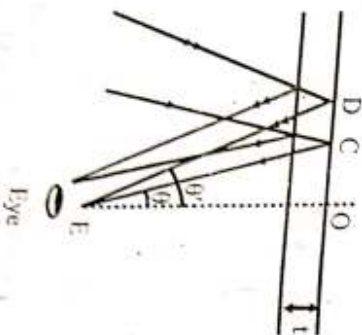
**Other Related Questions -**

**Q. Define Fringes of equal inclination.**

(2011)

**Ans. Haidinger's Fringes : Fringes of Equal Inclination:-**

The interference fringes can also be obtained from the films of uniform thickness. For a plane parallel sided film, the path difference  $2\mu t \cos r$  only. A very small change in  $r$  produces a large change in path difference provided that  $t$  is large. All directions equally inclined to the normal to the film will correspond to the same path difference, so that the circular fringes of centre with the point at infinity in the direction of normal will be seen. Such like fringes are called the fringes of equal inclination or Haidinger's fringes.



In this case all the pairs of interfering coherent rays of equal inclination pass through the plate as a parallel beam and hence meet at a point at infinity. The other pairs of different inclination meet at different points at infinity. Thus Haidinger's fringes can be viewed with a telescope at infinity. Fig. shows the formation of Haidinger fringes in a parallel sided air film. Let C be the point on the film which satisfies the condition of minimum intensity,  $2t \cos \theta = n\lambda$ , where  $\theta$  is the angle between normal to the film EO and EC, then the point C appears dark. Thus all points on the film having equal inclination  $\theta$  lie in a circle of radius OC around O and satisfy the condition of minimum intensity. As a result, a dark ring of angular radius  $\theta$  appears. As  $\theta$  increases  $2t \cos \theta$  decreases. Let D be the another point on the film at which the condition of maximum intensity is satisfied. Thus D appears bright and all points on the film having equal inclination  $\theta'$  lie in a circle of radius OD with O as centre and satisfy the condition of maximum intensity, therefore, a bright ring of angular radius  $\theta'$  appears. In this way, as  $\theta$  increases, we get alternate bright and dark rings of equal inclination at infinity and can be viewed with a telescope focussed at infinity.

**Q.7. What do you mean by fringes of equal thickness.**

(2009, 10, 12, 14, 15)



**Ans.** Now, let us assume that the two virtual images of the extended light source are not exactly parallel to each other, but from a small angle between them. Then, the center of the ring fringes in the fringe pattern displaces to one side. Then, the fringes will transform in arcs and may eventually become straight and parallel, when the angle is quite large. Under these conditions the optical path difference is a function of the local separation between the two images of the light source. The interference fringes thus observed are called "equal thickness fringes". Strictly speaking, the fringes are of the equal thickness type only if the eye or observing optical system is focused at an infinite distance. When the eye is not focused at an infinite distance, the observed fringes will be of an intermediate type to the equal inclination and the equal thickness fringes. The interference pattern is formed by localized fringes. The fringes are arcs with its convexity toward the thinner side of the wedge between the virtual images of the light source. A light ray exits one of the images of the light source at the point P on the other from the corresponding point at the other light source and with the same inclination respect to the image. The point where they cross is located near the light sources. For this reason, these are called **localized fringes**. Fig illustrates the interference fringes that can be obtained with a Michelson interferometer.

Q.8. Give theory of Newton's rings and obtain expressions for diameter of  $n^{\text{th}}$  dark and  $n^{\text{th}}$  bright fringes.

Other Related Questions -

Q. Discuss the formation of Newton's rings and obtain an expression for their diameters. Explain why Newton's ring are always seen in reflected light

(2011)

Ans. The formation of Newton's rings is a special case of interference in an air film of variable thickness. When a plano-convex lens of large focal length is placed on a plane glass plate, then a thin film of air is formed between the lower surface of the lens and the upper surface of the glass plate. When a monochromatic light is allowed to fall normally on the film, then circular fringes are observed. In reflected light the centre of the circular fringes are dark followed by alternatively bright and dark circular rings and vice-versa in transmitted light.

When white light is used in place of monochromatic light, a few coloured circular rings are seen around the point of contact with violet colour inside and red colour outside in each ring.

Experimental Arrangement:-

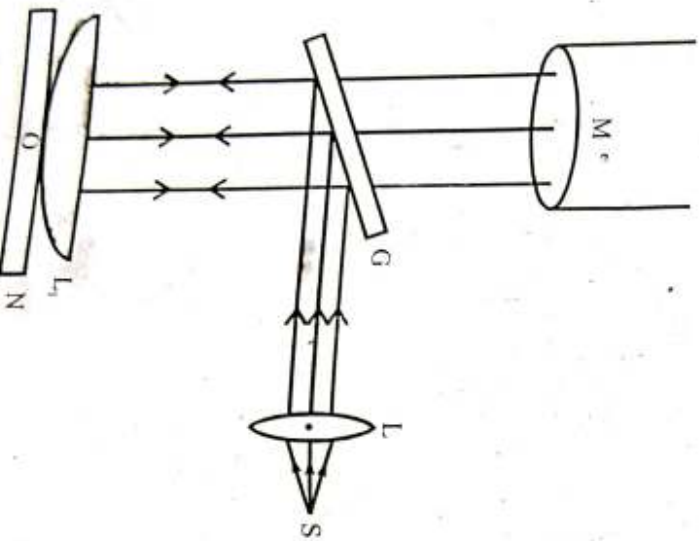


Figure 1

To observe the Newton's ring, the experimental arrangement is made as shown in figure (1). S is an extended monochromatic source, i.e., a sodium lamp.

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placed at the focus of a convex lens L. The horizontal parallel rays after the lens fall on a glass plate G inclined at  $45^\circ$ . The rays are partly reflected from the inclined glass plate and fall normally on a plano-convex lens of large focal length  $L_1$  placed over a plane glass plate N.

The air film is formed between the plano-convex lens  $L_1$  and glass plate around the point of contact, O. The interference takes place between the rays reflected from the upper and lower surfaces of the film and are viewed with a microscope M focussed on the air film.

Theory:-

The air film formed between the curved surface of the plano-convex lens and the upper surface of the glass plate is of wedge shape; therefore the path difference between the normal incidence ( $r = 0$ ) and for a very small angle of wedge ( $\theta \approx 0$ ), the path difference

$$2\mu t \cos\left(r + \theta\right) + \frac{\lambda}{2} = 2\mu t + \frac{\lambda}{2}$$

for constructive interference or for a bright ring

$$2\mu t \cos\left(r + \theta\right) + \frac{\lambda}{2} = 2\mu t + \frac{\lambda}{2}$$

or destructive interference or for a dark ring

$$2\mu t + \frac{\lambda}{2} = n\lambda$$

or for destructive interference or for a dark ring

$$2\mu t + \frac{\lambda}{2} = (2n + 1) \frac{\lambda}{2} \quad \text{or} \quad 2\mu t = n\lambda$$

the point of contact ( $t = 0$ ), the effective path difference  $= \frac{\lambda}{2}$ ; which is the condition for minimum intensity. Hence, the centre of the Newton's ring is dark.

Consider a plano-convex lens AOB placed on a plane glass plate MON (fig. 2). Let  $t$  be the thickness of the film at any point and  $R$  be the radius of the curved surface AOB. We know from the property of a circle that

$$PL \times QL = OL \times GL, \dots\dots\dots (i)$$

$$PL \times QL = OL \times GL = (2R - t) \times t$$

$$t \times (2R - t) = OL \times GL = (2R - t) \times t$$

$$t = (2R - t) \times t$$

$$t = (2R - t) \times t$$

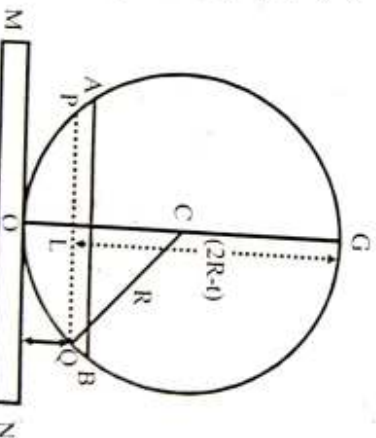


Figure (2)

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or  $r^2 = 2Rt - r^2$   
 Since,  $t \ll R$ ; hence  $r^2$  may be neglected in comparison to  $2Rt$ .

$$\text{So, } r^2 = 2Rt \Rightarrow t = \frac{r^2}{2R} \quad \dots\dots\dots\text{(ii)}$$

### Diameter of Bright Rings:-

Substituting the value of  $t = r^2 / 2R$  in the equation for the condition of maximum, we have

$$2\mu \frac{r^2}{2R} = (2n-1) \frac{\lambda}{2} \Rightarrow r_n^2 = \frac{(2n-1)\lambda R}{2\mu} \quad \dots\dots\dots\text{(iii)}$$

Let  $D_n$  be the diameter of the  $n^{\text{th}}$  bright ring, then

$$\left(\frac{D_n}{2}\right)^2 = \frac{(2n-1)\lambda R}{2\mu}$$

$$\text{or } D_n^2 = \frac{2(2n-1)\lambda R}{\mu} = 2(2n-1)\lambda R \quad (\text{for air } \mu = 1)$$

$$\text{or } D_n = \sqrt{2\lambda R(2n-1)}$$

Thus, it is clear from equation that the diameter of the bright ring is proportional to the square root of the odd natural numbers.

### Diameter of Dark Rings:-

Using the condition for minima, we have

$$2\mu \frac{r^2}{2R} = n\lambda \Rightarrow r_n^2 = \frac{n\lambda R}{\mu} \quad \dots\dots\dots\text{(v)}$$

If  $D_n$  be the diameter of the  $n^{\text{th}}$  dark ring (i.e.  $r_n = \frac{D_n}{2}$ ), then from eq. (v),

$$\frac{D_n^2}{4} = \frac{n\lambda R}{\mu} \Rightarrow D_n^2 = \frac{4n\lambda R}{\mu}$$

$$\text{or } D_n^2 = 4n\lambda R$$

$$\text{or } D_n = \sqrt{4n\lambda R} \quad (\text{for air } \mu = 1)$$

Thus, the diameter of the dark ring is proportional to the square root of the natural numbers. If  $D_n$  and  $D_{n+1}$  are diameters of  $n^{\text{th}}$  and  $(n+1)^{\text{th}}$  dark rings

$$D_{n+1} - D_n = \sqrt{4(n+1)\lambda R} - \sqrt{4n\lambda R} = \sqrt{4\lambda R} (\sqrt{n+1} - \sqrt{n})$$

The spacing between the consecutive rings may be obtained by putting

3, 4, 5 .... in eq. (vii). Then

$$D_2 - D_1 = \sqrt{4\lambda R} (\sqrt{2} - 1)$$

$$D_3 - D_2 = \sqrt{4\lambda R} (\sqrt{3} - \sqrt{2})$$

$$D_4 - D_3 = \sqrt{4\lambda R} (\sqrt{4} - \sqrt{3})$$

$$D_5 - D_4 = \sqrt{4\lambda R} (\sqrt{5} - \sqrt{4})$$

Hence, it is clear from the above values that the spacing between the consecutive rings decreases with the increasing order of the ring as shown in figure (3).



Figure 3

### 9. How you can locate the zero order fringe in biprism experiment?

(2013, 19)

Ans. Location of zero order fringe:-

In a biprism experiment with monochromatic light all bright fringes are like. However, with white light source zero order fringes are white. All other fringes are coloured. Thus by replacing monochromatic source by white light source we can easily place the cross wire in the position of the zero-order fringe. If now the white light is replaced with monochromatic light, the position of the crosswire in the eye-piece will give the zero-order band.

### 10. Briefly explain Fabry-Perot Interferometer.

Other Related Questions:-

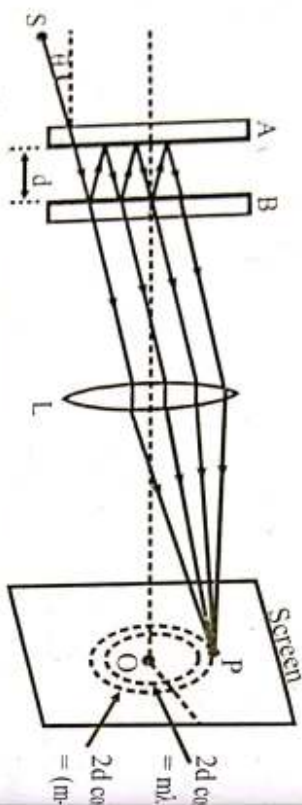
1. Discuss the formation of interference fringes in the light reflected by plane parallel plates after multiple reflections. Also plot variation of shapes of the fringes with the coefficient of reflection. (2014)
2. Discuss fringe formation in light transmitted by a plane parallel plate after multiple reflections. Also discuss variation of shape of fringes with coefficient of reflection. (2009)
3. Write short note on resolving power of FP etalon. (2011)

Ans. Fabry-Perot Interferometer and Etalon:-

The Fabry-Perot interferometer is a high resolving power instrument, which makes use of the "fringes of equal inclination", produced by the transmitted light after multiple reflections in an air film between two parallel highly reflecting glass plates.

The interferometer consists of two optically plane glass plates A and B with their inner surfaces silvered, and placed accurately parallel to each other. Crosses are provided to secure parallelism if disturbed. This system is difficult to manufacture and is no more in use. Instead an etalon which is much more easily manufactured is used. The etalon consists of two semi-silvered plates rigidly held

parallel at a fixed distance apart. The reflectance of the two surfaces can be as high as 99.99%. Although both reflected and transmitted beams interfere with each other, the Fabry-Perot interferometer is usually used in the transmissive mode.



S is a broad source of monochromatic light and  $L_1$  a convex lens (shown in Fig.) which makes the rays parallel. An incident ray suffers a large number of internal reflections successively at the two silvered surfaces shown in Fig. At each reflection a small fraction of the light is transmitted. Thus, each incident ray produces a group of coherent and parallel, transmitted rays with a constant path difference between any two successive rays where they interfere. Hence the rays from all points of the source produce an interference pattern on a screen placed in the focal plane of  $L_1$ .

#### Formation of Fringes:-

Let  $d$  be the separation between the two silvered surfaces and  $\theta$  the inclination of a particular ray with the normal to the plates. The path difference between any two successive transmitted rays corresponding to the incident ray is  $2d \cos \theta$ . The condition for these rays to produce maximum intensity is given by

$$2d \cos \theta = m \lambda$$

where  $m$  is an integer. The locus of points in the source, which give rays of constant inclination,  $\theta$  is a circle. Hence, with an extended source, the interference pattern consists of a system of bright concentric rings on a dark background, each ring corresponding to a particular value of  $\theta$ .

#### Determination of wavelength:-

When the reflecting surfaces A and B of the interferometer are adjusted exactly parallel, circular fringes are obtained. Let  $m$  be the order of the bright fringe at the centre of the fringe system. As at the centre  $\theta = 0$ , we have

$$2t = m \lambda$$

If the movable plate is moved a distance  $\lambda/2$ ,  $2t$  changes by  $\lambda$  and hence a bright fringe of the next order appears at the centre. If the movable plate is moved from the position  $x_1$  to  $x_2$  and the number of fringes appearing at the centre during this movement is  $N$ , then

$$N \cdot \frac{\lambda}{2} = x_2 - x_1 \Rightarrow \lambda = \frac{2(x_2 - x_1)}{N} \quad \dots\dots(i)$$

Measuring  $x_2, x_1$  and  $N$ , we can determine the value of  $\lambda$ .

#### Measurement of difference in wavelength:-

The light emitted by a source may consist of two or more wavelengths, as  $D_1$  and  $D_2$  lines in case of sodium. Separate fringe patterns corresponding to the two wavelengths are not produced in Michelson interferometer. Hence, Michelson interferometer is not suitable to study the fine structure of spectral lines. On the other hand, in Fabry-Perot interferometer, each wavelength produces its own ring pattern and the patterns are separated from each other. Therefore, Fabry-Perot interferometer is suitable to study the fine structure of spectral lines.

Difference in wavelengths can be found using coincidence method. Let  $\lambda_1$  and  $\lambda_2$  be two very close wavelengths in the incident light. Let us assume that  $\lambda_1 > \lambda_2$ . Initially, the two plates of the interferometer are brought into contact. Then the rings due to  $\lambda_1$  and  $\lambda_2$  coincide partially. Then the movable plate is slowly moved away such that the ring systems separate and maximum discordance occurs. Then the rings due to  $\lambda_2$  are half way between those due to  $\lambda_1$ . Let  $t_1$  be the separation between the plates when maximum discordance occurs. At the centre

$$2t_1 = m_1 \lambda_1 = \left( m_1 + \frac{1}{2} \right) \lambda_2 \quad \dots\dots(ii)$$

$$\text{or } m_1 (\lambda_1 - \lambda_2) = \lambda_2 / 2$$

$$\therefore m_1 = \frac{\lambda_2}{2(\lambda_1 - \lambda_2)} \quad \dots\dots(iii)$$

$$\text{Using the value of } m_1 \text{ in eq.(ii), we get } 2t_1 = \frac{\lambda_2}{2(\lambda_1 - \lambda_2)} \lambda_1$$

$$\therefore \lambda_1 - \lambda_2 = \frac{\lambda_1 \lambda_2}{4t_1} = \frac{\lambda_{\text{mean}}^2}{4t_1} \quad \dots\dots(iv)$$

(since  $\lambda_1 \lambda_2 = \lambda_{\text{mean}}^2$  as  $\lambda_1 - \lambda_2$  is very small.)

When the separation between the plates is further increased, the ring systems coincide again and then separate out and maximum discordance occurs once again. If  $t_2$  is the thickness now,

$$2t_2 = m_2 \lambda_1 = \left( m_2 + \frac{3}{2} \right) \lambda_2 \quad \dots\dots(v)$$

From eq.(v) and (ii), we get

$$2(t_2 - t_1) = (m_2 - m_1) \lambda_1 = (m_2 - m_1) \lambda_2 + \lambda_2 \quad \dots\dots(vi)$$

$$\text{or } (m_2 - m_1) = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

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Using the above expression into equ.(vi), we obtain

$$2(t_2 - t_1) = \frac{\lambda_1 \lambda_2}{(\lambda_1 - \lambda_2)}$$

$$\text{or } \lambda_1 - \lambda_2 = \frac{\lambda_1 \lambda_2}{2(t_2 - t_1)} \approx \frac{\lambda_{\text{mean}}}{2(t_2 - t_1)}$$

**Q.11. State and explain conditions for sustained interference pattern.**

**Ans. Conditions For sustained interference:-**

**(i) The waves from the two source must be of the same frequency:-**

If the light waves differ in frequency, the phase difference fluctuates irregularly with time. Consequently, the intensity at any point fluctuates with time and we will not observe steady interference.

**(ii) The two light waves must be coherent:-**

If the light waves are coherent, then they maintain a fixed phase difference over a time and space. Hence a stationary interference pattern will be observed.

**(iii) The path difference between the overlapping waves must be less than coherence length of the waves:-**

Light is emitted in the form of wave trains and a finite coherence length characterizes them. If we consider two interfering wave trains, having constant phase difference, as in figure, the interference effects occur due to parts PQ of wave 1 and ST of wave 2.

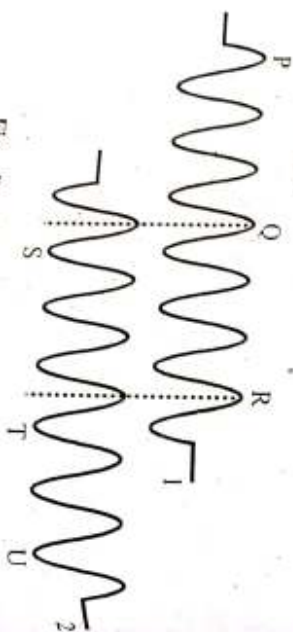


**Fig.- Partial overlap of wavetrains**

For the parts PQ and TU interference will not occur. Therefore, the interference pattern does not appear distinctly. When the entire wave train of one wave overlaps on the wave train of the other wave, the interference pattern will be distinct. On the other hand, when the path difference between the waves 1 and 2 becomes very large, the wave trains arrive at different times and do not overlap on each other. Therefore, in such cases interference does not take place. The interference pattern completely vanishes if the path difference is equal to the coherence length. It is hence required that

$$\Delta < l_{\text{coh}}$$

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**(iv) If the two sets of waves are plane polarized, their planes of polarization must be the same:-**

Waves polarized in perpendicular planes cannot produce interference effects.

**Q.12. Write short note on colours of thin film.**

**Ans. Colours in Thin films:-**

The colours exhibited in reflection by thin films of oil, mica, soap bubbles and coatings of oxides on heated metals etc are due to interference of light from an extended source such as sky. Thomas Young explained the origin of colours in thin films. It may be understood as follows. The films are usually observed by reflected light. The eye looking at the thin film receives light waves reflected from the top and bottom surfaces of the film.

The reflected rays are very close to each other and are in a position to interfere. The optical path difference between the interfering rays is  $\Delta = 2\mu t \cos r - \lambda/2$ . It is seen that the path difference depends upon the thickness  $t$  of the film, the wavelength  $\lambda$ , and the angle  $r$ , which is related to the angle of incidence of light on the film. White light consists of a range of wavelengths and for specific values of  $t$  and  $r$ , waves of only certain wavelengths constructively interfere. Therefore, only those colours are present in the reflected light. The other wavelengths interfere destructively and hence are absent from the reflected light. Hence, the film at a particular point appears coloured. As the thickness and the angle of incidence vary from point to point, different colours are intensified at different places. The colours seen are not isolated colours, as at each place there is a mixture of colours. The composition of colours is different at different places and contours of impressive hues are observed over the entire surface of the film.

**Q.13. Write short note on visibility of fringes and degree of coherence.**

**(2010)**

**Other Related Questions:-**

**Q. Write short note on visibility and degree of coherence.**

**(2009)**

**Ans. Visibility of Fringes:-**

The contrast of the interference fringes can be quantitatively described by the parameter called visibility. Visibility is defined as

$$V = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$

the value of visibility varies between 0 and 1. When the fringes are of maximum intensity in the bright areas and totally dark in the dark areas, the visibility is equal to 1. As the phase difference increases, the coherence between the light waves decreases and the visibility is reduced. Finally, when the coherence

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between the two light waves disappears,  $I_{\max}$  and  $I_{\min}$  become equal and visibility goes to zero. Then fringes are not observed and instead we observe uniform illumination.

From eq.  $I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$  and this is eq.  $I_{\min} = I_1 + I_2 + 2\sqrt{I_1 I_2}$ , the visibility of the fringes produced by two beams can be expressed as

$$V = \frac{(E_1 + E_2)^2 - (E_1 - E_2)^2}{(E_1 + E_2)^2 + (E_1 - E_2)^2} = \frac{2E_1 E_2}{E_1^2 + E_2^2} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} = \frac{2\sqrt{I_1} \sqrt{I_2}}{I_1 + I_2}$$

It is seen from the above equation that the closer the intensities of the waves, the higher is visibility of the fringes. When  $I_1 = I_2$ ,  $V = 1$ . It may be noted that  $V$  is always equal to 1 when monochromatic light is used.

#### Degree of Coherence:-

A dimensionless unit, expressed as a ratio, used to indicate the extent of coherence of an electromagnetic wave such as a lightwave.

**Note 1:** For lightwaves, the magnitude of the degree of coherence is equal to the visibility,  $V$ , of the fringes of a two-beam interference test, as given by

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

where  $I_{\max}$  is the intensity at a maximum of the interference pattern, and  $I_{\min}$  is the intensity at a minimum.

**Note 2:** Light is considered to be highly coherent when the degree of coherence exceeds 0.88, partially coherent for values less than 0.88 but more than nearly zero values, and incoherent for nearly zero and zero values.

**Q.14.** Two coherent light beams have intensity ratio 9 : 1. Find the ratio of maximum intensity to minimum intensity in the interference pattern. (2016)

#### Other Related Questions-

**Q.** Two coherent sources have intensity ratio 81:1, find the ratio of the maximum intensity to minimum intensity in interference pattern. (2016)

**Sol.** ∴ The ratio of maximum intensity to the minimum intensity is given by

$$\frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} \quad \dots\dots\dots(i)$$

$$\text{Here, } \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{9}{1}$$

$$\frac{a_1}{a_2} = \frac{3}{1} \text{ or } a_1 = 3a_2$$

.....(ii)

Substituting the value of  $a_1$  from eq.(ii) in eq.(i), we get

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$$\frac{I_{\max}}{I_{\min}} = \frac{(3a_2 + a_2)^2}{(3a_2 - a_2)^2} = \frac{(4)^2}{(2)^2} = \frac{16}{4} = \frac{4}{1}$$

**Q.15.** Define Fraunhofer Diffraction at a single slit.

#### Other Related Questions-

**Q.** Derive an expression for intensity distribution due to Fraunhofer diffraction at single slit and show that the relative intensities of the successive maxima are nearly in the ratio.

$$1 : \frac{1}{22} : \frac{1}{61} : \frac{1}{121} \quad (2017)$$

**Q.** Only plot the intensity distribution pattern due to Fraunhofer diffraction at single slit. Also show that the relative intensities in the successive maxima are nearly in the ratio.

$$1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} : \dots\dots\dots$$

**Q.** Demonstrate that in Fraunhofer diffraction at single slit, intensities ratio of principal maxima, first secondary maxima, second secondary maxima is given by  $1 : \frac{1}{22} : \frac{1}{61} : \frac{1}{121}$ . (2019)

#### Ans. Fraunhofer Diffraction At A Single Slit (Calculus Method):-

Let a monochromatic parallel beam of light be incident on the slit AB of width  $a$ . The secondary waves traveling in the same direction as the incident light come to focus at the point P. The secondary waves traveling at an angle  $\theta$  come to the focus at P' (figure 1). Let us consider the screen to be at a distance  $r$  from the slit. The center of the slit O is the origin of coordinates. Further, let us consider a small element  $dz$  of the wave front with the coordinates  $(0, z)$ . The coordinates of the point P' are  $(x_p, z_p)$  (figure 2). The distance of the element  $dz$  from the point P' is  $\rho$ .

