Writer : Ruchi Singh Kostha

Edition : First

Publisher : WINSOME PUBLICATION
Industrial Colony, Naini
Allahabad-211008
Uttar Pradesh
Mo.: 9935951901
Visit us at: www.winsomepublication.in

Composed by : Rudransh Graphics
Allahabad

Printers : Bhargava Press

© All rights reserved with the publishers.

Please give your valuable feedback on - winsomepublication@rediffmal.com

While all possible efforts have been made in the preperation of this book, neither the author nor publishers are responsible for any kind of errors and omissions. In case of any dispute it will be subject to Allahabad Jurisdiction only.

SYLLABUS

Unit I

Atomic Physics - Bohr-Sommerfield Model (History developments), Bohr model and the spectra of hydrogenic atoms, critical resonance and the ionisation potentials, Frank-Hertz experiment, characteristic and continuous X-rays, Moseley's law, Bragg's law.

Space Quantization, vector atom model and Quantum Numbers, Magnetic moment of the electrons and magneton, Larmor Precession, Electron spin, Stern-Gerlach experiment, Qualitative concept of various quantum number of an electron, Pauli's exclusion principle and electronic configuration of atoms.

Unit II

Magnetic Properties of Materials - Diamagnetism, Larmor's theory and dia-magnetic susceptibility, Paramagnetism, Langevin's theory and Curie-Weiss Law, Qualitative discussion of Ferromagnetism and antiferromagnetism.

Unit III

Quantum Concepts - Particle nature of radiation, Photoelectric effect and Compton effect, Wave nature of particles. De-Broglie waves, Davisson-Germer experiment, Wave packets, Phase velocity and group velocity, Heisenberg's uncertainty principle and applications, One dimensional Schrodinger's wave equation and concept of probabilities, amplitude application to one-dimen-sional potential step and barrier, quantum mechanical tunneling.

Unit IV

Nuclear Physics - Natural radioactivity, Laws of radioactive disintegration, radioactive series, detection of radiation, GM Counter and Bubble chamber Scintillation Counter.

Kinematics of nuclear reactions, artificial nuclear transmulation, discovery of neutron, radioactive tracers, transuranic elements, Cyclotron constitution of nucleus, Binding energy, liquid drop model and the semiempirical mass for-mula, Elementary theory of α-decay, β-decay and discovery of neutrino Magic numbers and the shell model, exchange forces in nuclei and Yukawa theory (qualitative), Thermonuclear energy, Classification of Elementary particles, Leptons, mesons and bayrons and their qunatum numbers, conservative laws.

Scanned by CamScanner

CONTENTS

S.N.	Details	Page No.
1.	Unit I	1 - 37
2.	Unit II	38 - 56
3.	Unit III	57 - 78
4.	Unit IV	79 - 124

Students preparing with WINSOME SERIES get excellent marks in exam

Our Publication

WINSOME SERIES
for ALLAHABAD STATE UNIVERSITY
&
WINSOME SERIES
For Allahabad University

Atomic Physics

Q.1. Give the postulates of Bohr for his atom model obtain expression for the radius and electron-energy of the n^{th} orbit. Explain how Bhor's atom model successfully accounts for the hydrogen spectrum. (2017)

Other Related Questions -

Q. States the various postulates of Bohr theory of hydrogenic atoms. Find the expression for the energy of the electron revolving in the n^{th} orbit. Calculate the energy of 1^{st} orbit of hydrogen atom. (2016)

Q. State the postulates of Bhor's theory of hydrogenic atom and derive an expression for the radius of n^{th} orbit. (2014)

Q. State the various postulates of Bohr's theory. Derive an expression for the radius of n^{th} orbit of Hydrogen like atom. What is Bohr radius? Calculate Bohr radius for hydrogen atom. (2012, 19)

Ans. Bohr Atom Model: -

Bohr assumed basically Rutherford nuclear - model of the atom and tried to overcome the defects of the model. He proposed two postulates.

Basic Postulates: -

(1) An electron cannot revolve round the nucleus in all possible orbits as suggested by the classical theory. The electron can revolve round the nucleus only in those allowed or permissible orbits for which the angular momentum of the electron is an integral multiple of $\hbar$.

Here, $\hbar$ is the Planck's constant - or rather, his original constant (h) divided by 2π:

$$\hbar = \frac{h}{2\pi} = 1.054572 \times 10^{-34} \text{ Js.}$$

$$h = 6.64 \times 10^{-34} \text{ Js.}$$

These orbits are called stationary orbits and an electron revolving in these orbits does not radiate energy.

Explanation: -

For an electron of mass m, moving with a speed v in an orbit of radius r, $L = mvr = mr^2\omega = n\hbar$. n is called the principal quantum number. It tankes integral values, excluding zero; n = 1, 2, 3, 4 .....

(2) An atom radiates energy only when an electron jumps from a stationary orbit of higher energy to one lower energy. If the electron jumps from an initial orbit of energy E_i to a final orbit of energy E_f ($E_i > E_f$), a photon frequency $\nu = \frac{E_i - E_f}{h}$ is emitted.

The Bohr formulae: -

Based on these postulates, Bohr derived the formulae for (i) the radii of

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

the stationary orbits and (ii) the total energy of the electron in the orbit.

Consider an atom whose nucleus has a positive charge Ze and mass M (Fig.1). For hydrogen, Z = 1. Let an electron of charge (-e) and mass m move round the nucleus in an orbit of radius r. Since M >> m, the nucleus is stationary. Hence the mass of nucleus does not come into the calculations.

The electrostatic force of attraction between the nucleus and the electron $= \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(-e)}{r^2}$ ..... (1)

The centrifugal force on the electron = mv^2/r ..... (2)

The system will be stable if $\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}$ ..... (3)

According to Bohr's first postulate, $mvr = n\hbar$ or $v = \frac{n\hbar}{mr}$

or $v^2 = \frac{n^2\hbar^2}{m^2r^2}$ ..... (4)

Substituting this value of v^2 in eq.(3),

$$\frac{m}{r}\left(\frac{n^2\hbar^2}{m^2r^2}\right) = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}$$

or $r = \frac{(4\pi\varepsilon_0)\hbar^2n^2}{Zme^2}$ ..... (5)

$\therefore$ Radius of the n^{th} permissible orbit for hydrogen $= r_n = \frac{4\pi\varepsilon_0\hbar^2}{me^2}n^2 (\because Z = 1)$

$\therefore \quad r_n = a_0n^2.$

Here the Bohr radius a_0 is defined as

$$a_0 = \frac{4\pi\varepsilon_0\hbar^2}{me^2} = \frac{4\pi(8.854\times10^{-12})(1.055\times10^{-34})^2}{(9.11\times10^{-31})(1.6\times10^{-19})^2}\,m = 0.053nm.$$

Calculation of total energy: -

The total energy of the electron in any orbit is the sum of its kinetic and potential energies. The potential energy of the electron is considered to be zero when it is at an infinite distance from the nucleus. P.E of an electron in an orbit is given by the work done in bringing the electron from infinity to that orbit. This amount of work is obtained by integrating the electrostatic force of attraction between the nucleus and electron from the limits ∞ to r.

P.E. of the electron $= \int_{\infty}^{r}\frac{Ze^2}{4\pi\varepsilon_0r^2}\,dr = \frac{-Ze^2}{4\pi\varepsilon_0r}$ ..... (6)

Visit us for more detail at : www.winsomepublication.in

K.E. of the electron $= \frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0r}$ ..... (7)

$\left[\because \text{from eq.(3)}, mv^2 = \frac{Ze^2}{4\pi\varepsilon_0r}\right]$

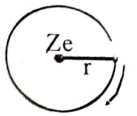


Figure 1

$\therefore$ Total energy of the electron in the n^{th} orbit = E_n = P.E. +K.E.

$$= \frac{-Ze^2}{4\pi\varepsilon_0r} + \frac{Ze^2}{8\pi\varepsilon_0r} = \frac{-Ze^2}{8\pi\varepsilon_0r}$$

Substituting the value of r from (5),

$$E_n = \frac{-me^4Z^2}{32\pi^2\varepsilon_0^2\hbar^2}\frac{1}{n^2}$$ ..... (8)

As the value of n increases, E_n increases. Hence, the outer orbits have greater energies than the inner orbits.

Bohr's interpretation of the Hydrogen spectrum: -

If an electron jumps from an outer initial orbit n_2 of higher energy to an inner orbit n_1 of lower energy, as in Fig.2, the frequency of the radiation emitted is given by $\nu = (E_{n_2} - E_{n_1})/h$.

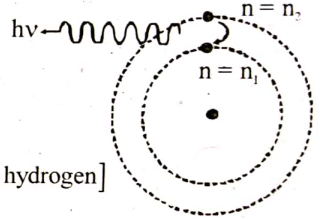


Figure 2

$$E_{n_2} = \frac{me^4}{32\pi^2\varepsilon_0^2\hbar^2}\frac{1}{n_2^2}$$

and $E_{n_1} = -\frac{me^4}{32\pi^2\varepsilon_0^2\hbar^2}\frac{1}{n_1^2}$ $[\because Z = 1 \text{ for hydrogen}]$

$\therefore \quad \nu = \frac{E_{n_2} - E_{n_1}}{h}$

$$\nu = \frac{me^4}{64\pi^3\varepsilon_0^2\hbar^3}\left\{\frac{1}{n_1^2} - \frac{1}{n_2^2}\right\}$$ ..... (9)

The wavenumber $\bar{\nu}$ of a rediation is defined as the reciprocal of its wavelength λ in vacuum and gives the number of waves contained in unit length in vacuum. $\bar{\nu} = 1/\lambda = \nu/c$.

$\therefore \quad \bar{\nu} = \frac{me^4}{64\pi^3\varepsilon_0^2\hbar^3c}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ ..... (10)

$\frac{me^4}{64\pi^3\varepsilon_0^2\hbar^3c} = R_\infty$ = Rydberg constant for infinite nuclear mass

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

$$= 1.097 \times 10^{7}\ m^{-1}$$

$$\therefore \quad \bar{\nu} = R_{\infty}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \qquad \ldots\ldots (11)$$

Spectral series of hydrogen atom: -

(1) Lyman series -

When an electron jumps from second, third, ... etc.., orbits to the first orbit , we get the Lyman series which lies in the ultraviolet region. Here, $n_1 =$ 1 and $n_2 = 2, 3, 4, 5$ .....

$$\therefore \quad \bar{\nu} = R_{\infty}\left(\frac{1}{1^2} - \frac{1}{n^2}\right); n = 2,3,4,5.....$$

(2) Balmer series -

When an electron jumps from outer orbits to the second orbit, $n_1 = 2$ and $n_2 = 3, 4, 5$ .... etc.

$$\bar{\nu} = R_{\infty}\left(\frac{1}{2^2} - \frac{1}{n^2}\right); n = 3,4,5 \ldots\ldots \text{ etc.}$$

This series is called Balmer series and lies in the visible region of the spectrum. The first line in the series (n = 3) is called the H_α line, the second (n = 4) the H_β line and so on .

(3) Paschen series -

Paschen series in the infrared region are given by $n_1 = 3$ and $n_2 = 4, 5, 6$ ....etc.

$$\therefore \quad \bar{\nu} = R_{\infty}\left(\frac{1}{3^2} - \frac{1}{n^2}\right); n = 4,5,6,7 \ldots..\text{etc.}$$

(4) Brackett series-

If $n_1 = 4$ and $n_2 = 5, 6, 7$ .....etc., we get the Brackett series.

$$\therefore \quad \bar{\nu} = R_{\infty}\left(\frac{1}{4^2} - \frac{1}{n^2}\right); n = 5,6,7 \ldots..$$

(5) Pfund series-

If $n_1 = 5$ and $n_2 = 6, 7, 8,$ ..... we get Pfund series.

$$\bar{\nu} = R_{\infty}\left(\frac{1}{5^2} - \frac{1}{n^2}\right); n = 6,7,8 \ldots..$$

Brackett and Pfund series lie in the very far infrared region of the hydrogen spectrum. By putting $n = \infty$ in each one of the series, we get the wavenumber of the series limit, i.e., the last line in the series. The electron jumps giving rise to the different series in hydrogen are shown in fig.3.

Visit us for more detail at : www.winsomepublication.in

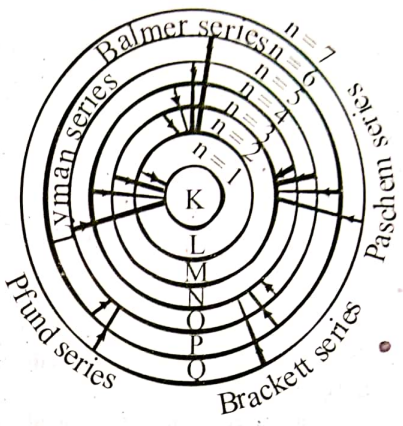


Figure 3

The energy - level diagram:-

The equation $E_n = -\dfrac{me^4Z^2}{32\pi^2\varepsilon_0^2\hbar^2}\dfrac{1}{n^2}$ can be diagrammatically represented. Then, it is called the energy level diagram. Substituting the values of m, e, ε_0 and $\hbar$ we get for hydrogen (Z = 1),

$$\frac{me^4Z^2}{32\pi^2\varepsilon_0^2\hbar^2} = \frac{(9.11\times10^{-31})(1.6\times10^{-19})^4(1^2)}{32\pi^2(8.854\times10^{-12})^2(1.055\times10^{-34})^2} = 21.76\times10^{-19}\ J$$

$$= \frac{21.76\times10^{-19}}{1.6\times10^{-19}}\ eV = 13.6\ eV$$

$$\therefore \quad E_n = (-13.6/n^2)\ eV \text{ for } n = 1, 2, 3, \ldots..$$

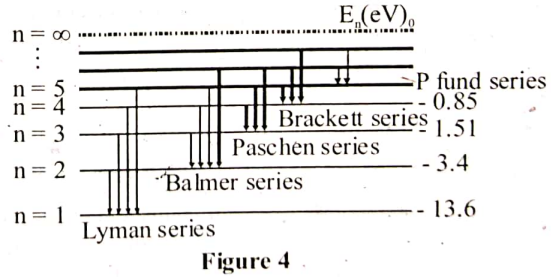


Figure 4

The lowest energy level E_1 is called the normal or the ground state of the atom and the higher energy levels E_2, E_3, E_4 ... are called the excited states. As n increases E_n increases. As n increases, the energy levels crowd and tend to

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

form a continuum.

In the energy - level diagram, the discrete energy states are represented by horizontal lines, and the electronic jumps between these states by vertical lines (Fig.4). Fig.4 shows schematically how spectral lines are related to atomic energy levels.

Q.2. What is Compton effect? Find expression for the Compton relation. Describe the experimental verification of Compton relation? (2016)

Other Related Questions -

Q. What is Compton effect? Derive the expression for the change in wavelength of scattered X-rays. Describe its experminetal verification. (2014)

Q. Write the formula for the change in wavelength due to Compton scattering. Describe its experimental verification. (2012)

Q. Give the theory of Compton effect and briefly explain its experimental verification. (2018)

Ans. The Compton Effect: -

Compton discovered that when X -ray of a sharply defined frequency were incident on a material of low atomic number like carbon, they suffered a change of frequency on scattering. The scattered beam contains two wavelengths. In addition to the expected incident wavelength, there exist a line of longer wavelength. The change of wavelength is due to loss of energy of the incident X - rays. This elastic interaction is known as Compton effect.

In the case of incoherent scattering, a scattered beam undergoes not only deviation in its direction but also change of wavelength occurs. In Compton effect, there is a change in wavelength of the scattered beam along with the change in its direction. Hence Compton effect is an incoherent scattering.

This effect was explained by Compton on the basis of quantum theory of radiation. The whole process is treated as a particle collision event between X- ray photon and a loosely bound electron of the scatterer. In this process, both momentum and energy are conserved. In the photon - electron collision, a portion of the energy of the photon is transferred to the electron. As a result, the X - ray proceeds with less than the original energy (and therefore has a lower frequency or a higher wavelength).

The incident photon with an energy $h\nu$ and momentum $h\nu/c$ strikes an electron at rest. The initial momentum of the electron is zero and its initial energy is only the rest mass energy, m_0c^2. The scattered photon of energy $h\nu'$ and momentum $h\nu'/c$ moves off in a direction inclined at an angle θ to the original direction. The electron acquires a momentum mv and moves at an angle ϕ to the original direction. The energy of the recoil electron is mc^2 (fig.1).