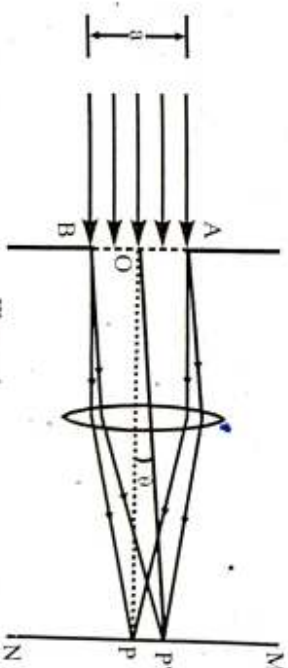
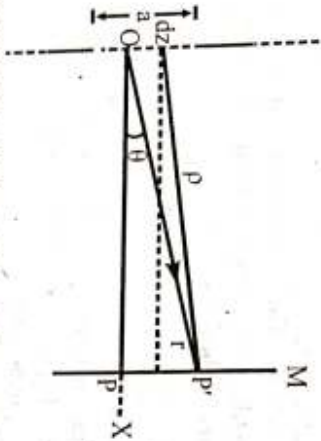


Figure 1

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The displacement at the point P' due to the element dz at any instant is given by

$$dy = Kdz \sin 2\pi \left( \frac{1}{T} - \frac{\rho}{\lambda} \right) \quad \dots\dots(i)$$



The resultant displacement at P' due to the whole wave front is obtained by integrating the above eq. (i)

$$y = K \int_{-\frac{a}{2}}^{\frac{a}{2}} \sin 2\pi \left( \frac{1}{T} - \frac{\rho}{\lambda} \right) dz \quad \dots\dots(ii)$$

Also,

$$\rho^2 = x_0^2 + (z_0 - z)^2 \quad \dots\dots(iii)$$

$$r^2 = x_0^2 + z_0^2$$

$$x_0^2 = r^2 - z_0^2$$

Substituting the value of  $x_0^2$  in eq. (iii), we get

$$\rho^2 = r^2 - z_0^2 + (z_0 - z)^2$$

$$\therefore \rho^2 = r^2 \left[ 1 - \frac{2zz_0}{r^2} + \frac{z^2}{r^2} \right] \quad \dots\dots(iv)$$

In the case of Fraunhofer diffraction, the screen is at a very large distance from the slit, therefore  $r \gg z$  and  $\frac{z}{r}$  is negligible.

$$\therefore \rho^2 = r^2 \left[ 1 - \frac{2zz_0}{r^2} \right]$$

$$\text{or } \rho = r \left[ 1 - \frac{2zz_0}{r^2} \right] \approx \left[ r - \frac{2zz_0}{r} \right]$$

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But  $\frac{z_0}{r} = \sin \theta$

$$\therefore \rho = r - z \sin \theta \quad \dots\dots(v)$$

Substituting the value of  $\rho$  in eq. (ii), we get

$$\begin{aligned} y &= K \int_{-\frac{a}{2}}^{\frac{a}{2}} \sin 2\pi \left( \frac{1}{T} - \frac{r}{\lambda} + \frac{z \sin \theta}{\lambda} \right) dz \\ &= -\frac{K\lambda}{2\pi \sin \theta} \left[ \cos 2\pi \left[ \frac{1}{T} - \frac{r}{\lambda} + \frac{a \sin \theta}{2\lambda} \right] - \cos 2\pi \left[ \frac{1}{T} - \frac{r}{\lambda} - \frac{a \sin \theta}{2\lambda} \right] \right] \\ &= \frac{K\lambda}{2\pi \sin \theta} \left[ 2 \sin 2\pi \left( \frac{1}{T} - \frac{r}{\lambda} \right) \sin 2\pi \left( -\frac{a \sin \theta}{2\lambda} \right) \right] \\ &= \frac{K\lambda}{\pi \sin \theta} \sin 2\pi \left( \frac{1}{T} - \frac{r}{\lambda} \right) \sin \left( \frac{\pi a \sin \theta}{\lambda} \right) \end{aligned}$$

$$\text{Let } \left( \frac{\pi a \sin \theta}{\lambda} \right) = \alpha$$

$$\therefore y = Ka \left( \frac{\sin \alpha}{\alpha} \right) \sin 2\pi \left( \frac{1}{T} - \frac{r}{\lambda} \right) \quad \dots\dots(vi)$$

The amplitude at P' is  $Ka \left( \frac{\sin \alpha}{\alpha} \right)$  and the intensity at P' is

$$I' = K^2 a^2 \left( \frac{\sin^2 \alpha}{\alpha^2} \right) = I_0 \left( \frac{\sin^2 \alpha}{\alpha^2} \right) \quad \dots\dots(vii)$$

Here  $I_0 = K^2 a^2$  and is the value of intensity at P, for  $\alpha = 0$ , as  $\frac{\sin \alpha}{\alpha} = 1$  when

$$\alpha \rightarrow 0.$$

(i) **Central Maximum:-**

For the point P on the screen (figure 2)  $\theta = 0$ ; and hence  $\alpha = 0$

The value of  $\frac{\sin \alpha}{\alpha}$  when  $\alpha \rightarrow 0$  is equal to 1. Hence, the intensity at

$$P = I_0 \left( \frac{\sin \alpha}{\alpha} \right)^2 = I_0 \text{ which is a maximum.}$$

(ii) **Secondary Maxima:-**

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The directions of secondary maxima are given by the equation,

$$\sin \theta_s = \frac{(2n+1)\lambda}{2a}$$

Substituting this value of  $\theta_s$  in equation  $\alpha = \frac{\pi}{\lambda} a \sin \theta$ , we get

$$\alpha = \frac{\pi}{\lambda} \cdot \frac{a(2n+1)\lambda}{2a} = \frac{(2n+1)\pi}{2} \quad \dots\dots\dots \text{(viii)}$$

Substituting  $n=1, 2, 3$  etc in eq.(viii), the values of  $\alpha$  are given by

$$\alpha = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \text{ etc}$$

(a) For the first secondary maximum,  $\alpha = \frac{3\pi}{2}$

$$\text{and } I = I_0 \left( \frac{\sin \alpha}{\alpha} \right)^2 = I_0 \left[ \frac{\sin \frac{3\pi}{2}}{\frac{3\pi}{2}} \right]^2 = I_0 \left[ \frac{1}{\frac{3\pi}{2}} \right]^2 = \frac{4I_0}{9\pi^2} = \frac{I_0}{22}$$

(b) For the secondary maximum,  $\alpha = \frac{5\pi}{2}$  and

$$I = I_0 \left( \frac{\sin \alpha}{\alpha} \right)^2 = I_0 \left[ \frac{\sin \frac{5\pi}{2}}{\frac{5\pi}{2}} \right]^2 = I_0 \left[ \frac{-1}{\frac{5\pi}{2}} \right]^2 = \frac{4I_0}{25\pi^2} = \frac{I_0}{61}$$

(c) For the tertiary maximum,  $\alpha = \frac{7\pi}{2}$  and

$$I = I_0 \left( \frac{\sin \alpha}{\alpha} \right)^2 = I_0 \left[ \frac{\sin \frac{7\pi}{2}}{\frac{7\pi}{2}} \right]^2 = I_0 \left[ \frac{-1}{\frac{7\pi}{2}} \right]^2 = \frac{4I_0}{121}$$

then the ratio of successive maxima is

$$1 : \frac{1}{22} : \frac{1}{61} : \frac{1}{121}$$

Thus, the secondary maxima are of decreasing intensity and the directions of these maxima are obtained from the equation given above.

tions of these maxima are obtained from the equation given above.

The intensity at P' is given by,  $I' = I_0 \left( \frac{\sin \alpha}{\alpha} \right)^2$

$$dI' = I_0 \left[ \frac{\alpha^2 2 \sin \alpha \cos \alpha - (\sin^2 \alpha) 2\alpha}{\alpha^4} \right] d\alpha$$

For  $I'$  to be a maximum,  $\frac{dI'}{d\alpha} = 0$

$$\alpha^2 (2 \sin \alpha \cos \alpha) - (\sin^2 \alpha) 2\alpha = 0$$

$$\therefore \tan \alpha = \alpha$$

If graphs are plotted for  $y = \alpha$ , and  $y = \tan \alpha$ , it will be found that the secondary maxima are not exactly midway between two minima. The positions of the secondary maxima are slightly towards the central maximum (figure 3).

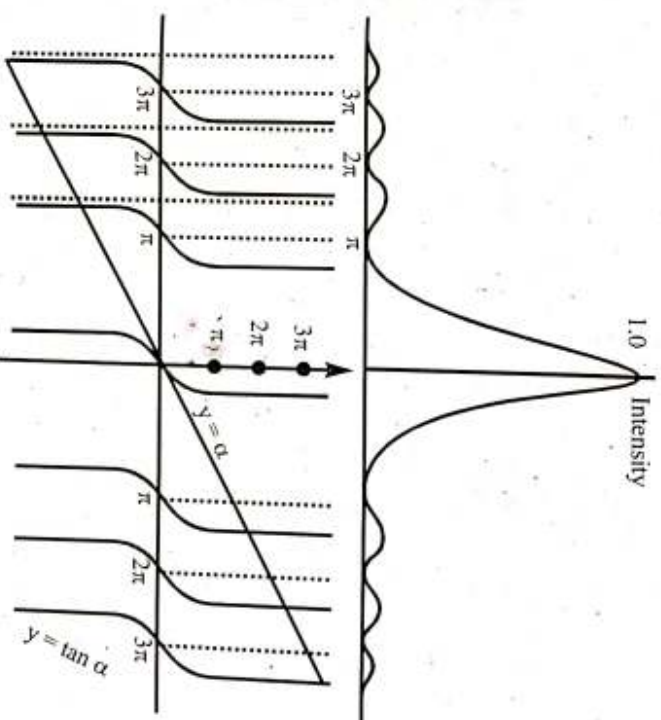


Figure 3

(iii) Secondary Minima:-

The directions of the secondary minima are given by the equation

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$$a \sin \theta = n\lambda$$

Substituting the value of  $a \sin \theta$  in eq. (ix), we get

$$\alpha = \frac{\pi}{\lambda} \cdot n\lambda = n\pi \quad \dots\dots (ix)$$

Substituting  $n = 1, 2, 3$  etc in eq. (ix), we obtain

$$\alpha = \pi, 2\pi, 3\pi \text{ etc.}$$

When these values of  $\alpha$  are substituted in the eq. (vii) for intensity, we get  $I = 0$

In figure (3), the positions of the secondary minima are shown for values of  $\alpha = \pi, 2\pi, 3\pi$  etc.

$\frac{\lambda}{a}, \frac{2\lambda}{a}, \frac{3\lambda}{a}$  etc. refer to the values of  $\sin \theta$  for these positions.

### Q.16. Discuss briefly Lloyd's single mirror.

Other Related Questions -

Q. Explain

(i) In Lloyd's mirror, central (zero<sup>th</sup> order) fringe is not visible.

(ii) In Lloyd's mirror central fringe is dark

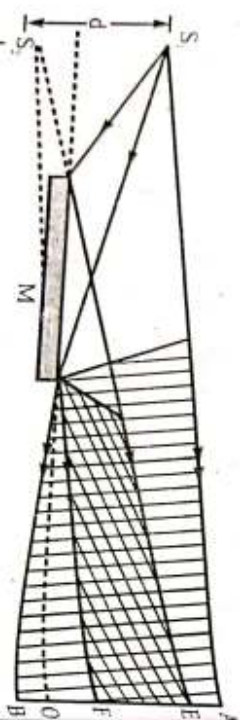
(iii) Difference between biprism and Lloyd's mirror fringes.

Q. Write short note on Lloyd's single mirror.

Ans. Lloyd's Single Mirror:-

(2018, 1

In 1834, Lloyd devised an interesting method of producing interference, using a single mirror and using almost grazing incidence. The Lloyd mirror consists of a plane mirror about 30 cm in length and 6 to 8 cm in breadth (figure). It is polished on the front surface and blackened at the back to avoid multiple reflections. A cylindrical wavefront coming from a narrow slit  $S_1$  falls on the mirror which reflects a portion of the incident wavefront, giving rise to a virtual image of the slit  $S_2$ .



Another portion of the wavefront proceeds directly from the slit  $S_1$  to the screen. The slits  $S_1$  and  $S_2$  act as two coherent sources. Interference fringes are observed on the screen.

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tween direct and reflected waves occurs within the region of overlapping of the two beams and fringes are produced on the screen placed at a distance  $D$  from  $S_1$  in the shaded portion EF.

The point O is equidistant from  $S_1$  and  $S_2$ . Therefore, central (zero-order) fringe is expected to lie at O (the perpendicular bisector of  $S_1S_2$ ) and it is also expected to be bright. However it is not usually seen since the point O lies outside the region of interference (only the direct light and not the reflected light reaches O).

By moving the screen nearer to the mirror such that it comes into contact with the mirror, the point O can be just brought into the region of interference. With white light the central fringe at O is expected to be white but in practice it is dark. The occurrence of dark fringe can be understood taking into the consideration of the phase change of  $\pi$  that light suffers when reflected from the mirror. The phase change leads to a path difference of  $\lambda/2$  and hence destructive interference occurs there.

**Determination of wavelength:-**

The fringe width is given by equation. Thus,

$$\beta = \frac{\lambda D}{d}$$

Measuring  $\beta$ ,  $D$  and  $d$ , the wavelength  $\lambda$  can be determined.

**Comparison between the fringes produced by biprism and Lloyd's mirror:-**

(i) In biprism the complete set of fringes is obtained. In Lloyd's mirror a few fringes on one side of the central fringe are observed, the central fringes being itself invisible.

(ii) In biprism the central fringes is bright whereas in case of Lloyd's mirror, it is dark.

(iii) The central fringe is less sharp in biprism than that is Lloyd's mirror.

**Q.17. Describe the procedure to determine the refractive index of materials in the form liquid using Newton's ring experiment. (2019)**

**Sol. Refractive index of a liquid:-**

The liquid, whose refractive index is to be determined, is filled in the gap between the lens and plane glass plate. Now the liquid film substitutes the air film. The condition for interference may then be written as where  $\mu$  is the refractive index of the liquid. For normal incidence the equation become

$$2\mu t = n\lambda$$

$$\text{As } t = \frac{r^2}{2R}, \quad \frac{2\mu r^2}{2R} = n\lambda$$

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$$\text{Or } r^2 = \frac{m\lambda R}{\mu} \quad \therefore D^2 = \frac{4m\lambda R}{\mu}$$

Following the above relation, the diameter of  $m^{\text{th}}$  dark ring may be expressed as

$$[D_m^2]_{\text{air}} = \frac{4m\lambda R}{\mu} \quad \dots\dots\dots(i)$$

Similarly, the diameter of the  $(m+p)^{\text{th}}$  ring is given by

$$[D_{m+p}^2]_{\text{air}} = \frac{4(m+p)\lambda R}{\mu} \quad \dots\dots\dots(ii)$$

Subtracting eq.(i) from eq.(ii), we get

$$[D_{m+p}^2]_{\text{air}} - [D_m^2]_{\text{air}} = \frac{4p\lambda R}{\mu} \quad \dots\dots\dots(iii)$$

But we know that

$$[D_{m+p}^2]_{\text{air}} - [D_m^2]_{\text{air}} = 4p\lambda R \quad \dots\dots\dots(iv)$$

$$\mu = \frac{(D_{m+p}^2)_{\text{air}} - (D_m^2)_{\text{air}}}{(D_{m+p}^2)_{\text{air}} - (D_m^2)_{\text{air}}} \quad \dots\dots\dots(v)$$

**Q.18. Find the expression for the displacement of fringes when a thin sheet of mica is introduced in the path of one of the interfering beam in biprism experiment.**

**Sol. Lateral Displacement of Fringes :-**

The biprism experiment can be used to determine the thickness of a given thin sheet of transparent material such as glass or mica. If a thin transparent sheet is introduced in the path of one of the two interfering beams, the fringe system gets displaced towards the beam in whose path the sheet is introduced. By measuring the amount of displacement, the thickness of the sheet can be determined.

Suppose  $S_1$  and  $S_2$  are the virtual coherent monochromatic sources of light. The point O is equidistant from  $S_1$  and  $S_2$ , where we obtain the central bright fringe. Therefore, the optical path  $S_1O = S_2O$ . Let a transparent plate G of thickness  $t$  and refractive index  $\mu$  be introduced in the path of one of the beams (figure). The optical path lengths  $S_1O$  and  $S_2O$  are now not equal and the central bright fringe shifts to P from O. The light waves from  $S_1$  and  $S_2$  travel paths in air and partly in the sheet G; the distance travelled in air is  $(S_1P - t)$  and the distance travelled in the sheet is  $t$ .

The optical path  $\Delta_{S_1P} = (S_1P - t) + \mu t = S_1P + (\mu - 1)t$

The optical path  $\Delta_{S_2P} = S_2P$

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The optical path difference at P is  $\Delta_{S_1P} - \Delta_{S_2P} = 0$ , since in the presence of the thin sheet, the optical path lengths  $S_1P$  and  $S_2P$  are equal and central zero fringe is obtained at P.

$$\therefore \Delta_{S_1P} = \Delta_{S_2P}$$

$$\Rightarrow [S_1P + (\mu - 1)t] = S_2P$$

$$\therefore S_2P - S_1P = (\mu - 1)t$$

But according to the relation  $S_2P - S_1P = \frac{x\lambda}{D}$  where  $x$  is the lateral shift of the central fringe due to the introduction of the thin sheet.

$$(\mu - 1)t = \frac{x\lambda}{D}$$

hence, the thickness of the sheet is  $t = \frac{x\lambda}{D(\mu - 1)}$

### Numerical

**(2015) In a Newton's ring experiment, what will be the order of a dark ring which will have double the diameter of that 20<sup>th</sup> dark ring? The wave length of**

**light is 5890 Å.**

**(2016)**

**Order of dark ring is 20**

**wavelength  $\lambda = 5890 \text{ Å}$**

**We know that the radius of the ring is given by 'r',**

$$r = \sqrt{\lambda \times R \times m}$$

$$\text{Diameter } (d) = 2r = 2\sqrt{\lambda \times R \times m}$$

$$D^2 = 4\lambda Rm$$

$$D_1 = 4\lambda R \times m_1$$

$$D_2 = 4\lambda R \times m_2 = 4\lambda R \times 79 = 80\lambda R$$

**Given that :**

$$D_1 = 2D_2$$

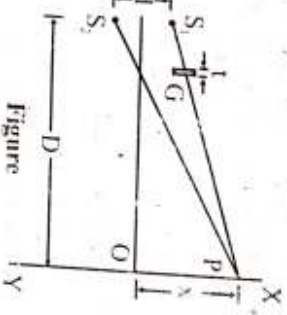
$$D_1^2 = D_2^2$$

$$4\lambda Rm_1 = 4 \times 80 \times \lambda R$$

$$m_1 = 80$$

**(2016)**

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Q.2. Two interfering beams have intensities in the ratio 1:4. Find the visibility of the interference fringes if the degree of coherence is  $\pi/2$ .

Other Related Questions -

Q. Two coherent light beams have intensities in the ratio 1:4. Calculate the visibility of the interference fringes if the degree of coherence is  $\pi/2$ .

Sol. The resultant intensity at a point in the region of superposition is given by

$$I = a_1^2 + a_2^2 + 2a_1a_2 \cos \delta$$

The above equation may also be written in terms of intensity as under:

$$I = I_1 + I_2 + 2\sqrt{I_1I_2} \cos \delta$$

where  $I_1 = a_1^2$  and  $I_2 = a_2^2$ .

$$\delta = \frac{\pi}{2}; I_1 = 4I \text{ and } I_2 = I$$

Substituting the above values in the expression for intensity,

$$I = 4I + I + 2\sqrt{4I \cdot I} \cos \frac{\pi}{2} = 4I + I + 0 = 5I$$

Q.3. Calculate the intensities in superposition of two interfering coherent light beams having intensities  $I_0$  and  $4I_0$  and phase differences  $2\pi/3$ . Same As -

Q. Calculate the intensity due to superposition of two interfering coherent beams having intensity  $I_0$  and  $4I_0$  and phase difference  $2\pi/3$ .

Ans. The resultant intensity at a point in the region of superposition is given by

$$I = a_1^2 + a_2^2 + 2a_1a_2 \cos \delta$$

The above equation may also be written in terms of intensity as under:

$$I = I_1 + I_2 + 2\sqrt{I_1I_2} \cos \delta$$

where  $I_1 = a_1^2$  and  $I_2 = a_2^2$ .

Given  $\delta = 2\pi/3, I_1 = 4I_0, I_2 = I_0$

Substituting the above values in the expression for intensity,

$$I = 4I_0 + I_0 + 2\sqrt{4I_0I_0} \cos \frac{2\pi}{3} \\ = 5I_0 - 2I_0 = 3I_0$$

Q.4. In a Newton's ring experiment, the diameter of the  $n^{\text{th}}$  ring is 0.3 cm and 0.25 cm when a transparent liquid is introduced between the lens and the plate. Calculate the value of refractive index of the liquid. (2014)

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$$\mu = \frac{(D_n^2)_{\text{air}}}{(D_n^2)_{\text{liq}}} = \frac{(0.30 \text{ cm})^2}{(0.25)^2} = 1.44$$

2.5. In a Newton's ring experiment, the diameter of 4<sup>th</sup> and 12<sup>th</sup> dark ring are 0.40 cm and 0.70 cm, respectively. Find out the diameter of 20<sup>th</sup> ring.

Sol. The difference of  $(n+p)^{\text{th}}$  and  $n^{\text{th}}$  dark ring is given by

$$D_{n+p}^2 - D_n^2 = 4pR\lambda \quad \dots\dots\dots(i)$$

here,  $(n+p) = 12$  and  $n = 4, D_{12} = 0.7 \text{ cm}$  and  $D_4 = 0.4 \text{ cm}$ .

$$D_{12}^2 - D_4^2 = 4 \times 8 \times R\lambda \quad \dots\dots\dots(ii)$$

and  $D_{20}^2 - D_4^2 = 4 \times 16 \times R\lambda \quad \dots\dots\dots(iii)$

Dividing eq. (iii) by eq. (ii), we get

$$\frac{D_{20}^2 - D_4^2}{D_{12}^2 - D_4^2} = \frac{4 \times 16}{4 \times 8} = 2$$

$$D_{20}^2 - D_4^2 = 2(D_{12}^2 - D_4^2)$$

$$D_{20}^2 = 2D_{12}^2 - D_4^2 = 2(0.7)^2 - (0.4)^2 = 0.98 - 0.16 = 0.82$$

$$D_{20} = \sqrt{0.82} = 0.9 \text{ cm}$$

## Chapter 4

Q.1. What is population inversion. Why it is essential for lasing action.

## Other Related Questions -

(2012,1)

Q. Population inversion is an essential condition for lasing action in conventional laser, explain.

Q. Explain population inversion.

Ans. Population Inversion:-

When the material is in thermal equilibrium condition, the population ratio is governed by the Boltzmann distribution law according to the following equation:

$$\frac{N_2}{N_1} = e^{-\frac{E_2 - E_1}{kT}}$$

.....(1)

It means that the population  $N_2$  at the excited level  $E_2$  will be far smaller than the population  $N_1$  at the level  $E_1$ . The condition in which there are more atoms in the lower energy level and relatively lesser number of atoms in a higher energy level is called normal condition or thermal equilibrium condition (1(a)). Thus, under thermal equilibrium,  $N_1 \gg N_2$ .

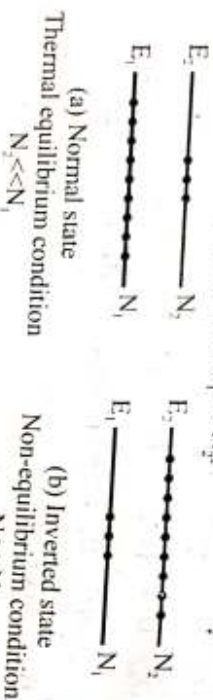


Figure 1

To achieve a high percentage of stimulated emission, a majority of atoms should be at the higher energy level than at the lower energy level. This may somehow enhance the number of atoms in the excited level, such that the population ratio  $N_2/N_1$  momentarily increases without change in temperature. This is a non-equilibrium condition of the material in which population of upper energy level  $N_2$  momentarily exceeds the population of the lower energy level  $N_1$  (figure 1(b)). That is

$$N_2 > N_1$$

.....(ii)

From eq.(1) it is seen that  $N_2$  can exceed  $N_1$  only if the temperature  $T$  is negative. In view of this, the state of population inversion is sometimes referred to as a **negative temperature state**. It does not mean that we can attain a state of population below absolute zero. The terminology underlines the fact that in a state of population inversion is a non-equilibrium state. It should be borne in mind that the population inversion is attained at normal temperatures.