According to the principle of conservation of energy,

$$h\nu + m_0c^2 = h\nu' + mc^2 \quad \text{.....(1)}$$

Visit us for more detail at : www.winsomepublication.in

Considering the x and y components of the momentum and applying the principle of conservation of momentum,

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + mv\cos\phi \quad \text{.....(2)}$$

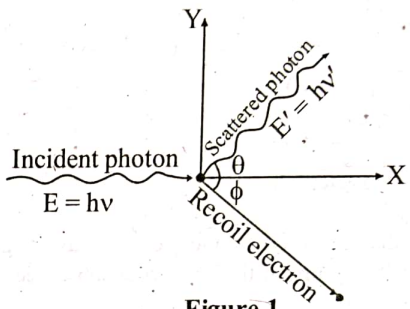


Figure 1

and $$0 = \frac{h\nu'}{c}\sin\theta - mv\sin\phi \quad \text{.....(3)}$$

from (2), $mvc\cos\phi = h(\nu - \nu'\cos\theta)$ .....(4)

from (3), $mvc\sin\phi = h\nu'\sin\theta$ .....(5)

Squaring and adding (4) and (5),

$$m^2v^2c^2 = h^2(\nu^2 - 2\nu\nu'\cos\theta + \nu'^2\cos^2\theta) + h^2\nu'^2\sin^2\theta$$

$$= h^2(\nu^2 - 2\nu\nu'\cos\theta) + h^2\nu'^2$$

$$= h^2(\nu^2 - 2\nu\nu'\cos\theta + \nu'^2) \quad \text{.....(6)}$$

From eq.(1), $mc^2 = h(\nu - \nu') + m_0c^2$

$$\therefore \quad m^2c^4 = h^2(\nu^2 - 2\nu\nu' + \nu'^2) + 2h(\nu - \nu')m_0c^2 + m_0^2c^4 \quad \text{.....(7)}$$

Subtracting (6) from (7),

$$m^2c^2(c^2 - v^2) = -2h^2\nu\nu'(1 - \cos\theta) + 2h(\nu - \nu')m_0c^2 + m_0^2c^4 \quad \text{.....(8)}$$

The value of $m^2c^2(c^2 - v^2)$ can be obtained from the relativistic formula

$$m = \frac{m_0}{\sqrt{(1 - v^2/c^2)}}$$

Squaring, $$m^2 = \frac{m_0^2}{1 - v^2/c^2} = \frac{m_0^2c^2}{c^2 - v^2}$$

$$\therefore \quad m^2c^2(c^2 - v^2) = m_0^2c^4 \quad \text{.....(9)}$$

From (8) and (9),

$$m_0^2c^4 = -2h^2\nu\nu'(1 - \cos\theta) + 2h(\nu - \nu')m_0c^2 + m_0^2c^4$$

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

$$\therefore \quad 2h(\nu-\nu')m_0c^2 = 2h^2\nu\nu'(1-\cos\theta)$$

$$\text{or} \quad \frac{\nu-\nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1-\cos\theta) \text{ or } \frac{1}{\nu'}-\frac{1}{\nu} = \frac{h}{m_0c^2}(1-\cos\theta)$$

$$\text{or} \quad \frac{c}{\nu'}-\frac{c}{\nu} = \frac{h}{m_0c}(1-\cos\theta)$$

$$\text{or} \quad \lambda'-\lambda = \frac{h}{m_0c}(1-\cos\theta) \qquad \text{.....(10)}$$

$$\therefore \quad \text{The change in wavelength} = d\lambda = \frac{h}{m_0c}(1-\cos\theta)$$

This relation shows that $d\lambda$ is independent of the wavelength of the incident radiations as well as the nature of the scattering substance. $d\lambda$ depends upon the angle of scattering only.

Case 1. When $\theta = 0$, $\cos\theta = 1$ and hence $d\lambda = 0$

Case 2. When $\theta = 90^0$, $\cos\theta = 0$ and hence

$$d\lambda = \frac{h}{m_0c} = \frac{6.63\times10^{-34}}{(9.11\times10^{-31})\times(3\times10^8)}m = 0.0243\,\text{Å} \qquad \text{.....(11)}$$

This is known as Compton wavelength.

Case 3. When $\theta = 180^0$, $\cos\theta = -1$ and hence $d\lambda = 2h/m_0c = 0.0485$ Å . $d\lambda$ has the maximum value at $\theta = 180^0$.

Experimental verification: -

Monochromatic X -rays of wavelength λ are allowed to fall on a scattering material like a small block of carbon (Fig.2) . The scattered X-rays are received by a Bragg spectrometer and their wavelength is determined. The spectrometer can freely swing in an arc about the scatterer. The wavelength of the scattered X -rays is measured for different values of the scattering angle. The experimental results obtained by Compton are shown in (Fig.3). In the scattered radiation in addition to the incident wavelength (λ), there exists a line of longer wavelength (λ'). The "Compton shift" $d\lambda$ is found to vary with the angle at which the scattered rays are observed.

Direction of Recoil electron: -

Dividing Eq. (5) by Eq. (4), we get

$$\tan\phi = \frac{h\nu'\sin\theta}{h(\nu-\nu'\cos\theta)} = \frac{\nu'\sin\theta}{(\nu-\nu'\cos\theta)}. \qquad \text{.....(12)}$$

Using Eq. (10), we get

$$\frac{1}{\nu'} = \frac{1}{\nu}+\frac{h}{m_0c^2}(1-\cos\theta) = \frac{1}{\nu}+\frac{h}{m_0c^2}.2\sin^2\frac{\theta}{2}$$

Visit us for more detail at : www.winsomepublication.in

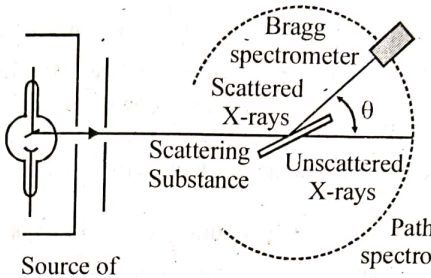


Fig.2

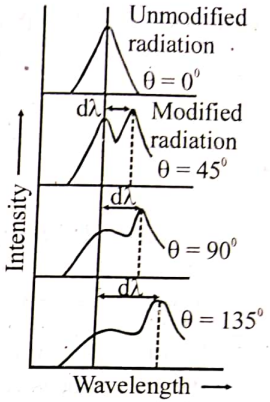


Fig.3

$$\text{or } \nu' = \frac{\nu}{1+\left(\frac{h\nu}{m_0c^2}\right)2\sin^2\frac{\theta}{2}} = \frac{\nu}{1+2\beta\sin^2\left(\frac{\theta}{2}\right)} \text{ where } \beta = \frac{h\nu}{m_0c^2}. \qquad \text{.....(13)}$$

Substituting this value of ν' in Eq. (12), we get

$$\tan\phi = \frac{\nu\sin\theta/\left[1+2\beta\sin^2\left(\frac{\theta}{2}\right)\right]}{\left[\nu-\left\{\nu\cos\theta/\left(1+2\beta\sin^2\frac{\theta}{2}\right)\right\}\right]} = \frac{\cot\left(\frac{\theta}{2}\right)}{(1+\beta)}$$

$$\therefore \quad \tan\phi = \frac{\cot\left(\frac{\theta}{2}\right)}{1+\left(\frac{h\nu}{m_0c^2}\right)} \qquad \text{.....(14)}$$

Kinetic Energy of Recoil electron: -

The Kinetic Energy of recoil electron is the difference between the energies of incident and scattered photons, i.e.

$$\text{K.E.} = h\nu - h\nu'$$

$$\text{K.E.} = h\nu - h\left[\frac{\nu}{1+2\beta\sin^2(\theta/2)}\right] = h\nu\left[\frac{2\beta\sin^2(\theta/2)}{1+2\beta\sin^2(\theta/2)}\right] \qquad \text{.....(15)}$$

Where $\beta = h\nu/m_0c^2$.

Visit us for more detail at : www.winsomepublication.in

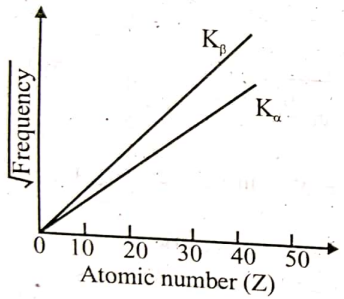
Scanned by CamScanner

Q.3. Deduce Moseley's law and show how it has been utilized in removing some of the defects of periodic classification of the elements. (2016, 11)

Ans. Moseley plotted the square root of the frequencies $\left(\sqrt{\nu}\right)$ of a given line (K_α) against the atomic numbers (Z) of the elements emitting that line . Moseley obtained a straight line as shown in Fig. The same linear relation was found to hold good for any line in any series. He concluded, therefore,that atomic number (and not atomic weight) is the fundamental property of elements.

The statement of Moseley's law: -

The frequency of a spectral line in X-ray spectrum, varies as the square of the atomic number of the element emitting it, or $\nu \propto Z^2$. Moseley's law may be written as $\sqrt{\nu} = a(Z-b)$. Here, Z is the atomic number of the element and a and b are constants depending upon the particular line.



Explanation, according to Bohr's theory: -

Bohr's theory of hydrogen spectrum gives the frequency of a spectral line as $\nu = Z^2 Rc\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$. Here, R is Rydberg' s constant and c the velocity of light.

K_α line originates from the transition of electron from second to first orbit.

Now, $n_1 = 1$ and $n_2 = 2$.

$\therefore$ frequency of K_α line $= \nu = Z^2 Rc\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4} cRZ^2$

This approximately corresponds to Moseley's law.

Importance of Moseley's law: -

(1) According to this law, it is the atomic number and not atomic weight of an element which determines its characteristic properties both physical and chemical.Therefore, the atoms must be arranged in the periodic table according to their atomic numbers and not according to their atomic weights. This would

Visit us for more detail at : www.winsomepublication.in

remove some discrepancies in the order of certain elements from the point of view of their atomic weights. For example, argon $_{18}Ar^{40}$ comes before potassium ($_{19}K^{39}$),cobalt ($_{27}Co^{58.9}$) comes before nickel ($_{28}Ni^{58.7}$),etc. So the arrangement is correct in the order of atomic number.

(2) Moseley's work has also helped to perfect the periodic table by (i) the discovery of new elments, e.g., hafnium (72), illinium (61), masurium (43), rhenium (75), etc. and (ii) the determination of the atomic numbers of rare-earths and fixing their positions in the periodic table.

Q.4. Define Stern and Gerlach experiment. (2015)

Other Related Questions -

Q. Describe Stern-Gerlach experiment. What interference is drawn from its results? What changes would be observed in the resultant pattern if - (i) Magnetic field is made homogeneous.

(ii) The electron spin were $\frac{3}{2}\bar{h}$ instead of $\frac{1}{2}\bar{h}$. (2013)

Q. Giving the principle of Stern-Gerlach experiment discuss the results in detail. (2009)

Q. Explain the significance of use of a beam of neutral atoms and not of ions in stern Gerlach experiment. (2011)

Q. Describe the Stern-Gerlach experiment and discuss the importance of the results obtained. (2018)

Ans. The stern and gerlach experiment: -

We have seen that the orbital and spin motions of the electrons in atoms endow the atoms with magnetic moments. Direct evidence for the existence of magnetic moments of atoms and their space quantisation is provided by the experiments of Stern and Gerlach.

Principle and Theory: -

The experiment is based on the behaviour of a magnetic dipole (atomic magnet) in a non - uniform magnetic field . In a uniform magnetic field (B), the dipole experiences a torque that tends to align the dipole parallel to the field. If the dipole moves in such a field in a direction normal to the field, it will trace a straight line path without any deviation. In an inhomogeneous magnetic field, the dipole experiences, in addition, a translatory force. If the atomic magnet flies across such an inhomogeneous magnetic field normal to the field direction, it will be deviated away from its rectilinear path. An expression for the deviation produced may be obtained as follows.

Let the magnetic field vary along the X - direction, so that field gradient is dB/dx and is positive (Fig.1). CD is the atomic magnet (of pole strength p, length *l*, dipole moment M) with its axis inclined at an angle θ to the field direction. If the field strength at the pole C is B, then the field strength at the

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

other pole D will be $B+\frac{dB}{dx}l\cos\theta$. Hence the forces on the two poles are pB and $p\left(B+\frac{dB}{dx}l\cos\theta\right)$. Hence the atomic magnet experiences not only a torque ($= pl B = \mu_s B$) but also a translatory force

$$F_x = \frac{dB}{dx} pl\cos\theta.$$

$$\therefore \quad F_x = \frac{dB}{dx}\mu_s\cos\theta \qquad \text{.....(1)}$$

Let V = velocity of the atomic magnet of mass m as it enters the field,
L = length of the path of the atom in the field and
t = the time of travel of the atom through the field = L/V.

The acceleration given to the atom along the field direction, by the translatory force F_x is F_x/m. The displacement of the atom along the field direction, on emerging out of the field is.

$$\alpha = \frac{1}{2}\left(\frac{F_x}{m}\right)t^2 = \frac{1}{2}\frac{F_x L^2}{mV^2}$$

$$= \frac{1}{2}\frac{dB}{dx}\frac{\mu_s\cos\theta}{m}\frac{L^2}{V^2} \qquad \text{.....(2)}$$

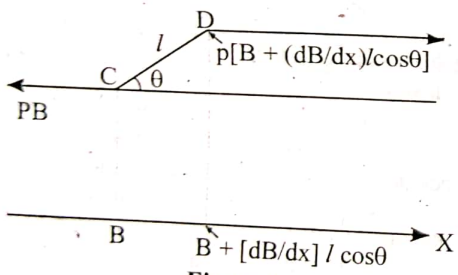


Figure 1

If μ is resolved component of the magnetic moment in the field direction, $\mu = \mu_s\cos\theta$

$$\therefore \quad \alpha = \frac{1}{2}\frac{dB}{dx}\frac{\mu}{m}\frac{L^2}{V^2} \qquad \text{.....(3)}$$

Experimental Arrangement: -

Silver is boiled in an oven (Fig.2(a)). Atoms of silver stream out from an

Visit us for more detail at : www.winsomepublication.in

opening in the oven. By the use of slits S_1 and S_2, a sharp linear beam of atoms is obtained. These atoms then pass through a very inhomogeneous magnetic field between the shaped poles of a magnet M M. A high degree of non-uniformity in the magnetic field is produced by making one of the pole-pieces of a powerful electromagnet a knife-edge shape and other flat with a groove cut in it opposite the knife edge [Fig.2(b)]. The lines of force are close together at the knife edge and the field there is much stronger than that at the other pole piece [Fig.2(c)]. The magnetic field is at right angles to the direction of movement of the atoms. Finally the atoms fall on a photographic plate P. The whole arrangement is enclosed in an evacuated chamber.

Results:- With no field, the beam produces a narrow continuous line on the plate [Fig.2(d)]. In terms of the vector atom model, those atoms, with electron spins directed parallel to the magnetic field, will experience a force in one direction, whereas those with oppositely directed spins will experience a force in the opposite direction. According to this, the beam of atoms should split into two beams in its passage through the inhomogeneous magnetic field. This splitting of the beam into two parts of approximately equal intensity was actually observed in these experiments. On applying the inhomogeneous magnetic field, it was found that the stream of silver atoms splits into two separate lines [Fig.2(d)]. Knowing dB/dx, L, V and α, μ was calculated. It was found that each silver atom had a magnetic moment of one Bohr magneton in the direction of the field.

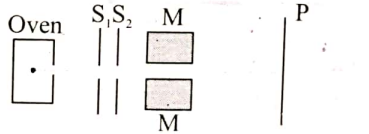


Figure 2(a)

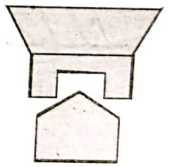


Figure 2(b)

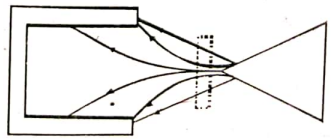


Figure 2(c)

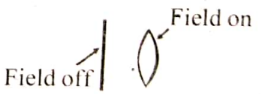


Figure 2(d)

Q.5. Sketch the pattern of X-ray spectra. Discuss the origin of characteristic and continuous X-rays. **(2013)**

Ans. X-ray spectra:-

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

Urey and his co-workers analysed the X-ray beam emitted from an X-ray tube using different potential differences and same target. Using tungsten as the target and different potential differences, the intensities of the rays produced are plotted against wavelength. (Fig.1) represents these graphs. For applied P.D.s 30 kV and 50kV, the spectrum is white. But, for the applied P.D. 70 kV, two sharp peaks are seen. The sharp peaks show the line or characteristic radiation. The line spectra are absent, till the P.D. is greater than a particular value. The smoothly varying curves represent the continuous spectrum. The superimposed lines on the continuous background constitute the characteristic spectrum.