The system shown in figure 1(a) has two energy levels. At thermal equilibrium, the population of the lower energy level is much greater than the population of the upper energy level. Visit us for more detail at : [www.winsomepublications.in](http://www.winsomepublications.in)

equilibrium, photon absorption and emission processes take place side by side, but because  $N_1 > N_2$ , the system absorbs photons rather than emit photons. Now suppose that the system is supplied with energy from an external source till  $N_2$  exceeds  $N_1$ . Then, the system is said to have attained that state of population inversion. The population inversion took place between the levels  $E_2$  and  $E_1$ .



Figure 2 - Amplification of a light wave in a medium with population inversion

When the system is in the population inversion condition, a few randomly emitted photons trigger stimulated emission of photons and those stimulated photons induce more stimulated emissions and so on. Consequently, light gets amplified (figure 2) and a cascade of light is produced. However, in this process atoms from  $E_2$  level make downward transitions and as soon as the population at lower level becomes equal or larger than that at the excited level, population inversion comes to an end. Energy is again to be supplied to the system to take it into the state of population inversion.

The non-equilibrium condition is attained by employing pumping techniques to transfer large number of atoms from lower energy level to higher energy level.

Q.2. Write short note on Michelson Interferometer.

(2017)

Ans. Michelson Interferometer:-

A schematic representation of the Michelson interferometer is shown in figure. It makes use of a broad source of quasi-monochromatic radiation. The incident beam is split into two by a beam splitter which is oriented at 45° with the direction of the beam. The two beams, reflected and transmitted, proceed to the two mirrors, which reflect them back. The reflected beams are combined by the beam splitter. A physical beam splitter has to have a finite thickness and one of its surfaces is coated for beam splitting. Thus one of the beams traverses the thickness three times while the other beam does so only once. This extra path traversed by the beam can be compensated by translating one of the mirrors; but the chromatic dispersion in glass cannot be compensated. In order to observe fringe in white light, a compensating glass plate identical to the beam splitter is placed in one arm. Both the beams have identical optical paths. When the mirrors are moved at 45°, the interference pattern is enclosed in a circular fringe.

micropub

formed by the beam splitter. The air film has constant thickness. Since the source is broad, circular fringes of constant inclination are formed.

The Michelson interferometer has a number of applications; notably among them are:

### Measurement of wavelength:-

The condition for interference in air ( $n=1$ ) is

$$2t \cos \theta_2 = m\lambda$$

When the observation is made at the center,  $\theta_2 = 0$  and hence

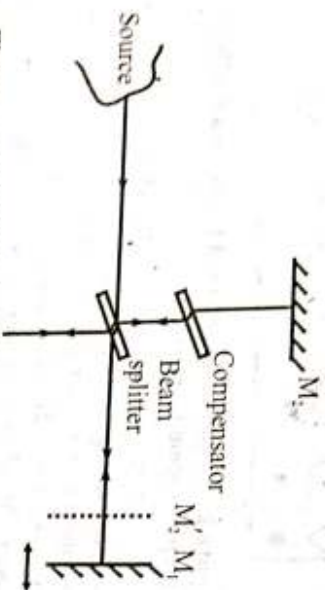


Figure - A schematic of a Michelson interferometer

$$2t = m'\lambda$$

When one of the mirrors is translated by distance  $t$  that can be read on a very accurate micrometer and the number of fringes  $m'$  passing through the center is observed, the wavelength  $\lambda$  is obtained as

$$\lambda = \frac{2t}{m'}$$

.....(ii)

The order  $m'$  is measured to full integer. Higher accuracy in the measurement of  $t$  is obtained by fractional measurement of fringe order. On the other hand if  $\lambda$  is known,  $t$  can be measured. The calibration of the standard meter involves the measurement of length in terms of wavelength. Michelson carried out this with the help of etalons.

### Q.3. Explain metastable state.

Ans. Metastable State:-

(2014)

An atom can be excited to a higher level by supplying energy to it of nanoseconds ( $10^{-9}$  s) through spontaneous emission. It means that atoms do not stay long enough at the excited state to be stimulated. As a result, even though the pumping agent continuously raises the atoms to the excited level, they undergo spontaneous transitions and rapidly return to the lower energy level. Population inversion cannot be established under such circumstances.

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In order to establish the condition of population inversion, the excited atoms are required to 'wait' at the upper energy level till a large number of atoms accumulate at that level. In other words, it is necessary that the excited state has a longer lifetime. A metastable state is such a state. Because of restrictions imposed by conservation of angular momentum, an electron excited to a metastable state cannot return to the ground state by emitting a photon, as it is generally expected to do. Such a state in which single-photon emission is impossible, has an unusually long time and is called a metastable state. Atoms excited to the metastable states remain excited for an appreciable time, which is of the order of  $10^{-4}$  to  $10^{-3}$  s. This is  $10^4$  to  $10^6$  times the lifetimes of the ordinary energy levels.

Therefore, the metastable state allows accumulation of a large number of excited atoms at that level. The metastable state population can exceed the population at a lower level and establish the condition of population inversion in the lasing medium. It would be impossible to create the state of population inversion without a metastable state. Metastable state can be readily obtained in a crystal system containing impurity atoms. These levels lie in the forbidden band gap of the host crystal. Population inversion readily takes place as the lifetimes of these levels are large, and secondly, there is no competition in filling these levels, as they are localized levels.

There could be no population inversion and hence no laser action, if metastable states do not exist.

### Q.4. Explain stimulated emission.

(2014)

#### Other Related Question:-

Q. Define adsorption, spontaneous emission and stimulated emission of light and explain that why stimulated emission of light is essential for lasing action? (2019)

Ans. An atom in the excited state need not "wait" for spontaneous emission of photon. Well before the atom can make a spontaneous transition, it may interact with a photon with energy  $h\nu = E_2 - E_1$ , and make a downward transition. The photon is said to stimulate or induce the excited atom to emit a photon energy  $h\nu = (E_2 - E_1)$ . The passing photon does not disappear and in addition to it there is a second photon which is emitted by the excited atom (figure 1). The phenomenon of forced photon emission by an excited atom due to the action of an external agency is called stimulated emission or induced emission. The process may be expressed as



The probability that a stimulated transition occurs is given by

$$(P_{21})_{\text{stimulated}} \propto \rho(\nu) \quad \text{.....(i)}$$

or  $(P_{21})_{\text{stimulated}} = B_{21}\rho(\nu)$  where  $B_{21}$  is the constant of proportionality and is known as the Einstein coefficient for stimulated emission. It is a constant characteristic of the atom and represents the properties of the energy states  $E_2$  and  $E_1$ .  $B_{21}$  represents the

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probability for induced transition from level  $2 \rightarrow 1$ . The number of stimulating transitions occurring in the material at any instant will be equal to the product of the number of atoms at the energy level  $E_2$  and the probability  $P_{21}$  for the stimulated transition. Thus, the rate of stimulated emission of photons is given by

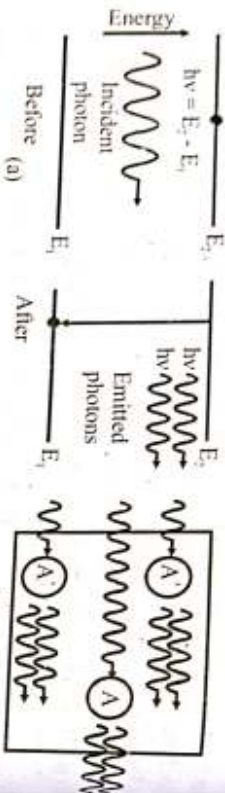
$$R_s = B_{21}(\nu)N_2 \dots\dots(ii)$$


Figure 1 - Stimulated emission (a) emission process (b) Material emits photons in a coordinated manner

In stimulated emission each incident photon encounters a previous excited atom, and the optical field of the photon interacts with the electron. The result of the interaction is a kind of resonance effect, which induced each atom to emit a second photon with the same frequency, direction, phase and polarization as the incident photon.

**Characteristics of stimulated emission:-**

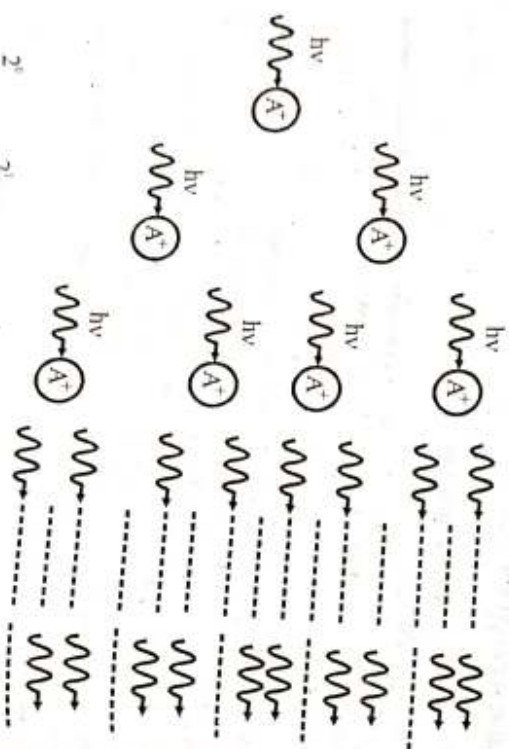


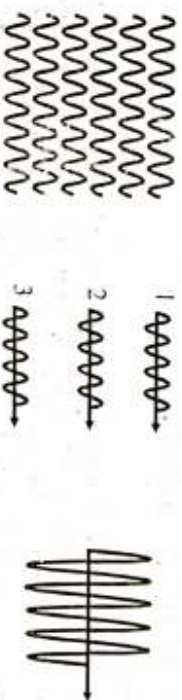
Figure 2 - Multiplication of stimulated photons into an avalanche

- The process of stimulated emission is controllable from outside.
- The photon induced in this process propagates in the same direction as that of stimulating photon.

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- The induced photon has features identical to that of the inducing photon. It has the same frequency, phase and plane of polarisation as that of the stimulating photon.

(iv) **Multiplication of photons** - The outstanding feature of this process is the multiplication of photons. For one photon interacting with an excited atom, there are two photons emerging. The two photon travelling in the same direction interact with two more excited atoms and generate two more photons and produce a total of four photons. These four photons in turn stimulate four excited atoms and generate eight photons, and so on. The number of photons builds up in an avalanche like manner, as shown in figure (2).



- Coherent radiation**
- Component waveform**
- Resultant waveform of waves of 1, 2 & 3**

Figure 3

- Light amplification** - All the light waves generated in the medium are due to one initial wave and all of the waves are in phase. Thus, the waves are coherent and interfere constructively (figure 3).
- The net intensity of light will be proportional to the square of the number of atoms radiating light. Thus,

$$I_{\text{total}} = N^2 I \dots\dots(iii)$$

The light emitted through the process of stimulated emission is of very high intensity and we say light is amplified.

**Stimulated emission of light is essential for laser action:-**

Stimulated emission is the basis for photon amplification and it is the fundamental mechanism underlying all laser action. In laser action the stimulating emission triggers a chain reaction in which the radiation from one atom stimulates another in succession until all the excited atoms in the system have returned to normalcy. In doing so, coherent monochromatic light (light of a single wavelength) is emitted.

**Absorption:-**

Suppose an atom is in the lower energy level  $E_1$ . If a photon of energy  $h\nu = (E_2 - E_1)$  is incident on the atom, it imparts its energy to the atom and disappears. Then we say that the atom absorbed an incident photon. As a result of absorption of adequate energy, the atom jumps to the excited state  $E_2$ . The transition is called **absorption transition**. It is also referred to as **induced absorption**. We may express the process as



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where A is an atom in the lower state and A\* is an excited atom.

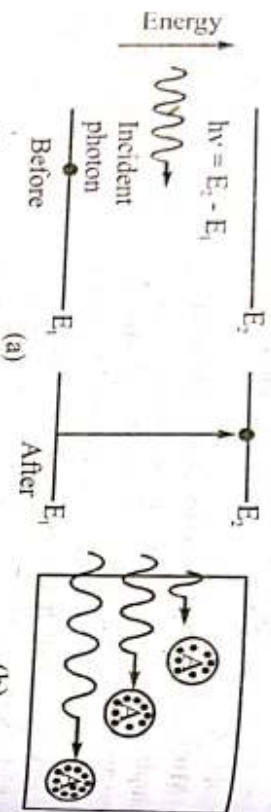


Figure - Absorption process (a) Induced absorption  
(b) Material absorbs photons

#### Spontaneous Emission:-

When an atom at lower energy level is excited to a higher energy level, it cannot stay in the excited state for a relatively longer time. In a time of about  $10^{-8}$  sec, the atom reverts to the lower energy state by releasing a photon of energy  $h\nu$ , where  $h\nu = (E_2 - E_1)$ . The emission of photon occurs on its own and without any external impetus given to the excited atom (figure 5). Emission of a photon by an atom without any external impetus is called **spontaneous emission**. We may write the process as  $A^* \rightarrow A + h\nu$

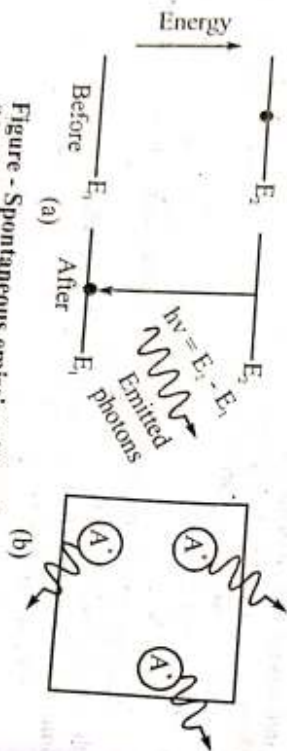


Figure - Spontaneous emission (a) emission process  
(b) Material emits photon haphazardly

#### Q.5. Draw a neat energy level diagram of He-Ne laser.

Other Related Questions:-

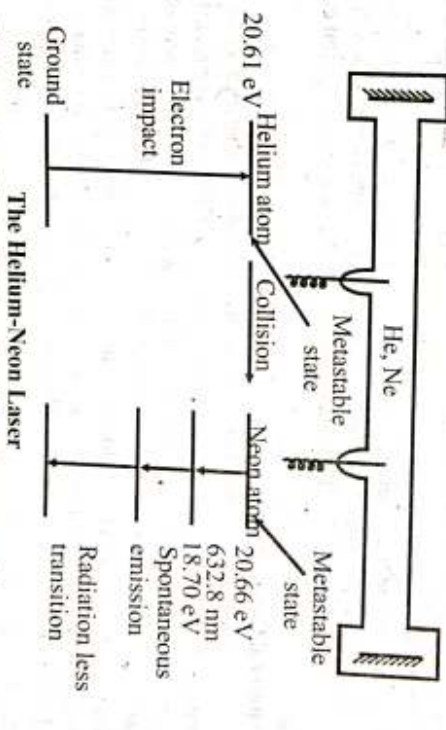
- Q. Write short note on He-Ne laser. (2017)
- Q. Draw relevant energy level diagram for He-Ne laser. (2009, 12, 16)
- Q. Describe construction and working of a He-Ne laser. (2014)
- Q. Explain working of a He-Ne laser using the relevant energy levels. (2011)
- Ans. Helium-Neon Laser:- Construction:- (2010)

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It consists of long discharge tube of length about 50 cm and diameter 1 cm. This tube is filled with a mixture of about 7 parts of He and 1 part of Ne at a low pressure ( $\approx 1$  mm of mercury). There are parallel mirrors at both ends of the tube, one of them partly transparent. The spacing of the mirrors is equal to an integral number of half wavelengths of the laser light.

#### Working:-

Electric discharge is produced in the gas by high frequency alternating current. Due to collisions with electrons from the discharge, He and Ne atoms are excited to metastable states of 20.61 and 20.66 eV respectively. He atoms transfer their energy to ground state Ne atoms. Thus with the help of He atoms population inversion is achieved in the Ne atoms.



The laser transition in Ne takes place from the metastable state at 20.66 eV to an excited state at 18.70 eV. A photon of wavelength 632.8 nm or 6328 Å is emitted in this transition. After that a spontaneous emission takes place from 18.70 eV to yield incoherent light. The remaining excitation energy is lost in collision with the tube walls.

#### Characteristics:-

- (1) He-Ne laser can be operated continuously.
- (2) It is more directional and more monochromatic.
- (3) Laser radiation is highly coherent.
- (4) Light from typical 1 mW laser beam is about 10,000 times brighter than ordinary light.

#### Q.6. Write short note on spatial and temporal coherence. (2015)

Other Related Questions:-

- Q. What is spatial coherence. (2017)
- Q. What is spatial and temporal coherence. (2018)
- Ans. Spatial Coherence:-

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Spatial coherence refers to the continuity and uniformity of a wave in a direction perpendicular to the direction of propagation. If the phase difference for any two fixed points in a plane normal to the wave propagation does not vary with time, then the wave is said to exhibit spatial coherence. It is also known as lateral coherence. Again looking at the point source S (figure),  $SP_1$ ,  $SP_2$  and therefore, the fields at points  $P_1$  and  $P_2$  would have the same phase. Thus, an ideal point source exhibits spatial coherence, as the waves produced by it are likely to have the same phase at points in space, which are equidistant from the source. On the other hand, an extended source is bound to exhibit lesser lateral spatial coherence.

Two points on the source separated by a lateral distance greater than one wavelength will behave quite independently. Therefore, correlation is absent between the phases of the waves emitted by them. The degree of contrast of the interference fringes produced by a source is a measure of the degree of the spatial coherence of its waves. The higher the contrast, the better is the spatial coherence.

#### Temporal Coherence:-

Temporal coherence is also known as longitudinal coherence. Let a point source of quasi-monochromatic light S (figure) emit light in all directions. Let it consider light travelling along the line  $SP_1P_2$ . The phase relationship between the points  $P_1$  and  $P_2$  depends on the distance  $P_1P_2$  and the coherence length of the light beam. The electric fields at  $P_1$  and  $P_2$  will be correlated in phase when the single wave train extends over greater length than the distance  $P_1P_2$ ; that is if the distance  $P_1P_2$  is less than the coherence length  $l_{coh}$ . Then, the waves are correlated in their rising and falling and they will preserve a constant phase difference. The points  $P_1$  and  $P_2$  would not have any phase relationship if the longitudinal distance  $P_1P_2$  is greater than  $l_{coh}$  since in such a case many wave trains would span the distance. It means different independent wave trains would be at  $P_1$  and  $P_2$  each other. The degree to which a correlation exists is known as the amount of longitudinal coherence.

**Q.7. Why continuous laser output cannot be obtained by Ruby laser?**  
**Other Related Questions:-**

**Q. Draw energy level diagram relevant for Ruby laser?**

**Q. Write short note on Ruby laser and explain its working.**

**Ans. Ruby laser:-**

(2013)

Ruby laser belongs to the class of solid state lasers. The term solid state one in which the active centers are fixed in a crystal or glassy material. Solid state lasers are electrically nonconducting. They are also called doped insulator lasers.

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Historically, the ruby laser was the first laser. It was invented in 1960 by Theodore Maiman, U.S.A. The ruby laser rod is in fact a synthetic ruby crystal,  $Al_2O_3$  crystal, doped with chromium ions at a concentration of about 0.05% by weight.  $Cr^{3+}$  ions are the actual active centers and have a set of three energy levels suitable for realizing lasing action whereas aluminium and oxygen atoms are inert.

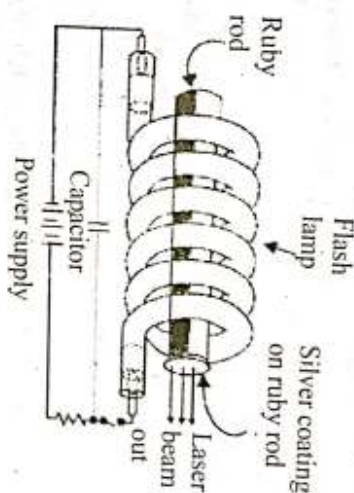


Figure (1)

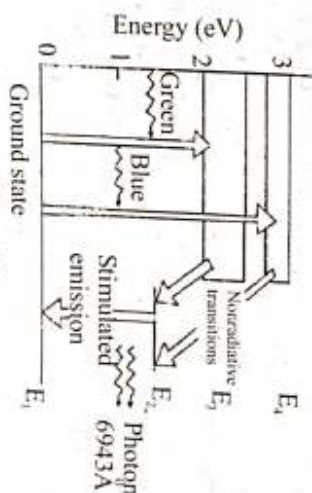


Figure (2)

**Construction:-**  
 The schematic of a ruby laser is shown in Fig (1). Ruby rod is taken in the form of a cylindrical rod of about 4 cm in length and 0.5 cm in diameter. Its ends are grounded and polished such that the end faces are exactly parallel and are also perpendicular to the axis of the rod. One face is silvered to achieve 100% reflection while the other is silvered to give 10% transmission and 90% reflection. The silvered faces constitute the Fabry-Pérot resonator. The laser rod is surrounded by a helical photographic flash lamp filled with xenon. Whenever activated by the power supply the lamp produce flashes of white light.

#### Working:-

Ruby laser uses a three-level pumping scheme. The energy levels of  $Cr^{3+}$  ions in the crystal lattice are shown in figure (2). There are two wide energy bands  $E_1$  and  $E_2$ , and a pair of closely spaced levels at  $E_3$ . When the flash lamp

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is activated, the xenon discharge generates an intense burst of white light lasting for a few milliseconds. The  $\text{Cr}^{3+}$  ions are excited to the energy bands  $E_2$  and  $E_3$  by the green and blue components of white light. The energy levels of these bands have a very small lifetime ( $\approx 10^{-9}$  s). Hence the excited  $\text{Cr}^{3+}$  ions rapidly lose some of the energy to the crystal lattice and undergo non-radiative transitions. They quickly drop to the levels  $E_2$ . The pair of levels at  $E_2$  and  $E_3$  are metastable states having a lifetime of approximately 1000 times more than the lifetime of  $E_3$  level. Therefore,  $\text{Cr}^{3+}$  ions accumulate at  $E_2$  level. When more than half of the  $\text{Cr}^{3+}$  ion population accumulates at  $E_2$  level, the state of population inversion is established between  $E_2$  and  $E_1$  levels. A chance photon emitted spontaneously by a  $\text{Cr}^{3+}$  ion initiates a chain of stimulated emission by other  $\text{Cr}^{3+}$  ions in the metastable state. Red photons of wavelength 6943 Å travelling along the axis of the ruby rod are repeatedly reflected at the end mirrors and light amplification takes place. A strong intense beam of red light emerges out of the front-end mirror.

Note that the green and blue components of light play the role of pumping agents and are responsible for causing population inversion. The spontaneous photons of  $\lambda = 6943$  Å, corresponding to red colour, act as input of the oscillator which actually gets amplified. The xenon flash lasts a few milliseconds. However, the laser does not operate throughout this period. Its output occurs in the form of irregular pulses of microsecond duration. It is because the stimulated transitions occur faster than the rate at which population inversion is maintained in the crystal. Once stimulated transition commences, the metastable state  $E_2$  gets depopulated very rapidly and at the end of each small pulse, the population at  $E_2$  has fallen below the threshold value required for sustained emission of light. As a result the lasing action and laser becomes inactive. The next pulse appears after the population inversion is once again restored. The process repeats.

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### Chapter 5

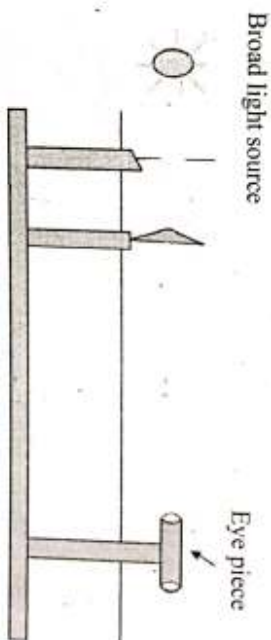
Q.1. Find the expression for the fringe width ( $\bar{X}$ ) with the help of Fresnel's biprism experiment.

Other Related Questions -

Q. Obtain an expression for the fringe width in the biprism experiment.

Ans. Experimental Arrangement:- (2016)

Light from monochromatic source is made to fall on a thin slit mounted vertically on a rigid optical bench fitted with a scale. The biprism and the screen (in this case an eye piece) are also mounted vertically. The eye piece can be moved in the plane perpendicular to the axis of bench using a micrometer based translation stage.



**Measurement of fringe width:** To get  $\bar{X}$ , fringes are first observed in the field of view of the microscope. The vertical wire of the eyepiece is made to coincide with one of the fringes and screw of micrometer is moved sideways and number of fringes is counted.

$$\bar{X} = \text{Distance moved} / \text{number of fringes passed}$$

Q.2. Give theory of Cornu's spiral and use it to explain diffraction at a straight edge. (2015)

Other Related Questions -

Q. Obtain an expression for the intensity distribution due to diffraction by a straight edge and also plot it. (2016)

Q. Write short note on Cornu's spiral. (2013, 19)

Q. What is Cornu's spiral? (2010)

Ans. Cornu's Spiral:-

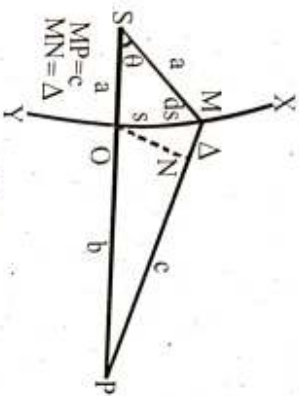
A narrow slit illuminated by light gives rise to a cylindrical wave front. Let S be a narrow slit perpendicular to the plane of the paper and illuminated by monochromatic light of wavelength  $\lambda$  figure (a).

For the cylindrical wave front XY, the slit is the axis of the cylindrical surface. The effect of the wave front at a point P in the plane is the same as that at all

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points along a line passing through the point P and parallel to the length of the segment PQ. Let  $y$  be the displacement at all points on the wavefront XY at time  $t$  such that

$$y = a \sin \omega t = a \sin \frac{2\pi t}{T}$$



Figure(a)

According to Huygens principle, every point on the primary wave front is a source of secondary disturbance and the resultant intensity at P can be obtained by combining the effect of all the secondary wavelets. Let us consider a small element  $ds$  of the wave front at M. The distance of the element measured along the curve is  $s$ . For the secondary disturbances from O to P, i.e., for distance  $OP = b$ , the phase difference is  $\frac{2\pi b}{\lambda}$ . The disturbance at the point due to a small element  $ds$  is given by

$$dy = K \sin 2\pi \left[ \frac{1}{T} - \frac{b}{\lambda} \right] ds$$

Similarly, the disturbance at P due to a small element at M is given by

$$dy = K \sin 2\pi \left[ \frac{1}{T} - \frac{c}{\lambda} \right] ds$$

where  $MP = c = b + \Delta$

$$dy = K \sin 2\pi \left[ \frac{1}{T} - \frac{b}{\lambda} - \frac{\Delta}{\lambda} \right] ds$$

$$dy = K \sin \left[ 2\pi \left( \frac{1}{T} - \frac{b}{\lambda} \right) - \frac{2\pi\Delta}{\lambda} \right] ds$$

$$dy = K \left[ \sin 2\pi \left( \frac{1}{T} - \frac{b}{\lambda} \right) \cos \frac{2\pi A}{\lambda} \right] ds - K \left[ \cos 2\pi \left( \frac{1}{T} - \frac{b}{\lambda} \right) \sin \frac{2\pi A}{\lambda} \right] ds \quad (6)$$

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$$y = K \sin 2\pi \left( \frac{t}{T} - \frac{b}{\lambda} \right) \int_0^s \cos \frac{2\pi \Delta}{\lambda} ds - K \cos 2\pi \left( \frac{t}{T} - \frac{b}{\lambda} \right) \int_0^s \sin \frac{2\pi \Delta}{\lambda} ds \quad \dots\dots\dots (V)$$

Let  $K \int_0^s \cos \frac{2\pi\lambda}{\lambda} ds = R \cos \phi$  and  $K \int_0^s \sin \frac{2\pi\lambda}{\lambda} ds = R \sin \phi$

Substituting these values in eq. (v), we obtain

$$y = R \sin 2\pi \left( \frac{t}{T} - \frac{b}{\lambda} \right) \cos \phi - R \cos 2\pi \left( \frac{t}{T} - \frac{b}{\lambda} \right) \sin \phi$$

$$= R \sin \left[ 2\pi \left( \frac{t}{T} - \frac{b}{\lambda} \right) - \phi \right]$$

$$\text{Also } R^2 = R^2 \sin^2 \phi + R^2 \cos^2 \phi$$

$$= K^2 \left[ \left\{ \int_0^s \sin \frac{2\pi\Delta}{\lambda} ds \right\}^2 + \left\{ \int_0^s \cos \frac{2\pi\Delta}{\lambda} ds \right\}^2 \right]$$

From the ASMP

$$c^2 = (a+b)^2 + a^2 - 2a(a+b) \cos \theta$$

$$= (a+b)^2 + a^2 - 2a(a+b) \left[ 1 - \frac{\theta^2}{2!} \right] = b^2 + a(a+b)\theta^2$$

$$\theta = \frac{s}{a} \therefore c^2 = b^2 + \frac{a(a+b)s^2}{a^2} = b^2 + \frac{(a+b)}{a} s^2$$

$$c = b \left[ 1 + \frac{(a+b)s^2}{a} \right]^{\frac{1}{2}} = b + \frac{(a+b)}{2ab} s^2$$

$$MP - NP = \Delta = c - b$$

$$= b + \frac{(a+b)}{2ab} s^2 - b$$

$$\Delta = \frac{(a+b)s^2}{2ab} \dots\dots\dots (vi)$$

Let  $\frac{2\pi\Delta}{\lambda} = \frac{\pi v^2}{2}$ , where  $v$  is a new variable depending on the values of  $a$ ,  $b$ ,  $\lambda$  and  $s$ . Substituting the value of  $\Delta$  from eq.(vi), we obtain

$$\frac{\pi}{\lambda} \left[ \frac{(b^2 - a^2)}{2ab} + \frac{1}{2} \right]$$

$$v^2 = v^2 \left[ \frac{1}{2(a+b)} \right]$$

$$s = v \sqrt{\frac{ab\lambda}{2(a+b)}}$$

.....(vii)

$$v = s \sqrt{\frac{2(a+b)}{ab\lambda}}$$

.....(viii)

$$ds = dv \sqrt{\frac{ab\lambda}{2(a+b)}}$$

Substituting the value of  $ds$  and  $\frac{2\pi\Delta}{\lambda}$  in the following relation

$$R^2 = K^2 \left[ \left\{ \int_0^s \cos \frac{2\pi\Delta}{\lambda} ds \right\}^2 + \left\{ \int_0^s \sin \frac{2\pi\Delta}{\lambda} \right\}^2 \right]$$

We get

$$R^2 = \frac{K^2 ab\lambda}{2(a+b)} \left[ \left( \int_0^u \cos \frac{\pi u^2}{2} du \right)^2 + \left( \int_0^v \sin \frac{\pi v^2}{2} du \right)^2 \right]$$

$$R^2 = K_1 [X^2 + Y^2]$$

$$X = \int_0^u \cos \frac{\pi u^2}{2} du \quad \text{and} \quad Y = \int_0^v \sin \frac{\pi v^2}{2} du$$

$$\int_0^u \cos \frac{\pi u^2}{2} du = M \cos \left( \frac{\pi u^2}{2} \right) + N \sin \left( \frac{\pi u^2}{2} \right)$$

and

$$\int_0^v \sin \left( \frac{\pi v^2}{2} \right) dv = M \sin \left( \frac{\pi v^2}{2} \right) - N \cos \left( \frac{\pi v^2}{2} \right)$$

$$\text{Here } M = \frac{\pi^0 u}{1} - \frac{\pi^2 u^3}{1.3.5} + \frac{\pi^4 u^5}{1.3.5.7.9}$$

$$\text{and } N = \frac{\pi u^3}{1.3} - \frac{\pi^3 u^5}{1.3.5.7} + \frac{\pi^5 u^7}{1.3.5.7.9.11}$$

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For a given value of  $s$  (the extent of the wave front contribution) the intensity at a point on the screen) the corresponding value of  $v$  is calculated from the relation,

$$v = \sqrt{\frac{2(a+b)}{ab\lambda}}$$

From the value of  $v$  thus obtained, the values of  $X$  and  $Y$  corresponding to the  $X$  and  $Y$  coordinates of a point on the spiral are obtained from Fresnel's integrals. Cornu's spiral is useful in understanding the Fresnel's diffraction patterns due to obstacles such as straight edge, thin wire, thick wire, narrow slit, wide aperture etc placed in the path of light.

#### Diffraction At a Straight Edge:-

$S$  is a narrow slit illuminated by monochromatic light of wavelength  $\lambda$ . The length of the slit is perpendicular to the plane of the paper.  $AD$  is a straight edge and the length of the edge is parallel length of the slit (figure (b)).  $XY$  is the incident cylindrical wave front. Cornu's spiral helps to obtain qualitatively the intensity distribution at a point on the screen.

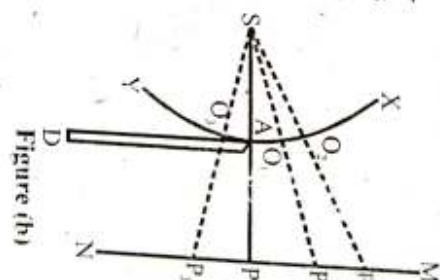


Figure (b)

Let us consider the points  $P_1$  and  $P_2$  in the illuminated portion and point  $P_3$  in the geometrical shadow region of the screen  $MN$ .  $A$ ,  $O_1$ ,  $O_2$  and  $O_3$  are the poles of the wave front for the points  $P_1$ ,  $P_2$ ,  $P_3$  and  $P_3$  respectively.

#### Intensity at the point $P_1$ :-

The pole of the wave front is the origin of coordinates. The lower half of the wave front is cut off by the obstacle and the intensity at  $P_1$  is due to the upper half of the wave front between  $A$  and  $X$ . This intensity is equal to  $I_0/4$ , where  $I_0$  is the intensity due to the whole wave front (figure (c)).

$$I = K_1 [X^2 + Y^2]$$

$$I = K_1 \left[ \left( \int_0^u \cos \frac{\pi u^2}{2} du \right)^2 + \left( \int_0^v \sin \frac{\pi v^2}{2} du \right)^2 \right] = K_1 [1 + 1] = 2K_1$$

Let  $I_0$  be the intensity due to the whole wave front.

$$\text{Then } I_0 = 2K_1 \Rightarrow K_1 = I_0/2$$

The intensity due to the upper half of the wave front alone is given by

$$I = \frac{I_0}{2} \left[ \left( \int_0^u \cos \frac{\pi u^2}{2} du \right)^2 + \left( \int_0^v \sin \frac{\pi v^2}{2} du \right)^2 \right] = \frac{I_0}{2} \left[ \frac{1}{4} + \frac{1}{4} \right] = \frac{I_0}{4}$$

This intensity is proportional to the square of the amplitude vector  $O_1P_1$ .

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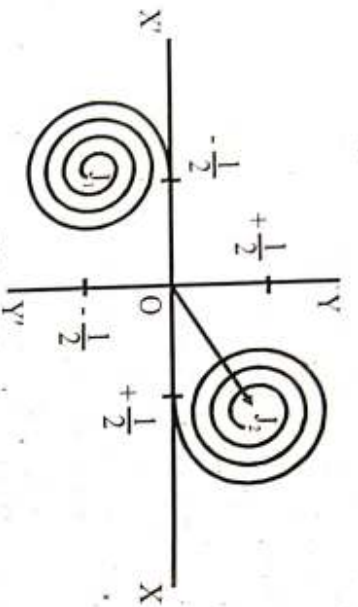


Figure (c)

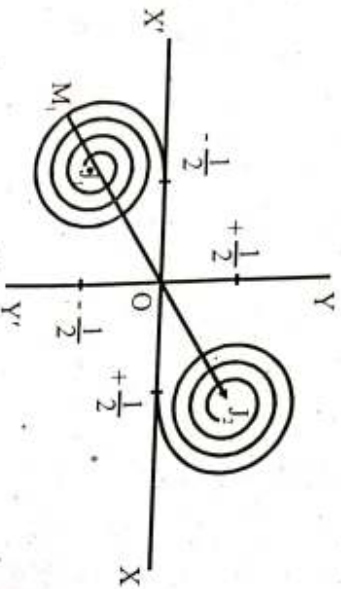
Intensity at the point  $P_1$  :-

Figure (d)

For the point  $P_1$ ,  $O_1$  is the pole of the wave front and let the exposed portion of the wave front between  $A$  and  $O_1$  correspond to the spiral  $O_1 M_1 J_1$ . The intensity in this case is proportional to the square of the vector  $M_1 J_1$ . Thus, the point  $P_1$  will be of maximum intensity.

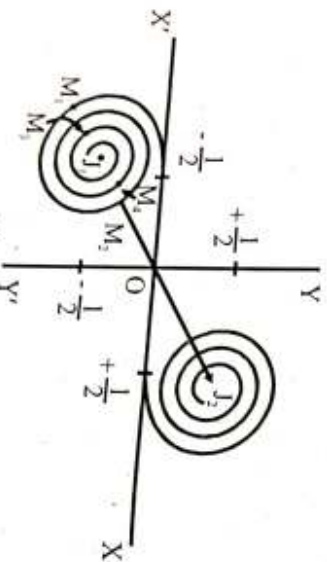
Intensity at the point  $P_2$  :-

Figure (e)

For the point  $P_2$ ,  $O_2$  is the pole of the wave front and let the exposed portion of the wave front between  $A$  and  $O_2$  correspond to the spiral  $O_2 M_2 J_2$ . The intensity in this case is proportional to the square of the vector  $M_2 J_2$ . Thus, the point  $P_2$  will be of maximum intensity.

portion of the wave front between  $A$  and  $O_2$  correspond to the length of the spiral  $OM_2$  (figure (e)). The intensity at  $P_2$  is proportional to the square of the amplitude vector  $M_2 J_2$  which is minimum.

Thus, when the point shifts away from  $P_1$ , the resultant intensity is proportional to the square of the amplitude vector whose magnitude passes through maxima and minima. When the point  $M$  shifts very near  $J_1$ , the difference in intensity between maxima and minima is very small and this results in uniform illumination.

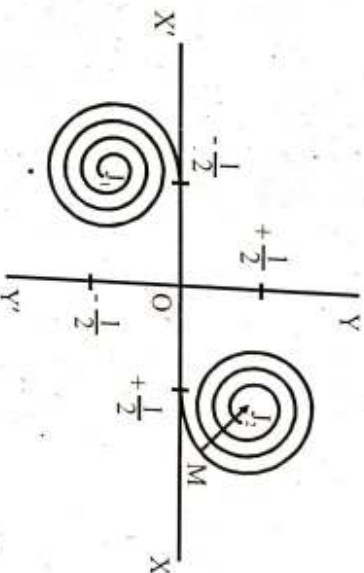
Intensity at the point  $P_3$  :-

Figure (f)

For the point  $P_3$ ,  $O_3$  pole of the wave front. The lower half of the wave front is cut off by the obstacle and the portion of the wave front between  $A$  and  $O_3$  is also obstructed corresponding to the length of the spiral  $OM$  (figure (f)). The intensity at  $P_3$  is proportional to the square of the amplitude  $MJ_2$ . As the point  $P_3$  shifts more and more into the region of geometrical shadow, the point  $M$  shifts more and more towards  $J_2$ . Thus, the magnitude of  $MJ_2$  gradually decreases. In other words, the intensity falls off gradually in the region of the geometrical shadow.

Q.3. Explain the construction of zone plate. (2013)

Other Related Questions-

Q. What is a zone plate? Give a simple theory and explain how it acts as a convergent lens and as a divergent lens at the same time. Compare performance of a zone plate with a lens. (2011)

Q. Give the theory of a zone plate and explain how it acts as a convergent lens as a divergent lens at the same time. What do you understand by phase reversal zone plate? (2009)

Q. Show that the radii of the half period zones are proportional to the square roots of the natural numbers ( $n$ ) while the area of the zones are independent of  $n$  (order of zones). (2017)

Q. Zone plate acts as convergent lens of multiple foci. (2016)

**Q. What is Zone plate? Explain that zone plate acts as a convergent lens multiple foci.**

**Ans. Zone plate:-**

A zone plate is a specially constructed screen such that light is obstructed from every alternate zone. It can be designed so as to cut off light due to the even numbered zones or that due to the odd numbered zones. The correctness of Fresnel's method in dividing a wavefront into half period zones can be verified with its help.



**Figure 1 - Zone Plate**

To construct a zone plate, concentric circles are drawn on white paper such that the radii are proportional to the square roots of the natural numbers. The odd numbered zones (i.e. 1<sup>st</sup>, 3<sup>rd</sup>, 5<sup>th</sup>, etc) are covered with black ink and a reduced photograph is taken. The drawing appears as shown in figure 1. The negative of the photograph will be as shown in figure 1. In the developed negative, the odd zones are transparent to incident light and the even zones will cut off light.

If such a plate is held perpendicular to an incident beam of light and a screen is moved on the other side to get the image, it will be observed that maximum brightness is possible at some position of the screen say  $b$  cm from the zone plate (figure 2). XO is the upper half of the incident plane wavefront. P is the point at which the light intensity is to be considered. The distance of the point from the wavefront is  $b$ .  $OM_1 (=r_1)$ ,  $OM_2 (=r_2)$  etc are the radii of the zones.

$$r_1 = \sqrt{b\lambda} \quad \text{and} \quad r_2 = \sqrt{2b\lambda}$$

where  $\lambda$  is the wavelength of light.

$$r_1 = \sqrt{nb\lambda} \quad \text{or} \quad b = \frac{r_n^2}{n\lambda}$$

.....(i)

If the source is at a large distance from the zone plate, a bright spot will be obtained at P. As the distance of the source is large, the incident wavefront can be taken as a plane one with respect to the small area of the zone plate. The even numbered zone cut off the light and hence resultant amplitude at P =  $A = m_1 + m_2 + m_3 + \dots$  etc. In this case the focal length of the zone plate  $f_n$  is given by

$$f_n = b = \frac{r_n^2}{n\lambda}$$

.....(ii)

Thus, a zone plate has different foci

.....(iii)

for different wavelengths. The radius of the  $n^{\text{th}}$  zone increases with increasing value of  $\lambda$ . It is very interesting to note that as the even numbered zones are opaque, the intensity at P is much greater than when the whole wavefront is exposed to the point P.

In the first case the resultant amplitude is given by

$$A = m_1 + m_2 + m_3 + \dots + m_n \dots (n \text{ is odd})$$

When the whole wavefront is unobstructed, the amplitude is given by

$$A = m_1 - m_2 + m_3 - m_4 \dots + m_n$$

$$= \frac{m_1}{2} \quad (\text{if } n \text{ is very large and } n \text{ is odd})$$

If a parallel beam of white light is incident on the zone plate, different colours come to focus at different points along the line OP. Thus, the function of a zone plate is similar to that of a convex lens and a formula connecting the distance of the object and image points can be obtained for a zone plate also.

**Action of a zone plate for an incident spherical wave front:-**

Let XY represent the section of the zone plate perpendicular to the plane of the paper. S is a point source of light, P is the position of the screen for a bright image. 'a' is the distance of the source from the zone plate and b is the distance of the screen from the plate.  $OM_1$ ,  $OM_2$ ,  $OM_3$ , ( $r_1$ ,  $r_2$ ,  $r_3$ ) etc are the radii of the 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup> etc. half period zones. The position of the screen is such that from one zone to the next there is an increasing path difference of  $\lambda/2$ .

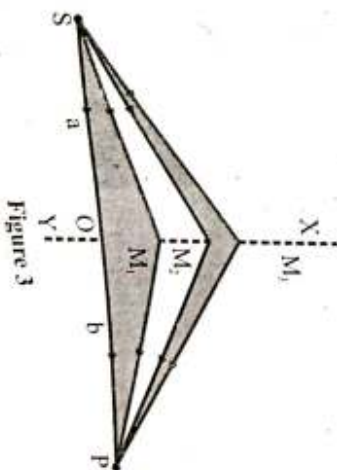


**Figure 3**

Thus,

$$SO + OP = a + b$$

.....(iii)

$$SM_1 + M_1P = a + b + \frac{\lambda}{2}$$

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$$SM_1 + M_1P = a + b + \frac{2\lambda}{2} \text{ and so on.}$$

From the  $\Delta SM_1O$

$$SM_1 = (SO^2 + OM_1^2)^{\frac{1}{2}} = (a^2 + r_1^2)^{\frac{1}{2}}$$

Similarly from the  $\Delta OM_1P$

$$M_1P = (OP^2 + OM_1^2)^{\frac{1}{2}} = (b^2 + r_1^2)^{\frac{1}{2}}$$

Substituting the values of  $SM_1$  and  $M_1P$  in equation (iii), we get

$$(a^2 + r_1^2)^{\frac{1}{2}} + (b^2 + r_1^2)^{\frac{1}{2}} = a + b + \frac{\lambda}{2}$$

$$a \left( 1 + \frac{r_1^2}{a^2} \right)^{\frac{1}{2}} + b \left( 1 + \frac{r_1^2}{b^2} \right)^{\frac{1}{2}} = a + b + \frac{\lambda}{2}$$

$$a + \frac{r_1^2}{2a} + b + \frac{r_1^2}{2b} = a + b + \frac{\lambda}{2}$$

$$\frac{r_1^2}{2} \left[ \frac{1}{a} + \frac{1}{b} \right] = \frac{\lambda}{2} \Rightarrow r_1^2 \left( \frac{1}{a} + \frac{1}{b} \right) = \lambda$$

Similarly for  $r_n$  i.e. the radius of the  $n^{\text{th}}$  zone, the relation can be written as

$$r_n^2 \left[ \frac{1}{a} + \frac{1}{b} \right] = n\lambda$$

Applying the sign convention,

$$\frac{1}{b} - \frac{1}{a} = \frac{n\lambda}{r_n^2} = \frac{1}{f_n}$$

$$\Rightarrow f_n = \frac{r_n^2}{n\lambda}$$

.....(vi)

eq. (vi) is similar to the equation  $\left( \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \right)$  in the case of lenses with  $a$  and  $b$  as the object and image distances and  $f_n$  the focal length. Thus a zone plate acts as a converging lens. A zone plate has a number of foci which depend on the number of zones used as well as the wavelength of light employed.

$$f_n \propto \sqrt{n}$$

This relations shows that the radii of half-period zones are proportional to  $\sqrt{n}$ .

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to the square root of natural numbers.

$$\pi (r_n^2 - r_{n-1}^2) = \pi \left\{ \frac{n\lambda uv}{u-v} - \frac{(n-1)\lambda uv}{u-v} \right\} = \frac{\pi \lambda uv}{u-v}$$

This is independent of  $n$ , hence for a given object and image, the areas of all the zones remain same. Further, the area diminishes as  $v$  or  $u$  decreases i.e., as the plate is approached by the object or the image. The numerical magnitudes of the displacements  $d_1, d_2, d_3, \dots$  at  $l$  due to wavelets from the various zones diminish only slightly with the order of the zone. Hence the resultant displacement at  $l$

$$D = d_1 - d_2 + d_3 + d_4 + d_5 + \dots$$

**Difference between a zone plate and a convex lens -**

For a given wavelength of light, a convex lens has only one focal length given by

$$\frac{1}{f} = (\mu - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

where  $f$  is the focal length of the lens,  $\mu$  is the refractive index of the material of the lens and  $R_1$  and  $R_2$  are the radii of curvature. In a convex lens, the violet rays of light come to focus nearer the lens than the red rays of light because for a given material the refractive index for violet rays of light is more than for red rays of light.

In the case of a zone plate, there are a number of foci between the point  $O$  and  $P$  (figure (3)). Each focus corresponds to the position where, with reference to  $P$  an odd number of half period elements can be constructed on each zone. As the screen is moved nearer the zone plate, the area of the half period elements decreases and more half period elements can be present on each zone. If  $P_m$  is the position on the image when  $(2m-1)$  half period elements can be present on each zone,  $f_m$  the focal length of the zone plate is given by

$$f_m = \frac{r_m^2}{(2m-1)n\lambda} \text{ .....(vii)}$$

Putting  $m = 1, 2, 3, \dots$ , etc., the different positions of the screen for a bright image can be obtained. In eq. (vii),  $r_m$  is the radius of the  $m^{\text{th}}$  zone of the wavefront,  $\lambda$  is the wavelength of light and  $(2m-1)$  is the number of odd half period elements present on each zone. For example, if the position of the screen is such that with reference to the point  $P$ , three half period elements can be constructed on each zone, then the focal length of the zone plate  $f_3$  is given by

$$f_3 = \frac{r_3^2}{5n\lambda}$$

With the decrease in the focal length of the zone plate, the brightness of the image

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decreases. Let the first zone contain only one period element. Then, the amplitude at P due to this zone is  $m_1$ . If the first zone contains three half-period elements for a particular position of the screen, then the amplitude at P due to the first zone

$$= m_1 - m_2 + m_3 = \frac{m_1}{2} + \left[ \frac{m_1}{2} - m_2 + \frac{m_3}{2} \right] + \frac{m_3}{2} = \frac{m_1}{2} + \frac{m_3}{2}$$

But,  $\frac{m_1 + m_3}{2}$  is less than  $m_1$ , because  $m_1 > m_3$ . Further, in a zone plate (for the same number of odd half-period elements contained in each zone) the focal length for violet light is more than for red light, which is reverse in the case of a convex lens.

#### Comparison between a zone plate and a convex lens:-

1. Both the zone plate and convex lens form a real image of the object and the equations connecting the conjugate distances are similar.
2. The focal lengths of both depend on the wavelength,  $\lambda$ , and hence suffer from chromatic aberration. The chromatic aberration in a zone plate is much more severe than in a convex lens.
3. A zone plate acts simultaneously as a convex lens and as a concave lens. In addition to a real image, a virtual image is also formed simultaneously. The convex lens forms only a real image.
4. In case of zone plate the image is formed by the diffraction phenomenon. In case of convex lens the image is formed due to refraction of light.
5. The zone plate has got multiple foci on either side of the plate. Hence, the intensity of the image will be much less.
6. Convex lens has only one focus. As all the light is focused at one point, the intensity of the image will be more.
7. In a zone plate, waves reaching the image point through any two alternate zones differ in path by  $\lambda$ , and in phase by  $2\pi$ . In case of a convex lens all the rays reaching the image point have zero path or phase difference.
7. A zone plate can be used over a wide range of wavelengths from microwaves to X-rays. Glass lens cannot be used beyond the visible region.