Main features of continuous X-ray spectrum:-

(1) For each anode potential, there is a minimum wavelength (λ_{min}) below which no radiation is emitted. Above this critical value, the intensity of the radiation increases rapidly with increasing wavelengths and after reaching a maximum, decreases gradually. The intensity never reaches zero showing that the radiation contains all possible wavelength above the minimum limit.

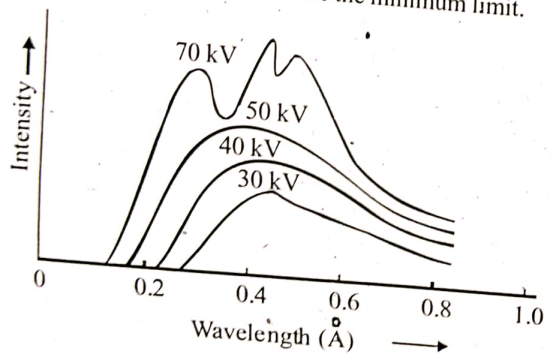


Figure 1

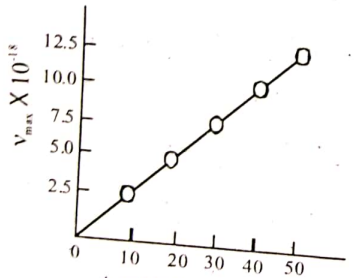


Figure 2

(2) When the voltage across the X-ray tube is increased, λ_{min} is shifted towards smaller values. Duane and Hunt showed that λ_{min} is inversely proportional to the applied voltage V or ν_{max} is directly proportional to V. If the limiting frequencies (ν_{max}) are plotted against the applied voltages (V), a straight line graph passing through the origin is obtained (Fig.2). This empirical law of Duane and Hunt is expressed analytically as

$$eV = h\nu_{max} = \frac{hc}{\lambda_{min}}$$

Visit us for more detail at : www.winsomepublication.in

Duane-Hunt Law: Statement:-

The short wavelength limit (λ_{min}) of the X-ray spectrum is inversely proportional to potential (V) applied across the tube terminals.

$$\lambda_{min} \propto \frac{1}{V}$$

Explanation:-

eV is the K.E. of the bombarding electron. If the entire K.E. of the electron striking the target is converted into the energy of the X-ray photon, then, $eV = h\nu_{max}$ according to Einstein's theory.

But $$\nu_{max} = \frac{c}{\lambda_{min}}$$

$$\therefore \quad eV = \frac{hc}{\lambda_{min}} \text{ or } \lambda_{min} = \frac{hc}{eV} \text{ or } \lambda_{min} \propto \frac{1}{V}$$

Most of the electrons that generate X-ray photons give up only a part of their energy in this way. Therefore, most of the X-radiation is of longer wavelength than λ_{min}. Thus, the continuous spectrum is the result of the inverse photoelectric effect, with electron kinetic energy (eV) being transformed into photon energy (hν).

Characteristic X-ray spectrum:-

There are two methods of producing characteristic X-rays.

(1) The characteristic X-rays of an element can be excited by using the element as the target in the X-ray tube and thus subjecting it to direct bombardment by electrons. For each target there is a minimum potential below which the line spectra do not appear. This critical P.D. below which the line spectra do not appear, is different for different targets. Molybdenum shows up the line spectra only if the P.D. is above 35 kV.

(2) Characteristic X-rays of an element can also be excited by allowing primary X-rays from a hard X-ray tube to fall on the element. The primary X-ray must be harder than the characteristic X-rays to be produced.

The peaks obtained in the X-ray spectrum (Fig.3) give us the line spectrum which is characteristic of the element used in the target. The group of lines of shortest wavelength is called the K-series. Usually two lines of this series are detected. These lines are termed as K_α and K_β lines in the order of decreasing wavelengths. The next group is called the L-series of longer wave-

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

lengths (L_α, L_β, L_γ, etc.). For heavier elements a third series, called the M-series has been detected.

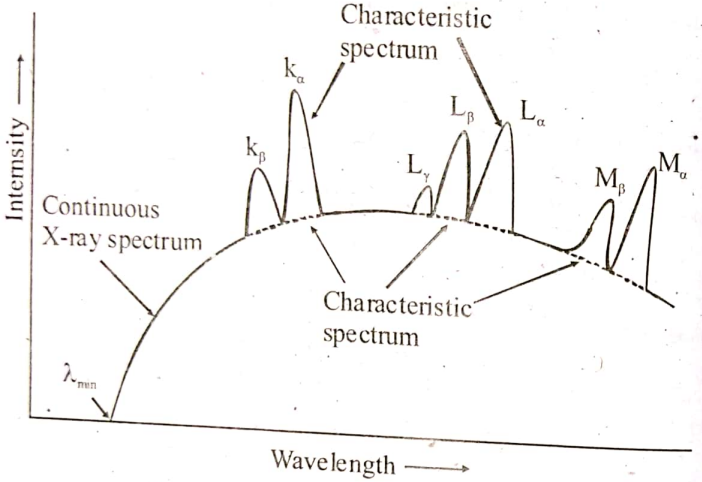


Figure 3

Origin of characteristic X-rays:-

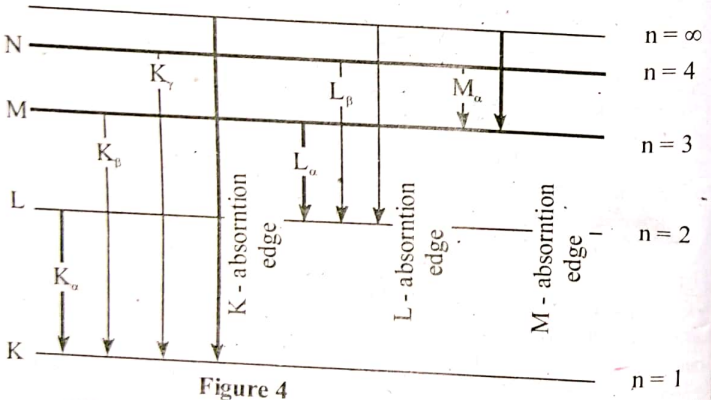


Figure 4

This can be understood in terms of Bohr's theory. Suppose an atom in the target of an X-ray tube is bombarded by a high-speed electron and a K-electron is removed. A vacancy is created in the K-shell. This vacancy can be filled up by an electron from either of L, M or N shells or a free electron. These possible transitions can result in the K_α, K_β lines and the limiting line. Similarly, the longer wavelength L-series originates when an L electron is knocked out of the atom, the M-series when an M electron is knocked out and so on (Fig.4).

It is clear that continuous spectra and line spectra both emitted by the same target are of different origin. The continuous spectrum is the result of the

Visit us for more detail at : www.winsomepublication.in

inverse photoelectric effect, with electron K.E. being transformed into photon energy hν. The line spectrum has its origin in electronic transitions within atoms that have been disturbed by the incident electrons.

Fig.3 shows a characteristic spectrum superimposed upon continuous spectrum.

Q.6. Write down semi-empirical mass formula and explain the physical significance of each term in it. **(2012,19)**

Other Related Questions -

Q. Write semi empirical mass formula. Explain its various terms. (2014)

Q. Write short note on semi-empirical mass formula. (2018)

Ans. Semi-empirical mass formula:-

The mass M of a neutral atom whose nucleus contains Z protons and (A - Z) neutrons is given by

$$_ZM^A = ZM_P + (A - Z)\,M_N + ZM_e - E_B$$
$$= ZM_H + (A - Z)\,M_N - E_B \quad \text{.....(i)}$$

where E_B (sometimes written as B) is the binding energy, may be regarded as being made up of a number of terms, each of which represents some general characteristic of nuclei. Von Weizsacker in 1935 obtained a formula, known as semi-empirical mass formula, for expressing the atomic mass of a nuclide, in terms of a series of binding energy terms. It is of great importance in nuclear stability problems. Its various terms are as follows:-

(a) Volume binding energy:-

The neutrons and protons are held together by short range attractive forces. These forces reduce the mass of the nucleus below that of its constituents by an amount proportional to the number of nucleons A. Since the volume of the nucleus is proportional to A, hence this term may be regarded as a volume binding energy often called exchange energy, and is given by

$$E_v = a_v A \quad \text{.....(ii)}$$

where a_v is a proportionality constant and the subscript v is for volume.