#### Q.4. Define Fresnel's Half period zones. Why zones are called Half period zones.

##### Other Related Questions:-

Q. Write short note on Fresnel's Half period zones. (2017)

Ans. Fresnel Half-Period Zones:-

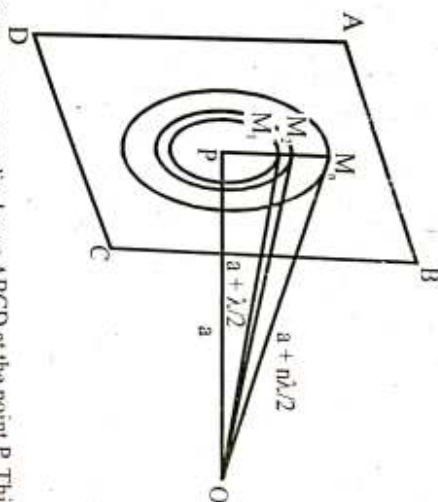
Consider a plane ABCD which represents a plane wavefront of monochromatic light of wavelength  $\lambda$ , travelling from left to right. According to Huygen's principle every point on the wave front is regarded as the origin of secondary wavelets and at a given instant every one of these secondary wave

(2018)

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lets pass through the point O. The resultant effect at O due to the whole wavefront will be equal to the total effect of all the disturbance reaching from different portions of the wave front. This is known as **Huygen's Fresnel theory**. To find the resultant effect we divide the entire wavefront into concentric zones as follows.



From O drop a perpendicular on ABCD at the point P. This point is the foot of the perpendicular from O on ABCD and is called the pole of the wave with respect to O. Let OP be equal to  $a$  and  $\lambda$  be the wavelength of light waves. With O as centre and radius  $(a + \lambda/2)$  draw a sphere, cutting the wavefront in a circle at  $M_1$ , then

$$OM_1 = a + \lambda/2 \text{ or } OM_1 - OP = \lambda/2$$

This simply means that the secondary wavelets originating from P and from the points on the circumference of the circle  $M_1$ , on reaching O, will differ in phase by

$$\frac{2\pi}{\lambda} (OM_1 - OP) = \pi$$

Now  $\pi$  radian is equivalent to phase difference  $T/2$ . Hence the area enclosed by the circle  $M_1$  is called **Fresnel's first half - period zone**.

Similarly we can draw other spheres of radii  $(a + 2\lambda/2)$   $(a + 3\lambda/2)$   $(a + 4\lambda/2)$  ..... which will intersect the wavefront in circles  $M_2, M_3, \dots$ . This construction divides the wavefront into a number of Fresnel's half - period zones. It may be noted that the first half - period zone is a circle, while the second half - period zone is an annular ring between  $M_1$  and  $M_2$ , and so on.

These are known as half - period zones, because the waves reaching O from P and  $M_1$  differ in phase by  $\pi$  or  $T/2$  or  $\lambda/2$ . Similarly the waves reaching O from  $M_1$  and  $M_2$  also differ in phase by  $\pi$  or  $T/2$  or  $\lambda/2$ .

Thus the phase difference between wavelets from successive half - period zones is  $\pi$  radian or half - period. This justifies their nomenclature as HPZ's.

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Q.5. Explain why the centre of shadow of an opaque circular disc is always bright of feeble intensity. (2016, 17)

Other Related Questions -

Q. The presence of a central bright spot (of feeble intensity) in the diffraction pattern of an opaque circular disc. (2012)

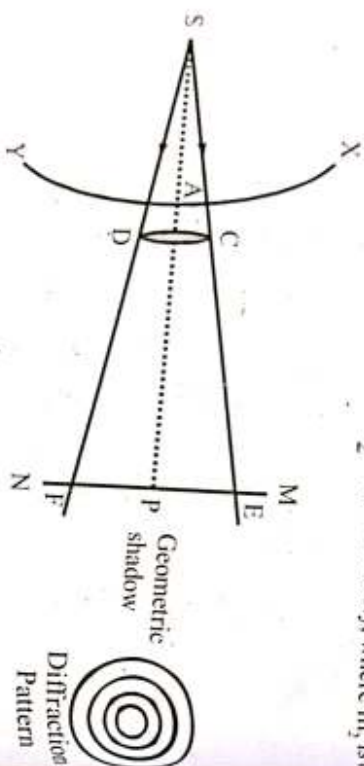
Q. Prove that there is always a bright spot at the centre of shadow of circular disc. (2016)

Q. Discuss Fresnel's diffraction at a small opaque disc illuminated by monochromatic light. Explain why the shadow of the disc is bright? What happens if the size of the disc is increased? (2013)

Ans. Diffraction At An Opaque Circular Disc:-

S is a point source of monochromatic light. CD is an opaque disc and MN is the screen. P is a point on the screen such that SAP is perpendicular to the screen. The screen is perpendicular to the plane of the paper. XY is the incident spherical wave front. EF is the geometrical shadow. With reference to the point P, the wave front can be divided into half period zones taking the centre of the disc (A) as the pole (Fig.). If one half period zone can be constructed on the surface of the disc, the rest of the zones are exposed to the

point P and the resultant amplitude at P =  $\frac{m_1}{2}$  approximately, where  $m_1$  is the



amplitude due to the second zone  $\left[ m_2 - m_3 + m_4 - \dots \right] \approx \frac{m_1^2}{2}$  approximately. Similarly, if two half period zones can be constructed on the surface of the disc

the resultant amplitude at P due to the exposed zones will be  $\frac{m_1}{2}$  and so on.

Thus, the point P will always be bright but the intensity at P decreases with increase in the diameter of the disc. That is, with a large diameter of the disc the most effective central zones will be cut off by the disc and the exposed outer zones are more oblique with reference to the point P. Thus (at P) the

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center of the geometrical shadow will be bright as if the disc were absent. The diffraction pattern consists of a central bright spot surrounded by alternate bright and dark rings, as shown in figure (b).

### Numerical

Q.1. The diameter of the first ring of a zone plate is 1 mm. If the plane waves ( $\lambda = 500 \text{ nm}$ ) fall on the plate, where should the screen be placed so that light is focused to a brightest spot? (2013)

Sol. Given  $r_1 = 0.5 \text{ mm} = 0.05 \text{ cm}$  and  $\lambda = 500 \text{ nm} = 5 \times 10^{-7} \text{ cm}$ ,  $m = 1$ . The plane wave falling on the plate will be focused to a brightest spot at the first focus of the plate.

Now, the first focal of the zone plate is given by

$$f = \frac{r_m^2}{m\lambda} = \frac{r_1^2}{\lambda} = \frac{(0.05 \times 0.05)}{5.0 \times 10^{-7}} = 50 \text{ cm}$$

Hence, the screen should be placed at a distance of 50 cm from the plate.

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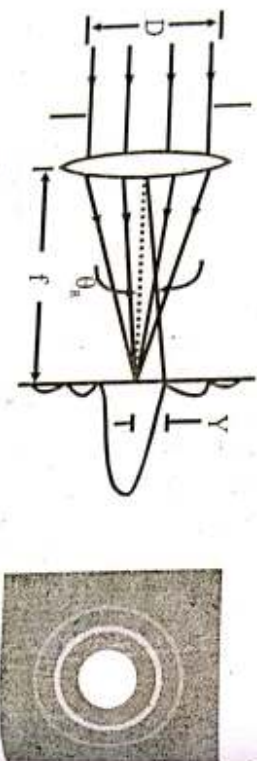
## Chapter 6

Q.1. What will be the effect on the fringe pattern on Young's double slit experiment if the slit widths are increased.

Ans. The fringe width  $\beta$  is given by  $\beta = \lambda D/d$ , where  $d$  is the separation of the slits. As the separation is increased, the fringe width decrease. (2016)

Q.2. Describe the modified Rayleigh's criterion for resolution. (2009, 11)  
Ans. Rayleigh's Criterion:-

When a beam of light from a point object passes through the objective of a telescope, the lens acts like a circular aperture and produces a diffraction pattern instead of a point image. This diffraction pattern is a bright disc surrounded by alternate dark and bright rings figure (1).



- (a) The image of a point source of monochromatic light, formed by a lens of diameter  $D$ , is a diffraction pattern.  
(b) The intensity pattern is a maximum at the center of the Airy disc.

Figure 1

It is known as Airy's disc. If there are two point objects lying close to each other, then two diffraction patterns are produced, which may overlap on each other and it may be difficult to distinguish them as separate (figure (2a)).



Figure (2)

To obtain the measure of the resolving power of an objective lens, Rayleigh suggested that the two images of such point objects lying close to each other may be regarded as separated if the central maximum of one falls on the first minimum of the other. In other words, when the central bright image of one falls on the first dark ring of the other, the two images are said to be just resolved (figure (2b)). This is equivalent to the condition that the distance between the centers of the patterns shall be equal to the radius of the central disc. This is called the Rayleigh criterion for resolution and is also known as Rayleigh's limit of resolution.

Q.3. Write expression for intensity distribution in Fraunhofer diffraction of light at  $N$  slits. Explain the principal maxima and find the number of secondary maxima between two successive primary maxima. (2015)

Other Related Questions:-

Q. Write expression only for intensity due to  $N$  parallel slits and prove also that intensity of principal maxima of a plane transmission grating is proportional to  $N^2$ . (2016)

Q. Using expression for intensity distribution function for Fraunhofer diffraction at a single slit, obtain an expression for intensity distribution function for diffraction at a periodic array of  $N$  slits. What are principal maxima? Show that there are  $N-1$  zeros of intensity between two successive principal maxima.

Ans. Fraunhofer Diffraction At  $N$  Slits:-

Fraunhofer diffraction at two slits consists of diffraction maxima and

minima given by  $\frac{\sin^2 \alpha}{\alpha^2}$  and the sharp interference maxima and minima, in each

diffraction maximum governed by  $\cos^2 \beta$  term.

To derive an expression for the intensity distribution due to diffraction at  $N$  slits, the expression for  $dy$  has to be integrated for  $N$  slits.

For a single slit,  $dy = K \int_{-\frac{a}{2}}^{+\frac{a}{2}} \sin 2\pi \left( \frac{1}{T} - \frac{r}{\lambda} + \frac{z \sin \theta}{\lambda} \right) dz$

Let  $\sin 2\pi \left( \frac{1}{T} - \frac{r}{\lambda} + \frac{z \sin \theta}{\lambda} \right)$  be equal to  $\phi(z)$  (i.e. function of  $z$ ).

For  $N$  slits

$$dy = K \left[ \int_{-\frac{a}{2}}^{+\frac{a}{2}} \phi(z) dz + \int_{\frac{a}{2}}^{+\frac{3a}{2}} \phi(z) dz + \int_{\frac{3a}{2}}^{+\frac{5a}{2}} \phi(z) dz + \dots + \int_{(N-1)\frac{a}{2}}^{+\frac{Na}{2}} \phi(z) dz \right]$$

On simplification

$$Y = K \frac{\sin \alpha}{\alpha} \left[ \sin 2\pi \left( \frac{1}{T} - \frac{r}{\lambda} \right) + \sin 2\pi \left( \frac{1}{T} - \frac{r}{\lambda} + \frac{d \sin \theta}{\lambda} \right) + \sin 2\pi \left( \frac{1}{T} - \frac{r}{\lambda} + \frac{2d \sin \theta}{\lambda} \right) + \dots + \sin 2\pi \left( \frac{1}{T} - \frac{r}{\lambda} + \frac{(N-1)d \sin \theta}{\lambda} \right) \right]$$

Here  $\alpha = \frac{\pi a \sin \theta}{\lambda}$ . For a general trigonometric summation

$$\sum_{m=0}^{m=N} \sin(x + pm) = \frac{\sin\left(x + \frac{m\pi}{2}\right) \sin\left[\left(\frac{n+1}{2}\right)m\right]}{\sin\left(\frac{m}{2}\right)}$$

Here  $x = 2\pi\left(\frac{1}{T} - \frac{r}{\lambda}\right)$ ;  $m = \frac{2\pi d \sin \theta}{\lambda} = 2\left[\frac{\pi d \sin \theta}{\lambda}\right] = 2\beta$

where  $\beta = \frac{\pi d \sin \theta}{\lambda}$  and  $n = (N-1)$

$$I = \frac{K a^2 \left(\frac{\sin \alpha}{\alpha}\right) \left[\sin\left(x + \frac{(N-1)m}{2}\right) \sin\left(\frac{Nm}{2}\right)\right]}{\sin\left(\frac{m}{2}\right)}$$

$$I = K a^2 \left(\frac{\sin \alpha}{\alpha}\right) \left(\frac{\sin N\beta}{\sin \beta}\right) \left[\sin 2\pi\left\{\frac{1}{T} - \frac{r}{\lambda} + \frac{(N-1)d \sin \theta}{2\lambda}\right\}\right]$$

The intensity at a point P' is given by

$$I = K^2 a^2 \left(\frac{\sin^2 \alpha}{\alpha^2}\right) \left(\frac{\sin^2 N\beta}{\sin^2 \beta}\right)$$

The maximum intensity, when  $\alpha = 0$  and  $\beta = 0$ , is  $I_0 = K^2 a^2$

$$I = I_0 \left(\frac{\sin^2 \alpha}{\alpha^2}\right) \left(\frac{\sin^2 N\beta}{\sin^2 \beta}\right)$$

The expression  $\frac{\sin^2 \alpha}{\alpha^2}$

represents the diffraction pattern due to a single slit.

The additional factor  $\frac{\sin^2 N\beta}{\sin^2 \beta}$

represents the interference effects due to the secondary waves from the N slits.

The numerator will be zero when,  $N\beta = 0, \pi, 2\pi, 3\pi, \dots$  etc =  $k\pi$

Denominator is also zero when,  $\beta = 0, \pi, 2\pi, 3\pi, \dots$  etc =  $k\pi$

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Since the quotient  $\frac{0}{0}$  is indeterminate,  $N\beta = k\pi$  gives the condition for minimum intensity for all values of k other than  $k = 0, N, 2N, 3N$  etc. The directions of principal maxima, correspond to the values of  $k = 0, N, 2N$  etc.

$$N\beta = \frac{N\pi d \sin \theta}{\lambda} \text{ or } k\pi = \frac{N\pi d \sin \theta}{\lambda}$$

For directions of principal maxima,  $k = 0, 1N, 2N, 3N$ , etc. =  $nN$

When  $n = 1, 2, 3$  etc.,

$$n\pi N = \frac{N\pi d \sin \theta}{\lambda}$$

$$\therefore d \sin \theta = n\lambda$$

( $n = 1, 2, 3, \dots$  etc)

If the width of the slit is a and the width of the opaque spacing is b,  $d = (a + b)$  and  $(a + b) \sin \theta = n\lambda$

Putting  $n = 1, 2, 3$  etc, the directions of principal maxima  $\theta_1, \theta_2, \theta_3, \dots$  etc can be determined. For values of k in between 0 and N between N and 2N, etc., there are (N-1) secondary minima and (N-2) secondary maxima.

**Intensity of Principal Maxima:-**

In a diffraction grating there are about 6000 narrow slits in one cm. For values of  $\beta = k\pi$  and  $\beta = 0, \pi, 2\pi, 3\pi$  etc.

$$\frac{\sin N\beta}{\sin \beta} = \frac{0}{0}$$

which is indeterminate.

To find the value of this limit, the numerator and the denominator are differentiated. Thus, we get

$$\frac{d}{d\beta} \left( \frac{\sin N\beta}{\sin \beta} \right) = \frac{N \cos N\beta}{\cos \beta} = \pm N$$

Thus, the resultant amplitude is proportional to N and resultant intensity is proportional to  $N^2$ .

$$I = N^2 I_0 \left( \frac{\sin^2 \alpha}{\alpha^2} \right)$$

These maxima are intense and are called **Principal maxima**.

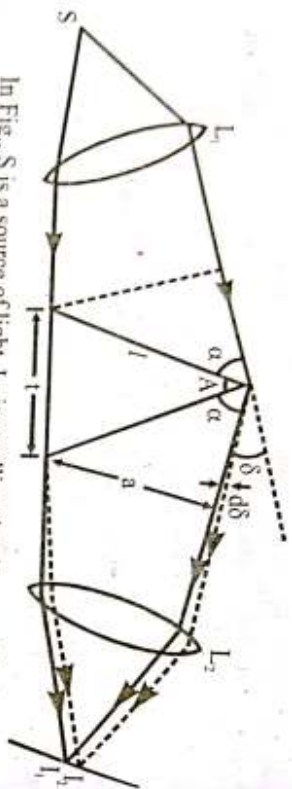
**Q4. Write short note on resolving power of prism. (2012, 14, 15, 16, 18)**

**Ans. Resolving Power of a Prism:-**

The term resolving power applied to the spectrographic devices (using a grating or a prism) signifies the ability of the instrument to form two separate images.

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rate spectral images of two neighboring wavelengths,  $\lambda$  and  $\lambda + d\lambda$  in the wavelength region  $\lambda$ .



In Fig., S is a source of light,  $L_1$  is a collimating lens and  $L_2$  is the telescope objective. As two wavelengths  $\lambda$  and  $\lambda + d\lambda$  are very close, if the prism is set in minimum deviation position would hold good for both the wavelengths. The final image  $I_1$  corresponds to the principal maximum for the wavelength  $\lambda$  and  $I_2$  corresponds to the principal maximum for the wavelength  $\lambda + d\lambda$ .  $I_1$  and  $I_2$  are formed at the focal plane of the telescope objective  $L_2$ . The fact of the prism limits the incident beam to a rectangular section of width  $a$ . Hence the Rayleigh criterion can be applied in the case of a rectangular aperture. In the case of diffraction at a rectangular aperture, the position of  $I_1$  will correspond to the first minimum of the image  $I_1$  for wavelength  $\lambda$ , provided

$$a \delta \sin \alpha = \lambda \quad \text{or} \quad \delta \sin \alpha = \frac{\lambda}{a} \quad \dots\dots(i)$$

Here  $\delta$  is the angle minimum deviation for wavelength  $\lambda$ .  
From the Fig.,  $\alpha + A + \alpha + \delta = \pi$

$$\therefore \alpha = \left[ \left( \frac{\pi}{2} \right) - \left( \frac{A + \delta}{2} \right) \right]$$

$$\therefore \sin \alpha = \sin \left[ \left( \frac{\pi}{2} \right) - \left( \frac{A + \delta}{2} \right) \right] = \cos \left( \frac{A + \delta}{2} \right)$$

$$\text{But } \sin \alpha = \frac{a}{l} \quad \therefore \cos \left( \frac{A + \delta}{2} \right) = \frac{a}{l} \quad \dots\dots(ii)$$

$$\text{Also, } \sin \frac{A}{2} = \frac{1}{2l} \quad \dots\dots(iii)$$

$$\text{In the case of a prism, } \mu = \frac{\sin \frac{A + \delta}{2}}{\sin \frac{A}{2}}$$

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$$\therefore \sin \frac{A + \delta}{2} = \mu \sin \frac{A}{2} \quad \dots\dots(iv)$$

Here  $\mu$  and  $\delta$  are dependant on wavelength of light  $\lambda$ .  
Differentiating eq.(iv) with respect to  $\lambda$ , we get

$$\frac{1}{2} \cos \left( \frac{A + \delta}{2} \right) \frac{d\delta}{d\lambda} = \frac{d\mu}{d\lambda} \left( \sin \frac{A}{2} \right) \quad \dots\dots(v)$$

Substituting the values of  $d\delta$  from equation (ii) and (iii), we obtain

$$\frac{1}{2} \left( \frac{a}{l} \right) = \frac{d\delta}{d\lambda} = \frac{d\mu}{d\lambda} \left( \frac{1}{2l} \right)$$

$$a \frac{d\delta}{d\lambda} = l \frac{d\mu}{d\lambda} \quad \dots\dots(vi)$$

Substituting the value of  $d\delta$  equation (i), we get

$$\frac{\lambda}{d\lambda} = l \frac{d\mu}{d\lambda} \quad \dots\dots(vii)$$

The expression  $\frac{\lambda}{d\lambda}$  measures the resolving power of the prism. It is defined as the ratio of the wavelength  $\lambda$  to the smallest difference in wavelength  $d\lambda$ , between this line and a neighbouring line such that the two lines appear just resolved, according to Rayleigh's criterion.

$$\text{So, resolving power of a prism} = l \frac{d\mu}{d\lambda} \quad \dots\dots(viii)$$

It means that the resolving power (1) is directly proportional to the length of the base of the prism and (2) rate of change of refractive index with respect to wavelength for that particular material.

**Q5. Write short note on Nicol Prism. (2016, 18)**

**Other Related Questions:-**

**Q. What is a Nicol prism? How it can be used as a polariser. (2019)**

**Ans. Nicol Prism:-**

Nicol prism is a polarizing device fabricated from a double refracting crystal. It is similar to a polaroid sheet in its action. A Nicol prism is made from calcite crystal. William Nicol designed it in 1820. A rhomb of calcite crystal about three times as long as it is thick, is obtained by cleavage from the original crystal. The ends of the rhombohedron are ground until they make an angle of  $60^\circ$  instead of  $71^\circ$  with the longitudinal edges. This piece is then cut into two along a plane perpendicular both to the principal axis and to the new end surfaces AB and CD. The two parts of the crystal are then cemented together

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with canada balsam, whose refractive index lies between the refractive indices of calcite for the o-ray and e-ray.  $\mu_o = 1.66$ ,  $\mu_e = 1.486$  and  $\mu_{\text{canada balsam}} = 1.55$ . The position of the optic axis is as shown in figure (a). The refractive index for e-ray depends upon the direction in which e-ray is propagating in the crystal. The difference between the refractive index between o-ray and that for e-ray goes on increasing with the angle between the two rays in the crystal. When this angle is  $90^\circ$ , the difference is a maximum. Thus, for a fixed value for  $\mu_o$ , the  $\mu_e$  has its maximum or minimum value in perpendicular direction. In the above  $\mu_e = 1.486$  represents the minimum value.

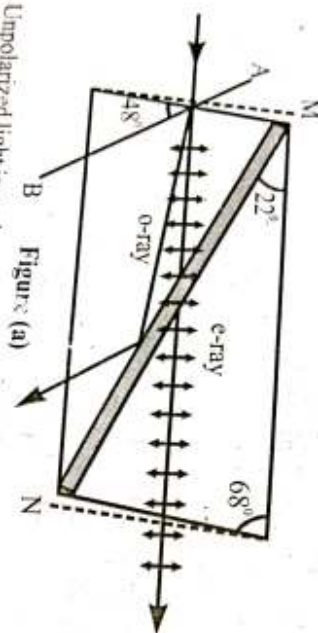


Figure (a)

Unpolarized light is made to fall on the crystal as shown in fig. (a) at an angle of about  $15^\circ$ . The ray after entering the crystal suffers double refraction and splits up into o-ray and e-ray. The two rays with their directions of vibration are as shown in the Fig (a). The values of the refractive indices and the angles of incidence at the canada balsam layer are such that the e-ray is transmitted while the o-ray is internally reflected. The face where the o-ray is blackened so that the o-ray is completely absorbed. The face where the plane polarized e-ray coming out of the Nicol. Thus, the Nicol works as a polarizer.

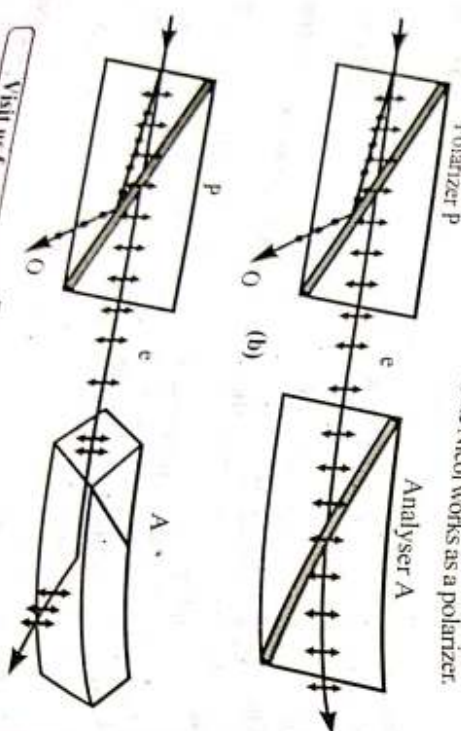


Figure (b)

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For studying the optical properties of transparent substances, two Nicol's are used - one as a polarizer and the other as an analyzer.

When two Nicol prisms P and A are placed adjacent to each other as shown in Fig. (b, c), one of them acts as a polarizer and the other acts as an analyzer. If unpolarized ray of light is incident on the Nicol prism P, a linearly polarized e-ray emerges from P with its vibration direction lying in the principal section of P. The state of the polarization of the light emerging from polarizer P can be examined with another polarizer A, which for convenience is called an analyzer. Let now this ray be incident on the second Nicol prism A, whose principal section is parallel to that of P. The vibration direction of the ray will be in the principal section of A and hence it is transmitted unhindered through the analyzer A.

If the Nicol prism A is gradually rotated, the intensity of the e-ray decreases in accordance with Malus law. When its principal section becomes perpendicular to that of the Nicol prism P (Fig. c), the vibrations of the ray, emerging from P and incident on A, will be perpendicular to the principal section of A. In this position the ray behaves as o-ray inside the prism A and is totally internally reflected by the canada balsam layer. Hence no light is transmitted by the prism A. In this configuration, the two Nicol prisms P and A are said to be crossed. If the Nicol prism A is further rotated through another  $90^\circ$ , the intensity of light emerging from A will go on increasing. The intensity will become a maximum when its principal section is again parallel to that of the prism P. Thus, the prism P produces linearly polarised light while the prism A detects it. Hence the prism P is called a polarizer and the prism A an analyzer.