(b) Surface binding energy:-

Since nucleus is finite, some nucleons are nearer to the surface so that they interact with few nucleons. Thus the binding energy E_v is reduced by an amount proportional to the surface area of the nucleus of radius R as the nucleons on the surface are less tightly bound than those in the interior. For light nuclei nearly all the nucleons are at the surface, while for heavy nuclei about half the nucleons are at the surface. Thus the surface energy is analogous to the surface tension of a liquid. It is proportional to R^2 or $A^{2/3}$.

or $$E_s = -a_s A^{2/3} \quad \text{.....(iii)}$$

where a_s is the proportionality constant and the subscript s means surface energy.

(c) Coulomb Energy:-

The only long range force in the nuclei is the Coulomb repulsion be-

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

tween protons. The Coulomb repulsion term is equal to the potential energy of Z protons packed together in a spherically symmetric assembly of mean radius

$R = R_0 A^{1/3}$ and of uniform charge density $\rho_c = \dfrac{Ze}{\frac{4}{3}\pi R_0^3 A}$.

The electrostatic energy is simply the work done against electrostatic forces in assembling such a sphere. Let dq be the charge on the shell of thickness dr on the sphere of radius r.

$$\therefore \quad dq = \rho_c \times 4\pi r^2 dr = \left(\frac{3Ze}{R_0^3 A}\right) r^2 dr$$

Hence the work in bringing this charge dq from infinity to r against the charge on the sphere of radius r is

$$\frac{3Ze}{R_0^3 A}.r^2 dr \times \frac{\frac{4}{3}\pi r^2 \rho_c}{4\pi\varepsilon_0 r} = \frac{3Z^2e^2}{4\pi\varepsilon_0 R^6 A^2}.r^4 dr$$

Integrating from 0 to R (= $R_0 A^{1/3}$) we have work done against Coulomb repulsion or the loss of binding energy E_c due to the disruptive Coulomb energy.

$$\therefore \quad E_c = -\frac{3}{5}\frac{Z^2e^2}{4\pi\varepsilon_0 R_0 A^{1/3}} = -a_c \frac{Z^2}{A^{1/3}} \qquad \text{.....(iv)}$$

where a_c is a constant and the subscript c designates Coulomb Energy.

(d) Asymmetry Energy:-

Study of no. of neutrons (N) and protons (Z) of stable nuclei shows that for nuclei with Z + N upto 18, N - Z = 0 or + 1, with the exception of hydrogen and $_2He^2$, i.e., there is a sharp tendency for neutrons and protons to pair up. As (Z + N) increases, nuclear forces do not increase much, but repulsion increases with charge. So proton increases in number less rapidly than neutrons, thus heavy nuclei have more neutrons than protons. Complete theory of this asymmetry term is shown in schematic diagram of nuclear energy levels near the highest filled levels separated by the same energy Δ. Keeping (N + Z) constant, if we remove a proton from level 3 and add a neutron to level 4, N - Z becomes 2 and the energy has increased or E_B has decreased by an amount Δ. Transferation of two nucleons in this way increases the energy by an amount 4 Δ (Δ for the first and 3Δ for the second). We thus find as a generalization of this process that the transfer of (N - Z)/2 nucleons decreases the binding energy by an amount $-\Delta(N - Z)^2/4$. Since Δ decreases as A increases, hence decrease in binding energy

$$E_{as} = -a_a \frac{(N-Z)^2}{A} = -a_a \frac{(A-2Z)^2}{A} \qquad \text{.....(v)}$$

Visit us for more detail at : www.winsomepublication.in

where a_a is a constant and subscript a is for asymmetry.

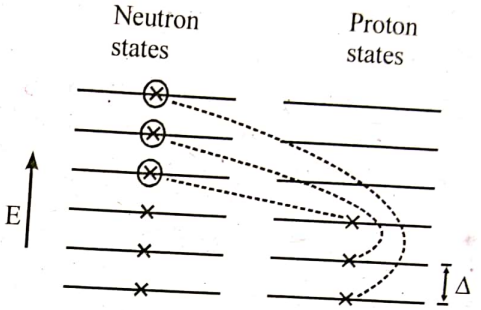


Figure 1- Schematic diagram of nuclear energy levels near the highest filled level

(e) Pairing Energy:-

The nuclides with even numbers of protons and neutrons are the most abundant and most stable. Nuclei with odd numbers of both neutrons and protons are the least stable, while nuclei for which either proton or neutron number is odd are intermediate in stability. To take account of this pairing effect an additional term is used. Conventionally this term is taken as zero for odd- A, -δ for odd-odd and +δ for even-even nuclides. On the basis of more detailed analysis, we have $\delta = a_p A^{-3/4}$ where a_p is the pairing energy constant. Hence we have

$E_p = 0$ for A odd, Z even N odd or Z odd N even ,
$E_p = a_p/A^{3/4}$, for A even, Z even and N even,
$E_p = -a_p/A^{3/4}$, for A even, Z odd and N odd.

(f) Magicity energy:-

When a shell is filled by having either the number of neutrons or the number of protons equal to one of the numbers 2, 8, 20, 28, 50, 82 or 126 which are known as magic numbers, a small local maximum in E_B as a function of A occurs.

$$\therefore \quad E_M = a_m A$$

where a_m is known as coefficient of magicity.

Collecting together the different stability or mass correction terms we can write Weizsacker's semi-empirical mass formula as:

$$zM^A = ZM_H + (A - Z)M_N - a_v A + a_s A^{2/3} + a_c Z^2 A^{-1/3} + a_a (A - 2Z)^2 A^{-1} \mp a_p A^{-3/4}$$

.....(vi)

This mass formula does not include closed shell effects or the magicity energy term. Experimentally it was observed that the mass of any isobar having Z or N equal to any magic number lies about 1 or 2 MeV below the value predicted by the mass formula. So we have to add this term to avoid the discontinuities in masses at the magic numbers.

The five constants of the semi-empirical mass formula can be evaluated

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

from five masses or five nuclear reactions. When inserted in mass formula, they should give other masses. A good representative set in MeV is

a_v	a_s	a_c	a_a	a_p
14.1	13	0.595	19	33.5

The contribution of the various terms to B_{ave} is shown in fig.2. The initial rise of B/A with A, obtained empirically, is mainly due to the decreasing importance of surface energy as A increases. At still larger A, the importance of the Coulomb energy becomes dominant causing a maximum at A ~ 60 and a subsequent decline at larger A. The semi-empirical mass formula matches the observed binding energies with in about 1% beyond A = 40. The discrepancies occur in neighbourhood of magic nuclei, having Z or N equal to 28, 50, 82 or 126. Mozer proposed a formula for odd A using a least-squares analysis of atomic masses.

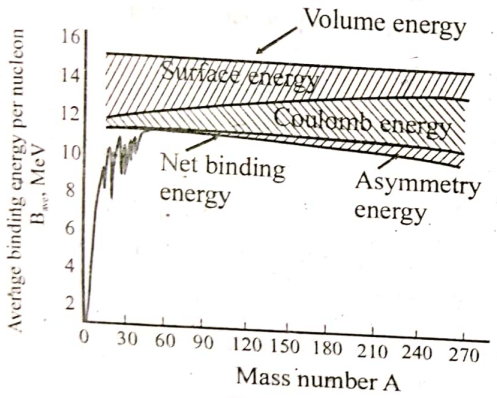


Figure 2

$$zM^A = ZM_H + (A - Z)M_N - a_vA + a_sA^{2/3} + a_cZ(Z - 1)A^{-1/3} + a_a(A - 2Z)^2A^{-1} + S(Z, A) + D(Z, A)$$

where S and D are the shell effect energy and the nuclear deformation energy terms respectively. Mozer found coefficients as

a_v	a_s	a_c	a_a
15.29	15.76	0.69	22.63

Q.7. Write short note on Sommerfeld's Relativistic model. (2012)

Ans. Sommerfeld's Relativistic Atom Model:-

Introduction:-

According to Bohr, the lines in the hydrogen spectrum should each have a well defined wavelength. Spectrographs of high resolving power showed that the H_α, H_β, and H_γ lines in the hydrogen spectrum are not single. Each spectral line actually consisted of several very close lines packed together. Michelson found that under high resolution, the H_α line can be resolved into two close components, with a wavelength separation of 0.13 Å. This is called the fine structure of the spectral lines. Bohr's theory could not explain this

Visit us for more detail at : www.winsomepublication.in

fine structure.

To explain the observed fine structure of spectral lines, Sommerfeld introduced to main modifications in Bohr's theory.

(1). According to Sommerfeld, the path of an electron around the nucleus, in general, is an ellipse with the nucleus at one of the foci. The circular orbits of Bohr are a special case of this.

(2). The velocity of the electron moving in an elliptical orbit varies considerably are different parts of the orbit. This causes relativistic variation in the mass of the moving electron. Therefore he took into account the relativistic variation of the mass of the electron with velocity. Hence this is known as the relativistic atom-model. Let us first analyse mathematically Sommerfeld's theory.

Elliptic orbits for hydrogen:-

Consider the electron moving in an elliptical orbit round the nucleus (N). Its position at any instant can be fixed in terms of polar coordinates r and ϕ (Fig.). In the case of circular orbits, there is only one coordinate that varies periodically, namely, the angle ϕ that the radius vector makes with the X-axis. In the case of elliptic motion, not only does the angle ϕ vary but the length of the radius vector r also varies periodically, as shown in Fig. We have now to quantise the momenta associated with both these coordinates (ϕ and r) in accordance with Bohr's quantum condition. The two quantisation conditions are

$$\int p_\phi d\phi = n_\phi h \quad .....(i) \quad \text{and} \quad \int p_r dr = n_r h \quad .....(ii)$$

n_ϕ is called the angular or azimuthal quantum number and n_r is called the radial quantum number. n_ϕ and n_r take only positive integral values and $n_\phi + n_r = n$, where n is the principal quantum number. It can take the integral values 1, 2, 3,.....etc.

To determine the allowed elliptical orbits, we have to evaluate the integrals in the eq. (i) and (ii). The momentum p_ϕ corresponding to the coordinate ϕ is merely the angular momentum p of the electron in the elliptic orbit and this, from Kepler's law, is a constant, i.e., $p_\phi = p$ = constant. Integrating eq. (i) from 0 to 2π,

$$\int_0^{2\pi} p_\phi d\phi = n_\phi h \quad \text{or} \quad p_\phi = \frac{n_\phi h}{2\pi} \quad .....(iii)$$

Evaluation of $\oint p_r dr = n_r h$

Now, momentum along the radius $= p_r = m\frac{dr}{dt}$

$$\text{Now} \quad p_r dr = m\frac{dr}{dt}dr = m\left(\frac{dr}{d\phi}\frac{d\phi}{dt}\right)\frac{dr}{d\phi}d\phi = m\left(\frac{dr}{d\phi}\right)^2\frac{d\phi}{dt}d\phi$$

$$\text{But} \quad p_\phi = mr^2\frac{d\phi}{dt}$$

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

$$\therefore \quad p_r dr = \left(\frac{1}{r}\frac{dr}{d\phi}\right)^2 p_\phi d\phi \qquad \text{.....(iv)}$$

The equation of an ellipse in polar coordinates is

$$\frac{1}{r} = \frac{1+\varepsilon\cos\phi}{a\left(1-\varepsilon^2\right)} \qquad \text{.....(v)}$$

where a is the semimajor axis and ε is the eccentricity.
Taking logarithm of eq. (v) and differentiating,

$$\frac{1}{r}\frac{dr}{d\phi} = \frac{\varepsilon\sin\phi}{1+\varepsilon\cos\phi} \qquad \text{.....(vi)}$$

$$\therefore \quad p_r dr = \frac{\varepsilon^2\sin^2\phi}{\left(1+\varepsilon\cos\phi\right)^2} p_\phi d\phi \qquad \text{.....(vii)}$$

Eq.(ii) now becomes $p_\phi \int_0^{2\pi} \frac{\varepsilon^2\sin^2\phi}{\left(1+\varepsilon\cos\phi\right)^2} d\phi = n_r h$ .....(viii)

The integral $I = \int_0^{2\pi} \frac{\varepsilon^2\sin^2\phi}{\left(1+\varepsilon\cos\phi\right)^2} d\phi$ can be integrated by parts.

$$\int u\,dv = uv - \int v\,du$$

Let $u = \varepsilon\sin\phi$, $du = \varepsilon\cos\phi\, d\phi$ and

$$dv = \frac{\varepsilon\sin\phi}{\left(1+\varepsilon\cos\phi\right)^2} d\phi \quad \text{so that} \quad v = \frac{1}{1+\varepsilon\cos\phi}$$

Then $I = \left[\frac{\varepsilon\sin\phi}{1+\varepsilon\cos\phi}\right]_0^{2\pi} - \int_0^{2\pi} \frac{\varepsilon\cos\phi}{1+\varepsilon\cos\phi} d\phi$

On substitution of the limits of integration, the first term on the R.H.S. becomes zero. Then

$$I = -\int_0^{2\pi} \frac{\varepsilon\cos\phi}{1+\varepsilon\cos\phi} d\phi = \int_0^{2\pi} \left(\frac{1}{1+\varepsilon\cos\phi} - 1\right) d\phi$$

Now $\int_0^{2\pi} \frac{d\phi}{1+\varepsilon\cos\phi}$ is a standard integral whose value works out to be $\frac{2\pi}{\left(1-\varepsilon^2\right)^{1/2}}$.

$$\therefore \quad I = \frac{2\pi}{\left(1-\varepsilon^2\right)^{1/2}} - 2\pi \qquad \text{.....(ix)}$$

Now, eq.(viii) can be written as

Visit us for more detail at : www.winsomepublication.in

$$\frac{2\pi p_\phi}{\left(1-\varepsilon^2\right)^{1/2}} - 2\pi p_\phi = n_r h$$

or $$\frac{n_\phi h}{\left(1-\varepsilon^2\right)^{1/2}} - n_\phi h = n_r h \left(\text{since } p_\phi = n_\phi \frac{h}{2\pi}\right) \qquad \text{.....(x)}$$

$$\therefore \quad n_r = \frac{n_\phi}{\left(1-\varepsilon^2\right)^{1/2}} - n_\phi \quad \text{or} \quad n_r + n_\phi = \frac{n_\phi}{\left(1-\varepsilon^2\right)^{1/2}}$$

But, $n_r + n_\phi = n$, the principal quantum number.

Hence, $n = \frac{n_\phi}{\left(1-\varepsilon^2\right)^{1/2}}$ or $1-\varepsilon^2 = \frac{n_\phi^2}{n^2}$ .....(xi)

For an ellipse $1 - \varepsilon^2 = b^2/a^2$ where a and b are the semi-major and semi-minor axes respectively.

Hence, $\frac{b^2}{a^2} = \frac{n_\phi^2}{n^2}$ or $\frac{b}{a} = \frac{n_\phi}{n}$ .....(xii)

Eq.(xi) is the condition that determines the allowed elliptical orbits. The allowed elliptical orbits are those for which the ratio of major and minor axes is that of two integers. When $n_\phi = n$, $b = a$, $\varepsilon = 0$ and the orbit becomes circular. n_ϕ can not be zero, since the ellipse would then degenerate into a straight line passing through the nucleus. Also n_ϕ can not be greater than n, since b is always less than a. Hence for a given value of n, n_ϕ can assume only n different values, which means there can be only n elliptical orbits of different eccentricities.

Another notation:-

In this notation, the value of azimuthal quantum number n_ϕ is described by the letters, s, p, d, f, etc. The value of n_ϕ corresponding to these letters is 1, 2, 3, 4 etc., respectively. In this notation, the orbit determined by $n = 3$ and $n_\phi = 1$ is represented by 3s. Similarly 4d will represent the orbit $n = 4$ and $n_\phi = 3$.

Total energy:-

The total energy E_n of a single electron = E_n = P.E. + K.E.

The P.E. is $\frac{-Ze^2}{4\pi\varepsilon_0 r}$.

The K.E. can be written as $\frac{1}{2}m\left[\left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\phi}{dt}\right)^2\right]$

$\frac{dr}{dt}$ = radial component of the velocity and $r\left(\frac{d\phi}{dt}\right)$ = transverse component of

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

the velocity.

$$\therefore \quad E_n = \frac{-Ze^2}{4\pi\varepsilon_0 r} + \frac{1}{2}m\left[\left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\phi}{dt}\right)^2\right] \qquad \text{.....(xiii)}$$

It can be shown that

$$E_n = -\left(\frac{me^4Z^2}{8\varepsilon_0^2h^2}\right)\frac{1}{n^2} \qquad \text{.....(xiv)}$$

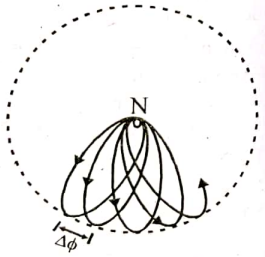
We find that the expression for the total energy is the same as that obtained by Bohr. This means that the theory of elliptical orbits introduces no new energy levels, other than those given by Bohr's theory of circular orbits. No new spectral lines, which would explains the fine structure, are to be expected because of this multiplicity of orbits. Hence, Sommerfeld proceeded further to find a solution to the problem of fine structure of spectral lines, on the basis of the variation of the mass of the electron with velocity.