Q.6. Find the missing order in double slit diffraction pattern.

Other Related Questions-

- Q. Find the missing orders in the diffraction pattern due to two slits if each slit is of width 'a' and are separated by '2a' distance. (2016)
- Q. Find the missing orders for a double slit Fraunhofer diffraction pattern if the slit widths are '2a' and they are 'a' distance apart. (2014)
- Q. Find the absent orders in double slit Fraunhofer diffraction if the opaque region between two slits is half as wide as is the thickness of any slit. (2019)

Ans. Missing orders in a double slit diffraction pattern:-

Let 'a' be the width of each slit and 'b' the distance between them.

The directions of interference maxima are given by

$$\dots\dots\dots (i)$$

(a + b) sin  $\theta$  = n $\lambda$

$$\dots\dots\dots (ii)$$

$$a \sin \theta = n\lambda$$

where n and p are integers. If the value of a and b are such that both the equations are satisfied for the same value of  $\theta$ , then a certain interference maximum will overlap the diffraction minimum and hence the spectrum order

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will be missing.

Dividing (i) by (ii), we get

$$\frac{a+b}{a} = \frac{n}{p}$$

If  $a = b$ , then

$$\frac{n}{p} = 2 \quad \text{or } n = 2p$$

If  $p = 1, 2, 3, \dots$ , etc., then  $n = 2, 4, 6$  etc.

This means that the 2, 4, 6 etc. orders of interference maxima will be missing in the diffraction pattern. There will be three interference maxima in the central diffraction maximum.

If  $b = 2a$ , then

$$\frac{a+2a}{a} = \frac{n}{p} \quad \text{or } n = 3p$$

If  $p = 1, 2, 3, \dots$ , then  $n = 3, 6, 9, \dots$

This means that 3<sup>rd</sup>, 6<sup>th</sup>, etc. orders of interference maxima will be missing in the diffraction pattern.

On both sides of the central maximum, the number of interference maxima is 2 and hence there will be five interference maxima in the central diffraction maximum. The position of the third interference maximum corresponds to the first diffraction minimum.

If  $b = 0$ , the two slits join and all the interference maxima will be missing and the diffraction pattern will be same as due to a single slit of width  $2a$ .

**Q.7. Differentiate between Fraunhofer class and Fresnel class of diffraction.**

**Ans.** There are usually two categories of diffraction namely (i) Fraunhofer diffraction and (ii) Fraunhofer diffraction. (2012, 14)

**Difference between Fraunhofer class and Fresnel class of diffraction:-**

(i) In Fresnel's diffraction, either the source or screen or both are at finite distances from the obstacle and thus the distances are important. Whereas in Fraunhofer's diffraction, the source of light and the screen are at infinite distances from the obstacle. In this class, the inclination is important.

(ii) In Fresnel's diffraction, the incident wavefront is either spherical or cylindrical while in Fraunhofer's diffraction, the incident wavefront is generally plane.

(iii) In Fresnel's diffraction, the centre of diffraction pattern may be bright or dark depending upon the number of Fresnel's zones, while in Fraunhofer's diffraction, the centre of the diffraction pattern is always bright.

**Q.8. Differentiate between prism and grating spectra. (2009, 10, 12, 14, 18)**

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**Ans. The Difference Between Prism And Grating Spectra:-**

The following are the points of difference between the spectra obtained with a grating and a prism:

- A prism produces only one spectrum, whereas the grating gives a number of spectra on either side of the central maximum.
- The prism spectrum is produced by the dispersion of light, whereas the grating spectrum is produced by the diffraction of light.
- The prism spectrum is brighter than the grating spectra.
- In the case of a prism, the deviation is the least for red and the greatest for violet; while in case of a grating, the deviation is the least for violet and the greatest for the red. Hence in prism spectrum, the spectrum colors are in the order from red to violet; whereas in grating spectral, the spectral colors are in the order from violet to red.
- The prism spectrum depends on the material of the prism, whereas the grating spectra are independent of the material of the grating.

(vi) The dispersive power of a grating is given by  $\frac{d\theta}{d\lambda} = \frac{n}{(a+b) \cos \theta}$  which is constant for a particular order; whereas the dispersive power of a prism is given by  $\frac{d\mu}{d\lambda}$ .

(vii) The resolving power of a grating  $\left( \lambda \frac{d\mu}{d\lambda} \right)$  is very large; whereas the resolving power of a prism  $\left( \lambda \frac{d\mu}{d\lambda} \right)$  is quite small.

(viii) In the grating spectra the spectral lines are almost straight; whereas in the prism spectrum the spectral lines are slightly curved.

**Q.9. Find the maximum number of order available with a grating. (2013)**

**Other Related Questions:-**

**Q. Find the maximum number of order available with a grating. Show that if the width of the grating element is less than twice the wavelength of light then only first order is available. (2018)**

**Ans. Maximum Number of Orders with a Diffraction Grating:-**

The maximum number of spectra available with a diffraction grating in the visible region can be evaluated by using the grating equation for normal incidence as

$$(a+b) \sin \theta_n = n\lambda \quad \text{or} \quad n = \frac{(a+b) \sin \theta_n}{\lambda}$$

where  $(a+b)$  is the grating element,  $\theta_n$  the angle of diffraction and the order of maximum.

The maximum possible value of angle of diffraction ( $\theta_n$ ) is  $90^\circ$ . There-

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fore, the maximum possible order available with grating is given by

$$n_{\max} = \frac{(a+b) \sin 90^\circ}{\lambda} = \frac{(a+b)}{\lambda}$$

Thus, the grating element determines the maximum possible order.

If the grating element  $(a+b)$  lies between  $\lambda$  and  $2\lambda$  or grating element  $(a+b) < 2\lambda$ , then

$$n_{\max} < \frac{2\lambda}{\lambda} < 2$$

That is, for normal incidence only first order will be obtained. Hence, if the width of a grating element is less than twice the wavelength of light, then only first order is possible.

If  $(a+b)$  is in between  $2\lambda$  and  $3\lambda$ , that is,  $(a+b) < 3\lambda$ ,  $n_{\max} < \frac{3\lambda}{\lambda} < 3$

Hence, for normal incidence, when  $(a+b) < 3\lambda$ , only two orders are available. Hence, if the width of grating element is less than thrice the wavelength of light, then only two orders are possible.

Q.10. Show that if the screen on which diffraction pattern is obtained is moved towards aperture, the intensity at the centre would be alternately maximum and minimum.

Ans. **Diffraction At A Circular Aperture:-**

Let AB be small aperture (a pin hole) and S is a point source of monochromatic light. XY is a screen perpendicular to the plane of the paper and P is a point on the screen. SP is perpendicular to the plane of the paper and P is the center of the aperture and r is the radius of the aperture.

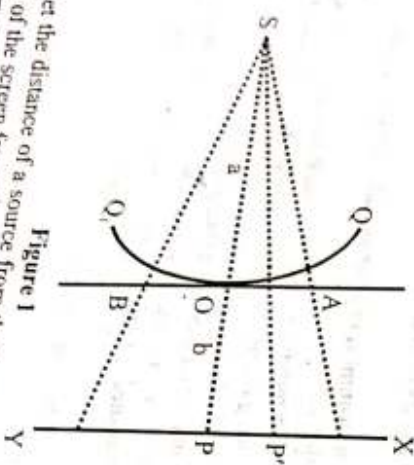


Figure 1

Let the distance of a source from the aperture be  $a$  ( $SO = a$ ) and the distance of the screen from the aperture be  $b$  ( $OP = b$ ).  $QOQ_1$  is the incident wavefront.

spherical wavefront and with reference to the point P, O is the pole of the wavefront (figure (1)). To consider the intensity at P, half period zones can be

constructed with P as center and radii  $b + \frac{\lambda}{2}, b + \frac{2\lambda}{2}$  etc. on the exposed wavefront AOB. Depending on the distance of P from the aperture (i.e. the distance b) the number of half period zones that can be constructed may be odd or even. If the distance a is such that only one half period zone can be

constructed, then the intensity at P will be proportional to  $m_1^2$  (where  $m_1$  is the amplitude due to the first zone at P). On the other hand, if the whole of the wavefront is exposed to the point P, the resultant amplitude is  $\frac{m_1}{2}$  or the

intensity at P will be proportional to  $\frac{m_1^2}{4}$ . The position of the screen can be altered so as to construct 2, 3 or more half period zones for the same area of the aperture. If only two zones are exposed, the resultant amplitude at P =  $m_1 - m_2$  (minimum) and if 3 zones are exposed, the amplitude =  $m_1 - m_2 + m_3$  (maximum) and so on. Thus by continuously altering the value of b, the point P becomes alternately bright and dark depending on whether odd or even number of zones are exposed by the aperture.

Now let us consider a point P' on the screen XY (figure 2). Let S to P' be joined. The line SP' meets the wavefront at O'. O' is the pole of the wavefront with reference to the point P'.

Now let us construct half period zones with the point O' as the pole of the wavefront. The upper half of the wavefront is cut off by the obstacle. If the first two zones are cut off by the obstacle between the points O' and A and if only the 3<sup>rd</sup>, 4<sup>th</sup> and 5<sup>th</sup> zones are exposed by the aperture AOB, then intensity at P' will be maximum. Thus if odd number of half period zones are exposed, point P' will be of maximum intensity and if even number of zones are exposed, point P' will be of minimum intensity. As the distance of P' from P increases the intensity of maxima and minima gradually decreases. It is because with the point P' far removed from P, the most effective central half period zones are cut off by the obstacle between the points O' and A. With the outer zones the obliquity increases with reference to the point P' and hence the intensity of maximum and minima also will be less. If the point P' happens to be of maximum intensity, then all the point lying on a circle of radius PP' on the screen also will be of maximum intensity. Thus with a circular aperture, the diffraction pattern

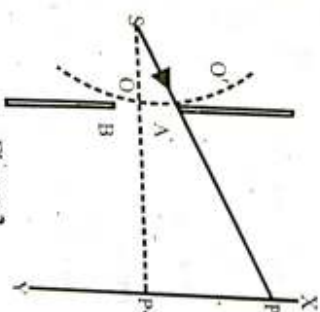


Figure 2

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will be concentric bright and dark rings with the centre P bright or dark depending on the distance b. The width of the rings continuously decreases.

**Q.11.** Obtain an expression for intensity distribution function for Fraunhofer diffraction at a plane transmission grating and also discuss the primary and secondary maxima and zeros.

**Other Related Questions -**

(2012)

**Q.** Write expression for intensity distribution in diffraction pattern formed by a plane transmission diffraction grating and discuss formation of principal and secondary maxima. Also find the ratio of intensity of a principal maximum with that of an adjacent secondary maximum for a very large number of slits.

**Ans. Plane Diffraction Grating:-**

(2011)

A diffraction grating is an extremely useful device and in one of its forms it consists of a very large number of narrow slits side by side. The slits are separated by opaque spaces. When a wave front is incident on a grating surface, light is transmitted through the slits and obstructed by the opaque surface. Such a grating is called a transmission grating. The secondary waves from the positions of the slits interfere with one another, similar to the interference of waves in Young's experiment. Joseph Fraunhofer used the first grating which consisted of a large number of parallel fine wires stretched on a frame. Now gratings are prepared by ruling equidistant parallel lines on a glass surface.

The lines are drawn with a fine diamond point. The space in between the two lines is transparent to light and the lined portion is opaque to light. Such a silvered surface (plane or concave) then light is reflected from the position of a mirror in between any two lines and such surfaces act as reflection gratings. If the spacing between the lines is of the order of the wavelength of light, then an appreciable deviation of the light is produced. Gratings used for the study of the visible region of the spectrum contain 10,000 lines per cm.

Gratings, with originally ruled surfaces are only few. For practical purposes replicas of the original grating are prepared. On the original grating surface a thin layer of colloidal solution is poured and the solution is allowed to harden. The film of colloidal solution is removed from the grating surface and then fixed between two glass plates. This serves as a plane transmission grating and then fixed between two glass plates. This serves as a plane transmission grating. A large number of replicas are prepared in this way from a single original ruled surface.

**Theory of plane transmission grating:-**

In figure 1, XY is the grating surface and MN is the screen, both perpendicular to the plane of the paper. The slits are all parallel to one another and perpendicular to the plane of the paper. The slits are all parallel to one another.

Here AB is a slit and BC is an opaque portion. The width of each slit is a and the opaque spacing between any two consecutive slits is b. Let a plane wave front be incident on the grating surface. Then all the secondary waves traveling in the same

direction as that of the incident light will come to focus at the point P on the screen. The screen is placed at the focal plane of the collecting lens. The point P where all the secondary waves reinforce one another corresponds to the position of the central bright maximum.

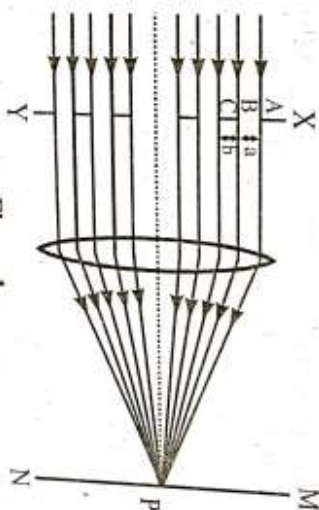


**Figure 1**

Now, consider the secondary waves traveling in a direction inclined at an angle  $\theta$  with the direction of the incident light (figure (2)). The collecting lens also is suitably rotated such that the axis of the lens is parallel to the direction of the secondary waves. These secondary waves come to focus at point  $P_1$  on the screen. The intensity at  $P_1$  will depend on the path difference between the secondary waves originating from the corresponding points A and C of two neighbouring slits. In figure (2),  $AB = a$  and  $BC = b$ . The path difference between the secondary waves starting from A and C is equal to  $AC \sin \theta$ .

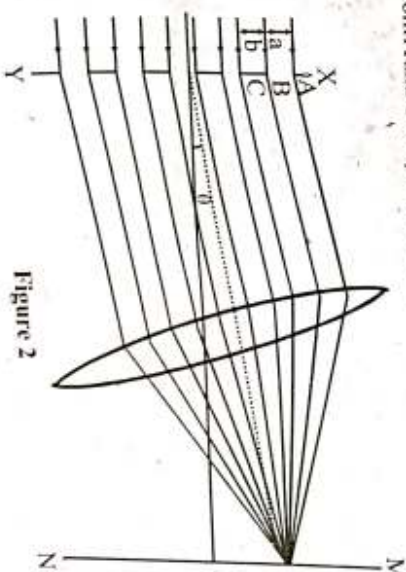


**Figure 2**

But  $AC = AB + BC = a + b$   
Path difference  $= AC \sin \theta = (a + b) \sin \theta$

The point  $P_1$  will be of maximum intensity if this path difference is equal to integral multiples of  $\lambda$ , where  $\lambda$  is the wavelength of light. In this case, all the secondary waves originating from the corresponding points of the neighbouring slits reinforce one another and the angle  $\theta$  gives the direction of maximum intensity. In general

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$$(a+b)\sin\theta_n = n\lambda$$

.....(1)

where  $\theta_n$  is the direction of the  $n^{\text{th}}$  principal maximum. Putting  $n = 1, 2, 3$ , etc. the angles  $\theta_1, \theta_2, \theta_3$  etc corresponding to the directions of the principal maxima can be obtained.

If the incident light consists of more than one wavelength, the beam gets dispersed and the angles of diffraction for different wavelengths will be different. Let  $\lambda$  and  $\lambda + d\lambda$  be two nearby wavelengths present in the incident light and  $\theta$  and  $(\theta + d\theta)$  be the angles of diffraction corresponding to these two wavelengths. Then, for the first order principal maxima

$$(a+b)\sin\theta = \lambda$$

$$\text{and } (a+b)\sin(\theta + d\theta) = \lambda + d\lambda$$

Thus, in any order, the number of principal maxima corresponds to the number of wavelengths present. A number of parallel slit images corresponding to the different wavelengths will be observed on the screen. In equ.(1),  $n = 1$  gives the direction of the first order image,  $n = 2$  gives the direction of the second order image and so on. When white light is used, the diffraction pattern on the screen consists of a white central bright maximum and on both sides of this maximum a spectrum corresponding to the different wavelengths of light present in the incident beam will be observed in each order.

**Secondary maxima and minima:-**

The angle of diffraction  $\theta_n$  corresponding to the direction of the  $n^{\text{th}}$  principal maxima is given by the equation

$$(a+b)\sin\theta_n = n\lambda$$

In this equation,  $(a+b)$  is called the grating constant. For a grating with 15,000 lines/inch, the value of

$$(a+b) = \frac{2.54}{15000} \text{ cm}$$

Now, let the angle of diffraction be increased by a small amount  $d\theta$  such that the path difference between the secondary waves from the points A and C increases by  $\lambda/N$  (figure 2).

Here N is the total number of lines on the grating surface. Then, the path difference between the secondary waves from the extreme points of the grating will be  $(\lambda/N) \cdot N = \lambda$ . Assuming the whole wave front to be divided into two halves, the path difference between the corresponding points of the two halves will be  $\lambda/2$  and all the secondary waves cancel one another's effect. Thus,  $(\theta_n + d\theta)$  will give the direction of the first secondary minimum.



after the  $n^{\text{th}}$  primary maximum. Similarly, if the path difference between the secondary waves from the point A and C is  $2\lambda/N, 3\lambda/N$  etc. Values of  $d\theta$ , these angles correspond to the directions of  $2^{\text{nd}}, 3^{\text{rd}}$  etc. secondary minima after the  $n^{\text{th}}$  primary maximum. If the value is  $2\lambda/N$ , then the path difference between the secondary waves from the extreme points of the grating surface is  $(2\lambda/N) \cdot N = 2\lambda$ , and considering the wave front to be divided into 4 portions of the  $2^{\text{nd}}$  secondary minimum can be understood. The number of secondary minima in between any two primary maxima is  $(N-1)$  and the number of secondary maxima is  $(N-2)$ .

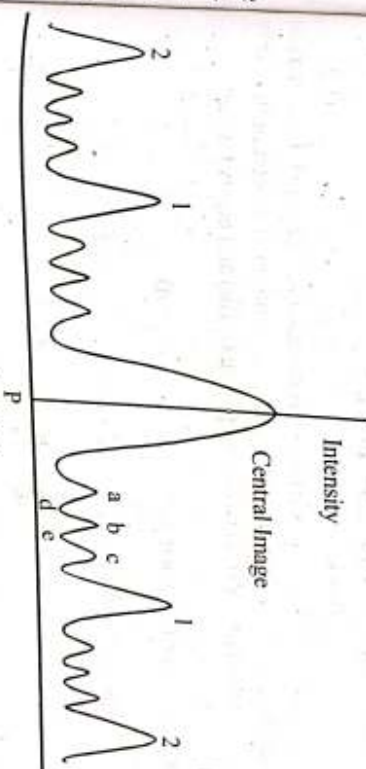


Figure 2

The intensity distribution on the screen is shown in figure (3). P corresponds to the position of the central maxima and 1, 2, etc. on the two sides of P represent the 1<sup>st</sup>, 2<sup>nd</sup>, etc principal maxima. a, b, etc are secondary maxima and d, e etc are the secondary minima. The intensity as well as angular spacing of the secondary maxima and minima is so small in comparison to the principal maxima that they cannot be observed. It results in uniform darkness between any two principal maxima.

**Q.12. Write short note on resolving power of the grating. (2013, 17)**

**Other Related Questions:-**

**Q. Obtain an expression on for resolving power of the grating and show that the resolving power of the grating is directly proportion to the total width of the ruled surface. (2019)**

**Ans. Resolving Power of Grating:-**

It is defined as the ability to form separate diffraction principal maxima of two wavelengths which are very close to each other.

**Derivation:-**

Let a light consisting of two wavelengths  $\lambda_1$  and  $\lambda_2$  is incident normally on a grating of grating element  $(a+b)$ .

The direction of the  $n^{\text{th}}$  principal maxima for wavelength  $\lambda_1$  is

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$$(a+b)\sin\theta = n\lambda_1$$

.....(i)

$$\text{Or } N(a+b)\sin\theta = Nn\lambda_1$$

(where  $N \rightarrow$  total numbers of lines on the grating)

The first minima in the direction  $(\theta + d\theta)$  is

$$N(a+b)\sin(\theta + d\theta) = n\lambda_1$$

.....(ii)

where  $m \in 0, N, 2N, \dots$ . Thus, the first minima adjacent to the  $n^{\text{th}}$  maxima in the direction  $(\theta + d\theta)$  is possible when  $m = (nN + 1)$

So from eq.(ii)

$$N(a+b)\sin(\theta + d\theta) = (nN + 1)\lambda_1$$

.....(iii)

The direction of principal maxima of the diffraction due to wavelength  $\lambda_2$  in the direction  $(\theta + d\theta)$  is  $(a+b)\sin(\theta + d\theta) = n\lambda_2$

$$\text{Or } N(a+b)\sin(\theta + d\theta) = nN\lambda_2$$

.....(iv)

For satisfaction of Rayleigh's criteria it is necessary that the principal maxima of wavelength  $\lambda_2$  should coincide with the first minima of wavelength  $\lambda_1$  in the same direction  $(\theta + d\theta)$ . Thus on comparing eq.(iii) and eq.(iv) we get

$$(nN + 1)\lambda_1 = Nn\lambda_2$$

.....(v)

Let us consider  $\lambda_1 = \lambda$  and  $\lambda_2 = \lambda + d\lambda \Rightarrow \lambda_2 - \lambda_1 = d\lambda$  from eq.(v)

$$(nN + 1)\lambda = Nn(\lambda + d\lambda)$$

$$\Rightarrow \lambda = Nnd\lambda \Rightarrow \frac{\lambda}{d\lambda} = Nn$$

.....(vi)

$\frac{\lambda}{d\lambda}$  is the resolving power of grating.

From eq.(i) and eq.(vi) we get,

$$\text{Resolving Power (R.P.)} = \frac{N(a+b)\sin\theta}{\lambda}$$

( $\because \lambda_1 = \lambda$ )

Thus, the resolving power of a grating is independent of the grating spectrum and (2) the total number of lines on the grating surface (i.e. the width of the grating surface). For a given grating, the distance between the spectral lines is double in the second order spectrum than that in the first order spectrum

## Chapter 7

Q1. Define circular and elliptical polarized light. (2017)

Ans. Circularly Polarized Light:-

A light wave is said to be circularly polarized, if in the course of wave propagation, the magnitude of the electric vector  $E$  stays constant but it rotates at a constant rate about the direction of propagation and sweeps a circular helix in space, as shown in figure 1. This is a picture in terms of the space variation of  $E$ . Alternately, if we could see the wave advancing towards our eyes, we would find that the tip of the  $E$  vector tracing a circle in space, completing one revolution within one wavelength. Hence, the state of polarization is called circular polarization. In, circularly polarized light, there is no preference to specific direction of oscillation.

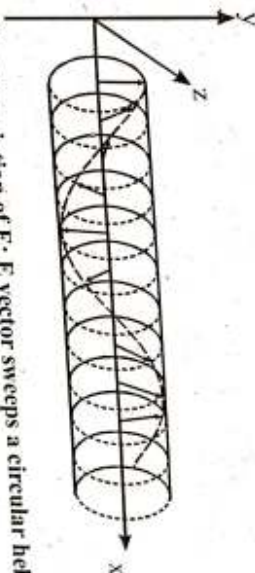


Figure 1 - Space variation of  $E$ ;  $E$  vector sweeps a circular helix in space

A circularly polarized light wave may be regarded as the resultant wave produced due to superposition of two coherent linearly polarized waves of equal amplitude oscillating in mutually perpendicular planes, and are out of phase by  $90^\circ$ .

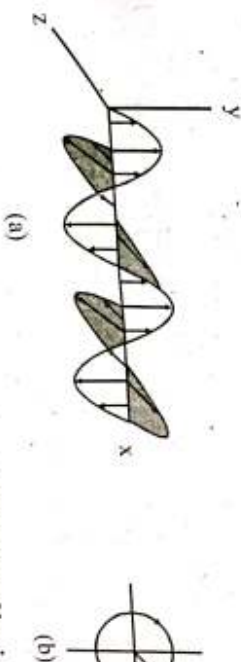


Figure 2 - Circularly polarized wave is a combination of horizontally polarized and vertically polarized waves that are out of phase by  $90^\circ$  and having equal amplitudes.

Let us again consider two linearly polarized waves having equal amplitude, out of which one is polarized in  $z$ -direction (horizontally polarized wave) and the other in  $y$ -direction (vertically polarized wave). Let us further assume that they are coherent and out of phase by  $90^\circ$  figure (2(a)). At some instant, the

E-field vector of z-polarized wave will have maximum amplitude and the E-field vector of y-polarized wave will be at zero. At that instant, the polarization is horizontal. A little time later, the z-polarized wave has decreased a little, while the y-polarized wave has started to increase. Then, the light looks like it is polarized at a slight angle- mostly horizontal, but with a small vertical component. A little while later, the z-component has decreased some more, and the y-component has increased some more, and the angle is greater. And so on. Eventually, the y-polarized wave is at a maximum, and the z-polarized wave is zero, and we have pure vertical polarization. If we stand at one point in space, and look at the direction of the wave, we will observe that the E-vector sweeps a circle in space. Hence, it is called circularly polarized wave. Note that the oscillations of the resultant E-vector do not take place in a single plane.

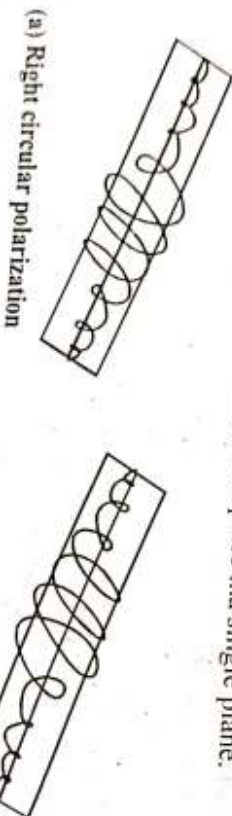


Figure 3 (a) Right circular polarization (b) Left circular polarization

If the rotation of the tip of E is clockwise as seen by an observer looking back towards the source, then the wave is said to be **right-circularly polarized** (figure 3a). If the tip of E rotates anti-clockwise, then the wave is said to be **left-circularly polarized**. This is a description of circular polarization in terms of the time variation of E. Elliptically Polarized Light:-

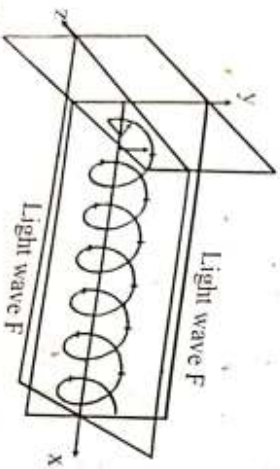


Figure 4 - Space variation of E, E vector sweeps a flattened helix in space

A light wave is said to be **elliptically polarized**, if the magnitude of electric vector E changes with time and the vector E rotates about the direction of propagation and sweeps a flattened helix in space, as shown in figure (4) of E. Alternately, if we imagine that we are looking at the light wave advancing towards us, we would observe that the tip of the E vector traces an ellipse in space.

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space. Hence, it is called elliptically polarized light. This is a description of elliptical polarization in terms of the time variation of E.

An elliptically polarized light wave may be regarded as the resultant wave produced due to superposition of two coherent linearly polarized waves of different amplitudes, oscillating in mutually perpendicular planes and are out of phase. If waves of differing amplitude are related in phase by  $90^\circ$ , or if the relative phase difference is other than  $90^\circ$  then the resultant light wave is elliptically polarized.

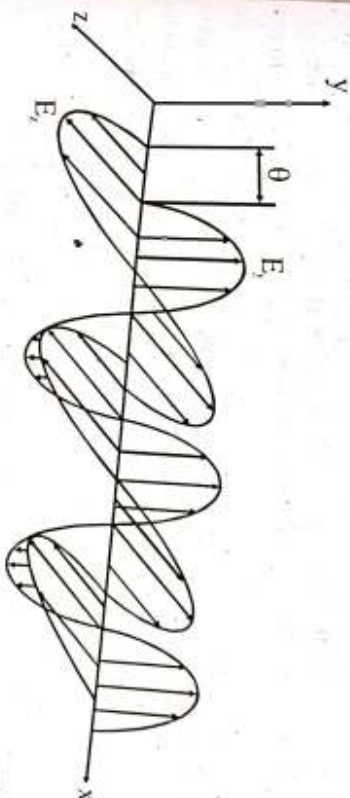


Figure (5) - Elliptically polarized wave is a combination of horizontally polarized and vertically polarized waves that are of different amplitudes and out of phase by  $\theta$

Let us consider two linearly polarized waves having different amplitudes, out of which one is polarized in x-direction and the other in y-direction. Let us further assume that they are coherent and out of phase by an arbitrary angle  $\theta$  (figure (5)). The E-field vector of y-polarized wave is a minimum amplitude at times when the E-field vector of x-polarized wave varies at each and vice versa (figure (5)). Then the oscillations of the resultant E-vector do not take place in a single plane. The magnitude of resultant E-vector varies at each point in space and the overall rotation of the E-vector has the appearance of a flattened helix. When we are looking back towards the source, if the rotation of E-vector occurs clockwise, it is said to be a right-elliptically polarized wave. If it rotates anti-clockwise, it is said to be a left-elliptically polarized wave.

(2013)

Q2. Explain Huygens' theory of double refraction. (2015)

Other Related Questions:-

(2015)

Q. Give a brief account of Huygens theory of double refraction in uniaxial crystals. (2011)

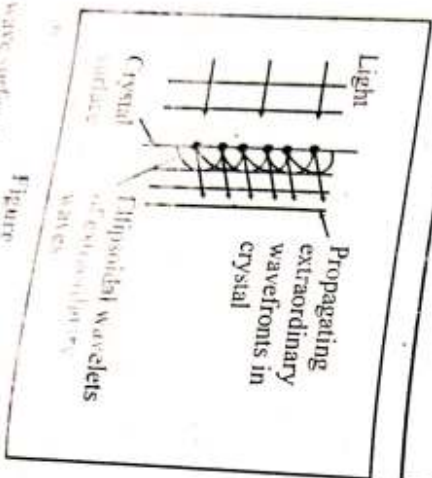
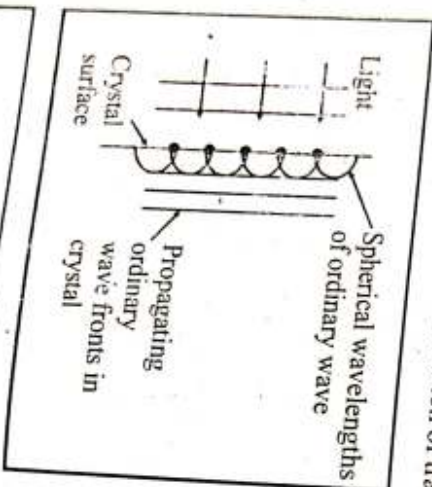
Q. Write short note on theory of double refraction in uniaxial crystal and with the help of diagram show double refraction for normal incidence. (2018)

Q. Give Huygens' theory of double refraction in uniaxial crystal and with the help of diagram show double refraction for normal incidence. (2018)

Ans. Huygens' theory of Double Refraction:-

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In order to explain the double refraction exhibited by anisotropic crystals, Huygens postulated that the incident light excites two separate wavelets within the crystal, one spherical wavelet associated with the ordinary waves and one ellipsoidal wavelet associated with the extraordinary waves. For example, the plane wavefront, in Fig., incident normally on the crystal surface generates spherical wavelets as well as ellipsoidal wavelets. The spherical wavelets propagate equally quickly in all directions. The wave surface corresponding to o-ray is therefore spherical. The tangent to these waves lies straight ahead and by successive application of the principle, the plane wave propagates straight ahead with speed  $v$ . The ellipsoidal wavelets propagate at different speeds in different directions. The wave surface corresponding to e-ray is therefore an ellipsoid of revolution about the optic axis. The common tangent to these ellipsoids after a little time is the new wavefront. The common tangent of generation of each ellipsoid to the tangent point on that ellipsoid is off at an angle and defines the direction of travel of the extraordinary wavefronts. The wavefronts are not perpendicular to their direction of travel.



refracted by the optic axis. As light propagates through the crystal, the two wave surfaces travel in different directions in the crystal. Ultimately, two refracted rays emerge from the crystal.

Q3. State the law of Malus.

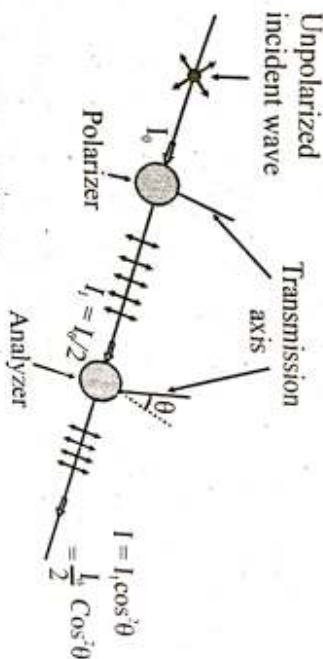
(2014)

Other Related Questions -  
Q. Discuss Malus' law.

Ans. Malus' Law:-

(2013)

The amount of light transmitted through a polarizer at an arbitrary angle is given by Malus' Law.



Figure

In 1809 Malus found that the intensity of polarized light transmitted through a polarizer is proportional to the square of cosine of the angle between the plane of polarization of the light and the transmission axis of the polarizer.

This statement is known as Malus' law.

If unpolarized light of intensity  $I_0$  is incident on a polarizer, plane polarized light of intensity  $I_0/2$  is transmitted by it. Let us denote  $I_0/2$  by  $I_1$ . Let this plane polarized light pass through an analyzer. The intensity of the light transmitted through the analyzer is given by

$$I = E_1^2 \cos^2 \theta = I_1 \cos^2 \theta = \frac{1}{2} I_0 \cos^2 \theta$$

Light transmitted through the analyzer at specific settings are as follows:

|                                   |                    |                             |
|-----------------------------------|--------------------|-----------------------------|
| Case (i): If $\theta = 0^\circ$   | axes parallel      | $I = I_1 = \frac{1}{2} I_0$ |
| Case (ii): If $\theta = 90^\circ$ | axes perpendicular | $I = 0$                     |
| Case (i): If $\theta = 180^\circ$ | axes parallel      | $I = I_1 = \frac{1}{2} I_0$ |
| Case (i): If $\theta = 270^\circ$ | axes perpendicular | $I = 0$                     |

Thus, we obtain two positions of maximum intensity and two positions of zero intensity when we rotate the axis of the analyzer with respect to that of the polarizer.

**Q.4. Write short note on Polaroids.****Ans. Polaroids:-****(2014)**

In 1928 E. H. Land invented Polaroid sheets, which utilize the phenomenon of selective absorption. The sheets are fabricated as follows. A clear plastic sheet of long chain molecules of PVA (polyvinyl alcohol) is heated and then stretched in a given direction to many times its original length. During the stretching process the PVA molecules become aligned along the direction of stretching. The sheet is then laminated to a rigid sheet of plastic to stabilise its size. It is then exposed to iodine vapour. The iodine atoms attach themselves to the straight long chain PVA molecules and consequently form long parallel conducting chains. The iodine atoms provide electrons, which can move easily along the aligned chains, but not perpendicular to them. When natural light is incident on the sheet, the electromagnetic vibrations that are in a direction parallel to the alignment of the iodine atoms are strongly absorbed because of those vibrations in a direction perpendicular to the chains. Consequently, only those vibrations in a direction perpendicular to the direction of molecular chains are transmitted. Thus, the light transmitted through the polaroid sheet is polarized. A sheet fabricated according to this process is known as H-sheet.

These sheet polarizers are inexpensive and can be made in large sizes. Polaroid sheets are widely used in sunglasses, camera filters etc to eliminate the unwanted glare from objects.

Polaroid sheets are extensively used for the production and detection of linearly polarized light.

**Q.5. Differentiate between Ordinary and extra ordinary rays. (2014)****Other Related Questions:-****Q. Describe polarization of ordinary and extra ordinary rays. (2014)**

**Ans. Comparison of ordinary and extraordinary light beams in a calcite crystal with respect to optics axis and direction of propagation. (2010)**

(i) o-ray obeys the laws of refraction and extra-ordinary rays:-

(ii) Both o-ray and e-ray does not follow the perpendicular planes.

(iii) Both o-ray and e-ray are plane polarized. They are polarized in mutually perpendicular planes. The electric vector of o-ray vibrates perpendicular to the principal section of o-ray while the vibrations of e-ray take place parallel to the principal section of e-ray.

(iv) Both o-ray and e-ray travel with the same speed in all direction within the crystal. The e-ray travels with different speeds along different directions in the crystal. However, the refractive index for e-ray varies from direction to direction. On the other hand, the refractive index for o-ray is defined as follows:

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$$\mu_0 = \frac{c}{v_0} = \frac{\text{velocity of light in a vacuum}}{\text{velocity of o-ray in the crystal}}$$

The principal refractive index for e-ray in **positive crystals** is defined as follows:

$$\mu_0 = \frac{c}{(v_e)_{\min}} = \frac{\text{velocity of light in a vacuum}}{\text{minimum velocity of e-ray in the crystal}}$$

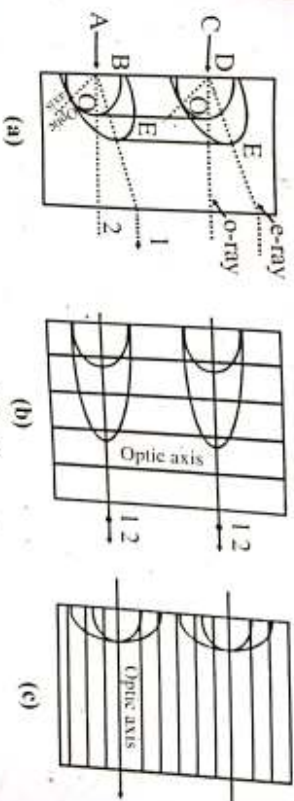
The principal refractive index for e-ray in **negative crystals** is defined as follows:

$$\mu_e = \frac{c}{(v_e)_{\max}} = \frac{\text{velocity of light in a vacuum}}{\text{maximum velocity of e-ray in the crystal}}$$

(v) When natural light is incident on an anisotropic crystal at an angle to the optic axis, it splits into o-rays and e-rays, which travel in different directions with different velocities figure (a).

When natural light is incident in a direction perpendicular to the optic axis, o-ray and e-ray propagate in the same direction in the crystal but with different velocities, as shown in figure (b). In a negative crystal e-ray leads o-ray and in case of a positive crystal o-ray leads e-ray.

When natural light is incident on the crystal in a direction parallel to the optic axis, it does not split into two rays. The o-rays and e-rays travel in the same direction with the same velocity, as shown in figure (c).

**Figure**

(vi) The distinction of o-ray and e-ray exists only within the crystal. Once they emerge from the crystal, they travel with the same velocity. The rays outside the crystal differ only in their direction of travel and plane of polarization. The designation of o-ray and e-ray has no meaning outside the crystal.

**Q.6. Write short note on production and detection of elliptically polarized light. (2016)****Ans. Production of Elliptically Polarized Light:-**

A quarter wave plate and a polarizer are the optical devices necessary to produce elliptically polarized light from unpolarized light.

Unpolarized light is first converted to plane polarized light by allowing

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it to pass through a polarizer (a polaroid sheet or a Nicol prism), as shown in figure (1). The plane-polarized light is then made incident on a quarter wave plate. The quarter wave plate or the polarizer is rotated such that the electric vector  $E$  of plane polarized light makes an angle  $\theta (\neq 45^\circ)$  with the optic axis of the quarter wave plate. The incident ray divides into o-ray and e-ray of amplitudes  $E \sin \theta$  and  $E \cos \theta$ . The rays travel along the same direction in the crystal with different velocities. The two rays are polarized in orthogonal planes. They are in phase at the front face but progressively get out of phase as they travel through the crystal. When they emerge out of the crystal they will have a path difference of  $\lambda/4$  or a phase difference of  $90^\circ$ . When they combine, they produce elliptically polarized light.

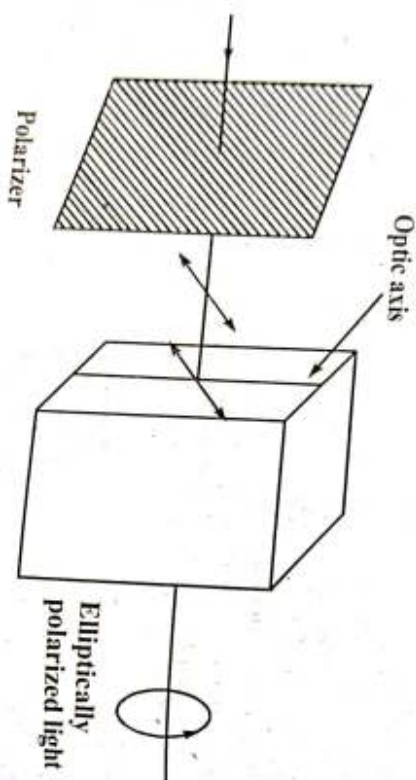


Figure 1  
Detection of Elliptically Polarized Light:-

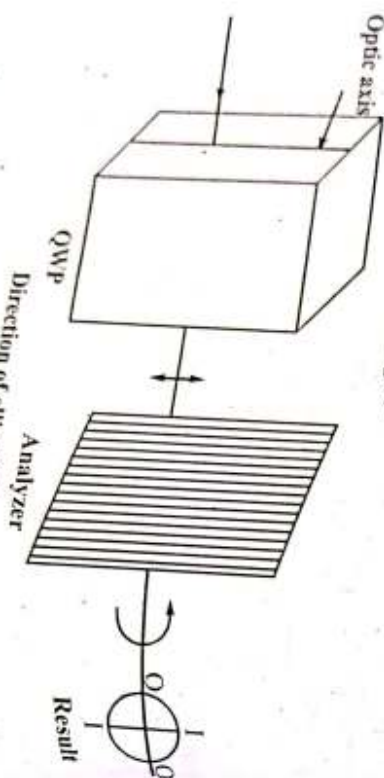


Figure 2  
Direction of elliptically polarized light

The light beam is allowed to pass through an analyzer (a polaroid sheet or a Nicol prism). If on rotating the analyzing polaroid sheet or Nicol, the intensity of the emerging beam varies from a maximum to a minimum value, but never

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reaching zero, then the incident light is elliptically polarized.

A similar result would be obtained if the incident light were partially polarized. The two cases may be distinguished by inserting a quarter wave plate in the path of light before it falls on the analyzer.

If the original light is elliptically polarized, it may be considered as resultant of two coherent plane polarized waves that is e-ray and o-ray, which are out of phase by  $90^\circ$ . If the light passes through the quarter wave plate, an additional phase difference of  $90^\circ$  is introduced between the e-ray and o-ray. Therefore, the total phase difference becomes  $180^\circ$  between the e-ray and o-ray. On emerging from the quarter plate, the e-rays and o-rays combine to produce plane-polarized light. If the light coming out of quarter wave plate is examined with an analyzer, light will be extinguished twice in one full rotation of the polarizer as shown in figure (2).

Q.7. Explain how linear and circular polarizations of light will change on passing through a half wave plate. (2015)

Other Related Questions -

Q. Differentiate between quarter wave plate and half wave plate. (2014, 18)

Q. Write short note on Quarter wave plate and its use. (2012)

Q. Discuss how the polarization states of (i) a plane polarized light and (ii) a circularly polarized light change in passage through a half waveplate. (2011)

Ans. Half wave plate:-

A half wave plate is a thin plate of birefringent crystal having the optic axis parallel to its refracting faces and its thickness chosen such that it introduces a half-wave ( $\lambda/2$ ) path difference (or a phase difference of  $180^\circ$ ) between e-ray and o-ray.

When a plane polarized light wave is incident on a birefringent crystal having the optic axis parallel to its refracting faces, it splits into two waves: o-waves and e-waves. The two waves travel along the same direction inside the crystal but with different velocities. As a result, when they emerge from the rear face of the crystal, an optical path difference would be developed between them.

$$\therefore (\mu_o - \mu_e) d = \frac{\lambda}{2}$$

$$\therefore d = \frac{\lambda}{2(\mu_o - \mu_e)}$$

A half wave plate introduces a phase difference  $\delta$ , between e-ray and o-ray given by  $\delta = (2\pi/\lambda) \Delta = \pi = 180^\circ$

Now let a plane polarized light be incident normally on the half-wave plate. Let the electric vector  $E$  make an angle with the optic axis of the half

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wave plate. The incident wave splits into two waves, e-waves and o-waves. The waves progressively develop path differences as they travel through the crystal and they emerge with a phase difference of  $180^\circ$ . When the two waves combine, they yield a plane-polarized wave, which has its plane of polarization rotated through an angle  $2\theta$ .

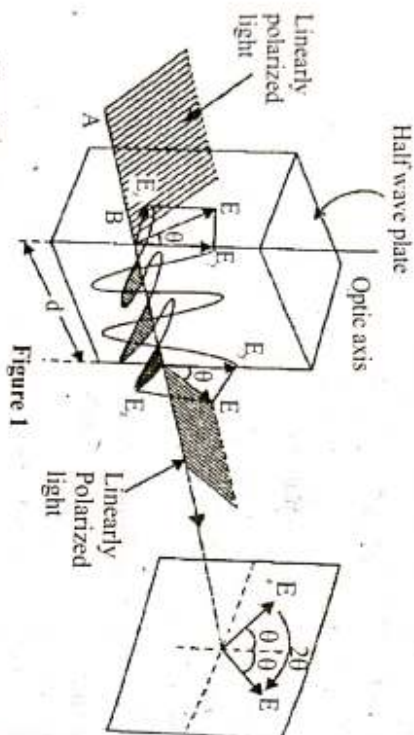


Figure 1

Therefore, a half-wave plate rotates the plane of polarization of the incident plane polarized light through an angle  $2\theta$ . The half wave plate will invert the handedness of elliptical or circular polarized light, changing right to left and vice versa.

Now we can understand what happens when e-ray and o-ray overlap on each other after emerging from an anisotropic crystal plate. It is obvious that they cannot produce interference fringes as in a double slit experiment. On the other hand, they combine to produce different states of polarization depending upon their optical path difference.

(i) When the optical path difference is 0 or an even or odd multiple of  $\lambda/2$ , the resultant light wave is linearly polarized.

(ii) When the optical path difference is  $\lambda/4$ , the resultant light wave is elliptically polarized.

(iii) In the particular instance when the wave amplitudes are equal and the optical path difference is  $\lambda/4$ , the resultant light wave is circularly polarized.

**Quarter wave plate:-**

A quarter wave plate is a thin plate of birefringent crystal having the optic axis parallel to its refracting faces and its thickness adjusted such that it introduces a quarter-wave ( $\lambda/4$ ) path difference (or a phase difference of  $90^\circ$ ) between the e-ray and o-ray propagating through it.

When a plane polarized light wave is incident on a negative birefringent crystal having the optic axis parallel to its refracting face, the wave splits into e-wave and o-wave figure. The two waves travel along the same direction but

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with different velocities. As a result, when they emerge from the rear face of the crystal, an optical path difference would be developed between them. Thus,

$$(\mu_o - \mu_e) d = \frac{\lambda}{4}$$

$$d = \frac{\lambda}{4(\mu_o - \mu_e)}$$

A quarter wave plate introduces a phase difference  $\delta$ , between e-ray and o-ray given by  $\delta = (2\pi/\lambda) \Delta = \pi/2 = 90^\circ$

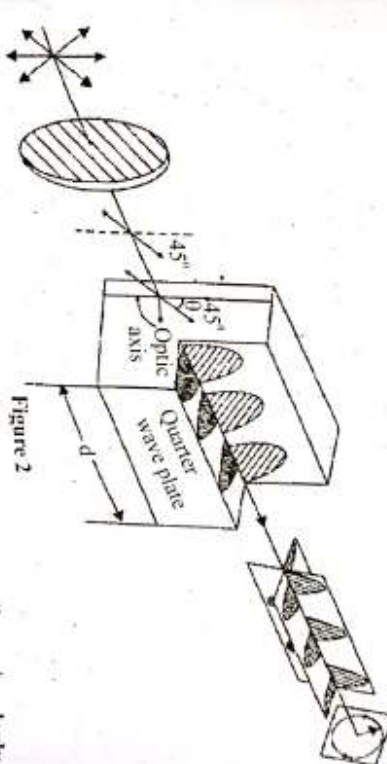


Figure 2

A quarter-wave plate is used for producing elliptically or circularly polarized light. It converts plane-polarized light into elliptically or circularly polarized light depending upon the angle that the incident light vector makes with the optic axis of the quarter wave plate.

### Numerical

Q.1 Calculate the thickness of a double refracting plate which can convert a circularly polarized light into plane polarized light. Given  $\mu_o = 1.658$ ,  $\mu_e = 1.486$  and  $\lambda = 5890 \text{ \AA}$ . (2014)

Sol. Plane polarized light gets converted into circularly polarized light by a suitably oriented quarter wave plate. Its thickness is given by

$$d = \frac{\lambda}{4(\mu_o - \mu_e)} = \frac{5890}{4(1.658 - 1.486)} \text{ \AA}$$

or  $d = 0.856 \text{ \mu m}$ .

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Q.2. Unpolarized light of intensity  $I_0$  is incident on polarizer. Deduce the intensity of light after passing from polarizer. (2013)

Other Related Questions -

Q. Unpolarized light of intensity  $I_0$  is incident on the polarizer. Calculate the intensity of light transmitted from the polarizer. (2019)

Ans. If incident beam is unpolarized, the intensity of the resultant beam will be  $I_0/2$ .

## Chapter 8

Q.1. Define optical rotation. Give the Fresnel's theory of optical rotation. (2016)

Other Related Questions -

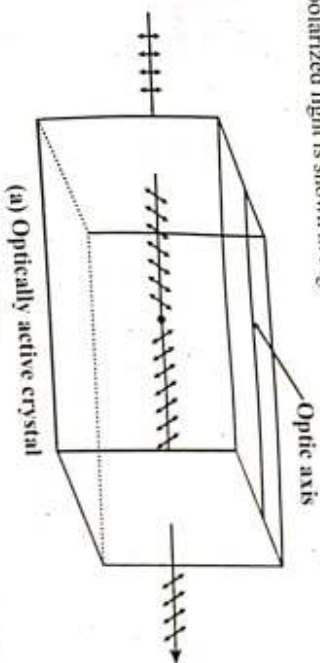
Q. Give Fresnel's theory of optical rotation. (2015)

Q. Obtain an expression for optical rotation produced giving by an optically active medium giving the meaning of the symbol used. (2012)

Q. Define optically active substance. Describe Fresnel's theory of optical rotation. (2019)

Ans. Optical Rotation:-

When a beam of plane polarized light propagates through a quartz crystal along the optic axis, the plane of polarization steadily turns about the direction of the beam. The optical rotation can be detected as follows. If two polaroid sheets or Nicol prisms are held in crossed configuration and if a beam of unpolarized light is viewed through them, the field of view appears to be completely dark. Now let a quartz crystal, cut with its faces perpendicular to the optic axis, be inserted between the polarizers such that light is incident normally on the crystal. The field of view now appears lit up indicating that the light is not cut off by the analyzer. In order to cut off the transmitted light, we find that the analyzer is to be rotated through a certain angle. The experiment establishes that the plane polarized light produced by the polarizer remains plane polarized while passing through the quartz crystal but the plane of polarization is rotated through an angle. This angle is the angle through which the analyzer is rotated in order to cut off the light totally. The optical rotation, i.e., rotation of the plane of polarized light is shown in Fig.



Figure

The ability to rotate the plane of polarization of plane polarized light by certain substance is called **optical activity**. Substances, which have the ability to rotate the plane of the polarized light passing through them, are called **optically active substances**. Quartz and cinnabar are examples of optically active crystals while aqueous solutions of sugar, tartaric acid are optically active solutions.

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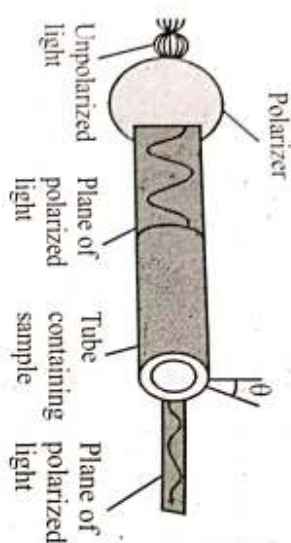
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(b) Optically active solution

Rotation of the plane polarized light. The plane is rotated by an angle after the light passes through an optically (a) active crystal or (b) solution.

Optically active substances are classified into two types.

(i) Dextrorotatory substances:-

Substances which rotate the plane of polarization of the light toward the right are known as right-handed or dextrorotatory.

(ii) Laevorotatory substances:-

Substances which rotate the plane of polarization of the light toward the left are known as left-handed or laevorotatory.

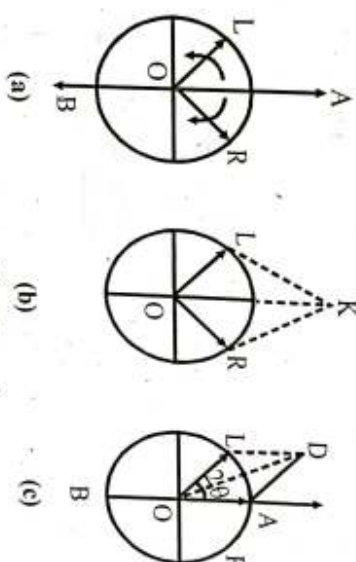
**Fresnel's Explanation:-**

A linearly polarised light can be considered as a resultant of two circularly polarised vibrations rotating in opposite directions with the same angular velocity. Fresnel assumed that plane-polarised light on entering a crystal along the optic axis is resolved into two circularly polarised vibrations rotating in opposite directions with the same angular frequency. In an optically inactive crystal like calcite, the two circularly polarised vibrations travel with the same angular velocity. On the other hand, in an optically active crystal like quartz, the two circularly polarised vibrations travel with the different angular velocities.

Figure (a) shows plane polarized light entering a calcite crystal along the optic axis AB and split up into two circular motions rotating in opposite directions. They are represented by OL and OR. OL is the circularly polarised vector rotating in the anticlockwise direction and OR is the circularly polarised vector rotating in the clockwise direction. If OL and OR start simultaneously from OA and rotate with the same angular velocity, then at any subsequent time, the resultant of OL and OR will lie along OA. Hence on emerging from the crystal, the two circular waves combine to produce a linear vibration along the initial direction OA. Figure (b). Therefore, crystals like calcite do not rotate the plane of vibration.

In case of quartz, the linearly polarised light, the component having components emerge out of the crystal, they are at an angle  $2\epsilon$ . The resultant of these two vectors (OR and OL) is now along OD figure (c). Before entering the crystal, the plane of vibration is along OA and after emerging from the crystal,

it is along OD which makes an angle  $\epsilon$  with the optic axis OA. After passing through the crystal, the circular components combine to produce a linear vibration whose direction is now along OD. Therefore, the plane of vibration has rotated through an angle  $\epsilon$ . The angle through which the plane of vibration is rotated depends on the thickness of the crystal.



Figure

Left and right circularly polarised light travel through the crystal at different speeds. Because one travels more slowly than the other, a phase shift builds up between them. When right and left-handed light are combined together at any point, the result is always linear polarization; but the angle of the polarisation depends on the phase shift between the two circular polarisations. If  $\mu_R$  and  $\mu_L$  are the refractive indices corresponding to the right and left circularly polarized light, the angle of polarisation,  $\epsilon$ , is given by:

$$\theta = \frac{\pi}{\lambda} (\mu_R - \mu_L) d$$

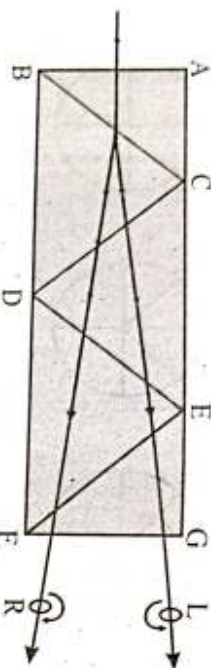
where  $d$  is the thickness of the material. As a result, the direction of polarisation spirals around  $z$  in space.

The cause of this special behaviour for circularly polarised light is that the optically active molecules are chiral, meaning that they have helical twist in them. Any arrangement of atoms with a helical structure can form left-handed helices or right-handed helices. One is a mirror image of the other. Now there is a curious property of helices not possessed by rotating circles. If we look at a wheel rotating clockwise from one side, the same wheel appears to go around clockwise from the other side. Helices are different. If we turn round a right-handed helix (in which the spiral appears to go around clockwise as we look into the helix), then it is still a right-handed helix. So, helical molecules may interact with circularly polarised light differently, depending on whether their chirality is the same handedness as the circular polarisation or opposite handedness. This is the fundamental reason for the two refractive indices and it is interesting that the cause of the phenomenon lies at a molecular level and

does not have anything to do with spatial ordering of molecules. This is why the phenomenon occurs with liquids as well as solids.

**Q.2. Write short note on Laurent's half shade polarimeter. (2014, 17)**  
**Ans. Laurent's Half Shade Polarimeter:-**

A polarimeter is an instrument used for determining the optical rotation of solutions. When used for determining the optical rotation of sugar it is called a saccharimeter.



Figure

#### Construction:-

A polarimeter consists of a glass tube for holding the solution under test held between crossed Nicol prisms. Beyond the polarizing Nicol prism a half-shade plate is located which is used for accurately adjusting the two Nicol for crossed position. Light from a monochromatic source is rendered parallel by the lens L and is incident on the polarizer,  $N_1$ . The light transmitted by the polarizer is plane polarized. The polarized beam then passes through the half shade plate and a glass tube G containing the solution. The light emerging from the solution will be incident on the analyzer  $N_2$ . The light is observed through a telescope T. The analyzing Nicol  $N_2$  can be rotated about the axis of the tube the rotation can be measured with the help of a graduated circular scale.

#### Working:-

To find the specific rotation of a solution, the analyzer is first adjusted such that field of view is completely dark. Then the glass tube is filled with the solution and is held in position. The field of view now becomes illuminated. The field can be again be made dark by rotating the analyzer through a certain angle which gives the optical rotation of the solution. The practical difficulty in this method is in determination of the exact position for which complete darkness is achieved. The difficulty is overcome by using what is known as a Laurent's half-shade device.

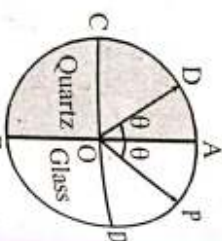


Figure 2

It consists of a semicircular half wave plate ACB of quartz cemented to a semicircular plate ADB of glass figure (2). The optic axis of the wave plate is parallel to the line of separation AB. The half wave plate introduces a phase

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difference of  $180^\circ$  between e-ray and o-ray passing through it. The thickness of the glass plate is such that it transmits the same amount of light as done by the quartz half wave plate. One half of the incident light passes through the quartz plate ACB and the other half through the glass plate ADB. When the light after passing through the polarizer is incident normally on the half shade plate and has vibrations along OP. On passing through the glass, half the vibrations will remain along OP but on passing through the quartz half, the vibrations will split into e-rays and o-rays. The o-vibrations are along OD and e-vibrations are along OA. The half wave plate introduces a phase difference of  $\pi$  rad between the two vibrations. The vibrations of o-ray will occur along OC instead of OD on emerging from the plate. Therefore the resultant vibration will be along OQ whereas the vibrations of the beam emerging from glass plate will be along OP. In effect, the half wave plate turns the plane of polarization of the incident light through an angle  $2\theta$ .

If the principal plane of the Nicol  $N_2$  is aligned parallel to OP, the plane polarized light emerging from the glass tube will pass through the glass plate of the half shade plate and that part appears brighter. On the other hand light coming out of the quartz plate is partially obstructed and the corresponding field of view appears less bright. If the principal plane of  $N_2$  is parallel to OQ the quartz half will appear brighter than the glass half. Thus, the two halves of the plate are unequally illuminated. When the principal plane of  $N_2$  is parallel to AB, the two halves appear equally bright and when it is parallel to CD, the two halves are equally dark.

To find the specific rotation of a solution, the analyzer is first in the position for equal darkness without solution in the tube G. The reading on the circular scale is noted. Next, the tube is filled with the optically active solution of known concentration. The field of view is now partially illuminated. The analyzer is rotated till the field of view becomes equally dark.

The reading on the circular scale is noted again. The difference between the two scale readings gives the angle of rotation of the plane of polarization caused by the solution. Knowing the values of  $\theta$ ,  $l$ , and C, the specific rotation is obtained using the formula.

$$[S]_D = \frac{\theta}{KCl} \quad \dots\dots(i)$$

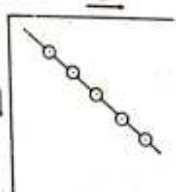


Figure 3

In the actual experiment, different concentrations of solutions are taken and the corresponding angles of rotation are determined. A graph is plotted between concentration C and the angle of rotation  $\theta$ . The graph is a straight line figure (3). Using the value of the slope in eq. (i), the specific rotation of the optically active substance is calculated.

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Q.3. Describe the construction and theory of Babinet compensator and explain how the Babinet compensator is used to produce elliptically polarized light.

Other Related Questions -

Q. Write short note on fringes formation in Babinet's compensator. (2019)

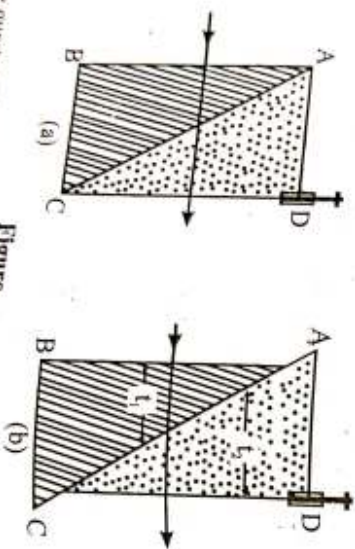
Q. Discuss formation of fringes by Babinet compensator. (2015)

Ans. Babinet Compensator:- (2011)

A compensator is an optical device whose function is to compensate a path difference. It is used in conjunction with a polarizer and analyzer combination to investigate elliptically polarized light. The compensator helps in determining the axis of the ellipse and the ratio of their lengths. Elliptically polarized light may be considered as the resultant of two coherent plane polarized vibrations occurring in mutually orthogonal directions and with an initial path difference of  $\lambda/4$ . When the elliptically polarized light passes through the device such that it introduces a further path difference of  $\lambda/4$ . The total path difference between the perpendicular vibrations become  $\lambda/2$  and the vibrations recombine after emerging from the device to produce plane polarized light. From the analysis of this plane polarized light, the information regarding the incident elliptically polarized light can be obtained.

**Construction:-**

The Babinet compensator is made of two wedge-shaped quartz sections, ABC and ADC, having equal acute angles. The wedges are placed against each other such that they form a small rectangular block as shown in figure.



Figure

One of the quartz wedges is fixed and the other can be displaced along their plane of contact with the help of a micrometer screw arrangement. Thus, the combination acts as a plate of variable thickness. The optic axis of the first section is parallel to its refracting edge AB and the optic axis of the second section is in a direction perpendicular to the

edge. The two optic axes are perpendicular to each other and also to the perpendicular to the incident beam.

**Production of Polarized Light:-**

Let plane polarized light be incident normally on the face AB of the compensator. It splits into e-ray and o-ray parallel and perpendicular to AB respectively. The e-ray travels slower than o-ray in the first section, since quartz is a positive uniaxial crystal. When these rays enter the second section, the e-ray becomes o-ray since the optic axis in the second section is in a direction normal to that in the first prism. Similarly, o-ray becomes e-ray. Thus, the two rays exchange their velocities in passing from one section to the other section. The net effect is that one section cancels the effects of the other.

If  $d_1$  is the thickness of the first section, and  $\mu_e$  and  $\mu_o$  are the refractive indices, of quartz for e-rays and o-rays respectively, the path difference between the e-rays and o-rays in the first section will be

$$\Delta_1 = [\mu_e - \mu_o] d_1$$

As the principal planes of the two sections are at right angles, the e-rays and o-rays change their roles in going from the first section to the second section. The velocities of e-ray and o-ray are interchanges and if the thickness of the second section is  $d_2$ , then the path difference between the rays in the second prism will be

$$\Delta_2 = [\mu_o - \mu_e] d_2$$

As the compensator is thin, the separation of the rays is negligible. The net path difference between the two rays after emerging from the crystal will be

$$\Delta = \Delta_1 + \Delta_2$$

$$\Delta = (\mu_e - \mu_o) d_1 + (\mu_o - \mu_e) d_2 = (\mu_e - \mu_o) (d_1 - d_2)$$

The net phase difference is

$$\delta = \frac{2\pi}{\lambda} (\mu_e - \mu_o) (d_1 - d_2)$$

For a ray passing through the centre of the compensator where  $d_1 = d_2$ , the net path difference and hence the phase difference is zero. It means that the effect of one wedge is exactly cancelled by the other. This is true for all wavelengths and the incident vibration is transmitted as such. Plane polarized light incident on the compensator will emerge as plane polarized light with its plane of vibration parallel to that of the incident light.

Any desired thickness difference ( $d_1 - d_2$ ) can be achieved at the centre of the compensator by moving the second section relative to the first section. Thus, any desired value of phase difference can be obtained between the e-rays and o-rays. Therefore, the light emerging will be either plane or circular or elliptically polarized light, depending on the phase difference.

Thus the compensator has the same effect as that of a wave plate of varying thickness. The advantage of compensator is that it can be arranged to suit any wavelength while a quarter wave plate is designed to suit only one particular wavelength.

**Analysis of Elliptically Polarised Light:-**

Using the compensator, one can determine the characteristic of elliptically polarised light.



Figure 1

Let the compensator be placed between crossed polarizer P and analyzer A, as shown in Figure (1). Let the transmission axis of polarizer be oriented at  $45^\circ$  with respect to the optic axis of wedge ABC. If the compensator A is independent of the light emerging from the compensator is plane polarized in the same plane as transmitted by P and therefore it will be extinguished by the analyzer A. Similarly, as darkness from the midpoint for which the rotation is  $180^\circ$  or  $360^\circ$ , no net the emergent light is plane polarized in the same plane as transmitted by P and hence will be extinguished by the analyzer. So the field of view is covered by a series of equidistant parallel dark bands.



Figure (2)

As pressure between them, where the path difference corresponds to an odd multiple of  $\lambda/2$ , i.e.,  $\lambda/2, 3\lambda/2, 5\lambda/2, \dots (2m+1)\lambda/2$ , the transmitted light is plane polarized. The analyzer transmits the light completely and those regions will be bright. In all other cases, the emerging light is elliptically polarized with varying parameters of the ellipse, as shown in Figure (2). If white light is used, the central band will be dark while others will be coloured.

By using white light source, the compensator is adjusted such that the central dark band is under cross wire and the micrometer reading is noted. The micrometer wire is then turned through an angle such that the compensator introduces a phase difference of  $\pi/2$  at cross wire. Then elliptically polarized light

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is made to be incident on the compensator. The central dark band undergoes a shift with respect to the cross wire. The compensator is rotated through an angle  $\alpha$  in its own plane until the central dark band is on the cross wire. The axes of the incident elliptically polarized light are parallel to the optic axes of the wedges of the compensator.

**Phase difference:-**

The elliptical vibration can be regarded as made of two mutually perpendicular linear vibrations, which are having a phase difference,  $\delta$ .  $\delta$  can be determined as follows.

First the compensator is illuminated with white plane-polarised light and the micrometer is adjusted to bring the central dark band on the cross-wire. The white light is then replaced by elliptically polarised light. The central band shifts to a point where the original phase difference  $\delta$  between the two component vibrations of elliptical polarised light is exactly balanced by the phase difference introduced by the compensator. This phase difference is determined by rotating the screw until the central dark band is again on the cross-wire. If this rotation is  $\phi$ , then

$$\frac{\delta}{2\pi} = \frac{\phi}{\alpha} \quad \text{or} \quad \delta = \frac{2\pi\phi}{\alpha}$$

**Position of axes:-**

The position of the major and minor axes of the given elliptical vibration can be found as follows. The compensator is illuminated with white polarised light and the micrometer screw is adjusted to bring the central dark band on the light and the micrometer screw is then turned through an angle  $\alpha/4$  so that the compensator introduces a phase difference of  $90^\circ$ . The central dark band now not on the cross-wire. The elliptically polarised light is made incident on the compensator. Then the compensator is rotated in its own plane until the central dark band again comes on to the cross-wire. The axes of the incident light are parallel to the optic axes of the wedges of the compensator.

**Ratio of the axes:-**

From figure (3) OA and OB represent the optic axes of the two wedges of the compensator. OC is the direction of the principal section of the analyzer. DE is the direction of vibration of light emerging from the compensator at the cross-wire. The tangent of the angle  $\theta$  that the principal section of the analyzer makes with the optic axes of the wedge gives the ratio of the axes. Thus,

$$\tan \theta = a/b$$

$\theta$  is determined by rotating the analyzer until the bands disappear.

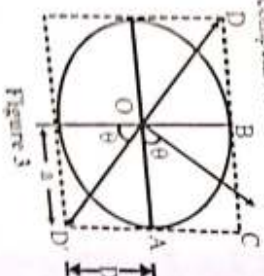


Figure 3

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Thus the compensator has the same effect as that of a wave plate of varying thickness. The advantage of compensator is that it can be arranged to suit any wavelengths while a quarter wave plate is designed to suit only one particular wavelengths.

### Analysis of Elliptically Polarized Light: -

Using the compensator, one can determine the characteristics of elliptically polarized light.

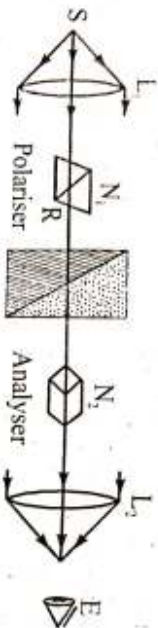


Figure 1

Let the compensator be placed between crossed polarizer P and analyser A, as shown in figure (1). Let the transmission axis of polarizer be oriented at  $45^\circ$  with respect to the optic axis of wedge ABC of the compensator. At midpoint R the light emergent from the compensator is plane polarized in the same plane as transmitted by P and therefore it will be extinguished by the analyser A. Similarly, at distances from the midpoint for which the retardation is  $1\lambda, 2\lambda, 3\lambda, \dots, m\lambda$  and the emergent light is plane polarized in the same plane as transmitted by P and hence will be extinguished by the analyser. So the field of view is crossed by a series of equidistant parallel dark bands.

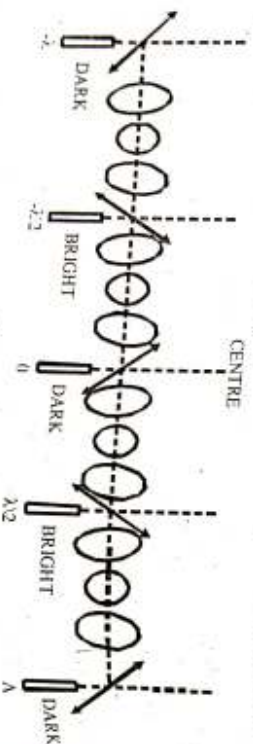


Figure (2)

At position between them, where the path difference corresponds to an odd multiple of  $\lambda/2$ , i.e.,  $\lambda/2, 3\lambda/2, 5\lambda/2, \dots, (2m+1)\lambda/2$ , the transmitted light is plane polarized. The analyser transmits the light completely and those regions will be bright. In all other cases, the emerging light is elliptically polarized with varying parameters of the ellipse, as shown in figure (2). If white light is used, the central band will be dark while others will be coloured.

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The elliptical vibration can be regarded as made of two mutually perpendicular linear vibrations, which are having a phase difference,  $\delta$ .  $\delta$  can be determined as follows.

First the compensator is illuminated with white plane-polarised light and the micrometer is adjusted to bring the central dark band on the cross-wires. The white light is then replaced by elliptically polarised light. The central band shifts to a point where the original phase difference  $\delta$  between the two component vibrations of elliptical polarised light is exactly balanced by the phase difference introduced by the compensator. This phase difference is determined by rotating the screw until the central dark band is again on the cross-wires. If this rotation is  $\phi$ , then

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Form figure (3) OA and OB represent the optic axes of the two wedges of the compensator. OC is the direction of the principal section of the analyser. DE is the direction of vibration of light emerging from the compensator at the cross-wires. The tangent of the angle  $\theta$  that the principal section of the analyser makes with the optic axes of the wedge gives the ratio of the axes. Thus,

$$\tan \theta = a/b$$

$\theta$  is determined by rotating the analyser until the bands disappear

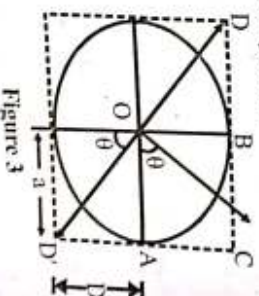


Figure 3

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and the field becomes uniformly illuminated. The angle of rotation is  $\theta$ .

**Advantages:-**

A quarter wave plate produces a fixed phase difference between o-ray and e-ray and can be used only for monochromatic light of one particular wavelength. In case of a compensator, the phase difference between the rays can be varied continuously and hence a compensator, made of a combination of wedges, can be used for light of any given wavelength.

**Q.4. Obtain an expression for rotation of plane of polarization produced by an optically active substance.**

**Other Related Questions-**

(2014)

**Q. Define specific rotation.**

(2018)

**Ans. Specific Rotation:-**  
A measure of the optically activity of a sample is the rotation produced for a 1mm slab for a solid, or a 100 mm path length for a liquid. This measure is called the specific rotation. Liquids usually rotate the light much less than solids. Solutions of solids will obviously show an effect that depends on the concentration of active material and, to a small extent, both on temperature and the solvent.

If an optically active material is kept between two crossed polarizers, the field of view becomes bright. In order to get darkness once again, the analyzer has to be rotated through an angle. The angle through which the analyzer is rotated equals the angle through which the plane of polarization is rotated by the optically active substance. This angle depends on

- The thickness of the substance,
- Density of the material or concentration of the solution,
- Wavelength of light, and
- The temperature.

The amount of rotation  $\theta$  caused by crystalline materials is given by,

$$\theta = \alpha l$$

where  $\alpha$  is called the **rotational constant**  
In solutions the amount of rotation  $\theta$  is given by

$$\theta = s c l$$

where  $c$  is the concentration and  $s$  is called the **specific rotation**.  
The specific rotation for a given wavelength of light at a given temperature is defined conventionally as the rotation produced by one decimetre long column of the solution containing 1 gm of optically active material per c.c. of solution.

$$[s]_D = \frac{\theta}{l \times c} = \frac{\text{Rotation in degrees}}{\text{Length in decimeters} \times \text{conc. in gm/cc.}}$$

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### Numerical

**Q.1. 80 gm of impure sugar when dissolved in one litre of water gives an optical rotation  $9.9^\circ$  when placed in a tube of length 20 cm. If the specific rotation of sugar is  $65^\circ$  (decimeter)<sup>-1</sup>(gm/cc)<sup>-1</sup>. Find the percentage of impurity in the sugar sample.**

**Sol.** The specific rotation is given by

(2016)

$$[\alpha]_D = \frac{100}{l \cdot c} \Rightarrow c = \frac{100}{l \cdot \alpha}$$

Here,  $\theta = 9.9^\circ$ ,  $\alpha = 65^\circ$  and  $l = 20$  cm

$$\therefore c = \frac{10 \times 9.9}{20 \times 65} = 0.076 \text{ gm/cc.} = 76 \text{ gm/litre}$$

It means 80 gm of impure sugar is dissolve in 1 litre in place of 76 gm of pure sugar. Therefore, the percentage purity of sugar sample

$$= \frac{76}{80} \times 100 = 95\%$$

Hence impurity is = 100 - 95 = 5 %.

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