Sommerfeld's relativistic theory:-

The velocity of the electron in a circular orbit is constant. But the velocity of the electron in an elliptical orbit varies, begin a maximum at the perihelion and a minimum at the aphelion. Furthermore this velocity is quite large $\left(\approx \frac{1}{137}c\right)$. Therefore, Sommerfeld modified his theory, taking into account variation of the mass of the electron with velocity. He showed that the relativistic equation describing the path of an electron is

$$\frac{1}{r} = \frac{1+\varepsilon\cos\psi\phi}{a\left(1-\varepsilon^2\right)} \qquad \text{.....(xv)}$$

where $\psi^2 = i - \frac{Z^2e^4}{16\pi^2\varepsilon_0^2p^2c^2}$



The path of the electron given by eq.(xv) is a rosette - a precessing ellipse . i.e., an ellipse whose major axis precesses slowly in the plane of the ellipse about an axis through the nucleus (Fig.).

It can be shown that the total energy E_n in the relativistic theory is,

$$E_n = \frac{me^4Z^2}{8\varepsilon_0^2h^2n^2} - \frac{me^4Z^4\alpha^2}{8\varepsilon_0^2h^2}\left(\frac{n}{n_\phi} - \frac{3}{4}\right)\frac{1}{n^4} \qquad \text{.....(xvi)}$$

where $\alpha = \frac{e^2}{2\varepsilon_0 ch} \approx \frac{1}{137}$

Visit us for more detail at : www.winsomepublication.in

α is a dimensionless quantity and is called the fine structure constant. The first term in eq.(xvi) is the energy of the electron in the orbit with the principal quantum number n. The second term is the Sommerfeld relativity correction. This term shows that the energy does depend on the azimuthal quantum number n_ϕ. This results in a splitting of the energy levels of the atom, for a given value of n, into n components, corresponding to the n permitted values of n_ϕ.

Fine structure of the H_α line:-

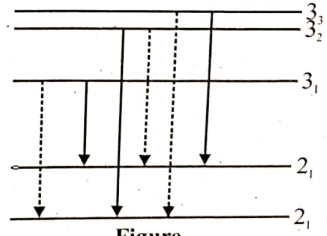
H_α line is due to the transition from n = 3 state to n = 2 state of the hydrogen atom.

For n = 3, there are three possible energy levels corresponding to the three values of n_ϕ, 1, 2, and 3. Similarly there are two possible energy levels for n = 2. Theoretically, six transitions are possible:

$$3_3 \to 2_2;\ 3_3 \to 2_1;\ 3_2 \to 2_2;\ 3_2 \to 2_1;\ 3_1 \to 2_2;\ 3_1 \to 2_1,$$

and they are shown in Fig.

Actually the H_α line has only three components. To make experiment and theory agree, some of the transitions have to be ruled out by some selection rule. The selection rule is that n_ϕ can change only by +1 or -1, i.e., $\Delta n_\phi = \pm 1$. There is no such restriction on Δn. On the basis of this selection rule,there are only three allowed transitions. In Fig., the allowed transitions are shown by continuous lines and the forbidden lines by dotted lines.



Figure

Q.8. Describe vector atom model and explain various quantum numbers. (2011, 18)

Ans. The vector atom model:-

Introduction:-

(1) Bohr's theory explain only the series spectra of the simplest hydrogen atom. It could not explain the multiple structure of spectral lines in the simplest hydrogen atom. Sommerfeld theory was able to give an explanation of the fine structure of the spectral lines of hydrogen. However, Sommerfeld's theory could not predict the correct number of the fine structure lines. Moreover, it gave no information about the relative intensities of the lines. Sommerfeld's theory could not explain the complex spectra of alkali metals like sodium.

(2) These older theories were inadequate to explain new discoveries like Zeeman

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

Effect and Stark Effect in which the spectral lines could be split up under the influence of magnetic and electric fields.

(3) Another drawback of the Bohr model was that it could not explain how the orbital electrons in an atom were distributed around the nucleus.

Therefore, in order to explain the complex spectra of atoms and their relation to atomic structure, the vector atom model was introduced. The two distinct features of the vector atom model are:

(1) The conception of spatial quantisation, and

(2) The spinning electron hypothesis.

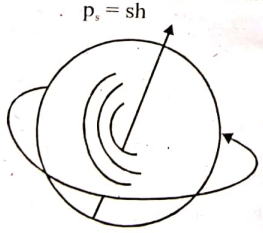
(1) Spatial quantisation:-

According to Bohr's theory, the orbits are quantised as regards their magnitude (i.e., their size and form) only. But according to quantum theory, the direction or orientation of the orbits in space also should be quantised. To specify the orientation of the electron orbit in space, we need a fixed reference axis. This reference line is chosen as the direction of an external magnetic field that is applied to the atom. The different permitted orientations of an electron orbit are determined by the fact that the projections of the quantised orbits on the field direction must themselves by quantised. The idea of space quantisation leads to an explanation of Zeeman effect. The Stern-Garlach experiment provided an excellent proof of the space quantisation of atom.

(2) Spinning electron:-

To account for the observed fine structure of spectral lines and to explain the anomalous Zeeman effect, the concept of spinning electron was introduced by Uhlenbeck and Goudsmit in 1926. According to their hypothesis, the electron spins about an axis of its own, while it also moves round the nucleus of the atom in its orbit (Fig.). So the electron is endowed with a spin motion over and above the orbital motion. According to the quantum theory, the spin of the electron also should be quantised. Hence a new quantum number called the spin quantum number (s) is introduced. Since the orbital and spin motions are both quantised in magnitude and direction according to the idea of spatial quantisation, they are considered as quantised vectors. Hence the atom model based on these quantised vectors is called the "vector atom model", to which vector laws apply.

According to the older theories, the electron was supposed to have only orbital motion round the nucleus. Hence, only the orbital angular momentum and orbital magnetic moment were considered. The spin endows the electron with a spin angular momentum sh and a spin magnetic moment. Hence the total angular momentum of an atom should be the vector sum of the orbital angular momentum and spin angular momentum. Similarly, the total magnetic moment



Figure

Visit us for more detail at : www.winsomepublication.in

of an atom should be the vector sum of the orbital and spin magnetic moments.

Quantum numbers associated with the vector atom model:-

(1) The principal quantum number (n):- This is identical with the one used in Bohr-Sommerfeld theory. The serial number of the shells starting from the innermost is designated as its principal quantum number (n). It can take only integral values excluding zero. i.e., n = 1, 2, 3, 4,.....

(2) The orbital quantum number (l):- This may take any integral value 0, 1, 2, 3, 4,....,(n - 1). Thus, if n = 4, l can take the four values 0, 1, 2, 3. By convention, an electron for which $l = 0$, is called s electron; if $l = 1$, p electron; $l = 2$, d electron; $l = 3$, f electron etc. The orbital angular momentum p_l of the electron is given by $p_l = l\hbar$.

According to the wave mechanics, $p_l = [l(l+1)]^{1/2}\hbar$

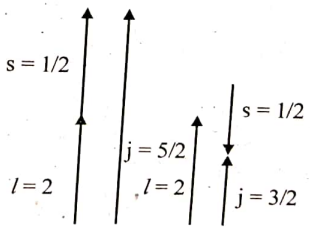
(3) The spin quantum number (s):- This has only one value, s = 1/2. The spin angular momentum $p_s = s\hbar$ where s = 1/2. According to wave mechanics, $p_s = [s(s+1)]^{1/2}\hbar$

(4) Total angular momentum quantum number (j):- It represents the total angular momentum of the electron which is the sum of the orbital angular momentum and spin angular momentum. The vector $\vec{j}$ is defined by the equation $\vec{j} = \vec{l} + \vec{s}$ with the restriction that $\vec{j}$ is positive. The spin angular momentum $s = \pm\frac{1}{2}$, $\therefore j = l \pm s$, plus sign when s is parallel to l and minus sign when s is antiparallel. Thus for $l = 2$ and s = 1/2, j can have the values 5/2 and 3/2 (Fig.).

The total angular momentum of the electron = $p_j = j\hbar$.

According to wave mechanics, $p_j = \sqrt{j(j+1)}\,\hbar$.

To explain the splitting of spectral lines in a magnetic field, three more magnetic quantum numbers are introduced.



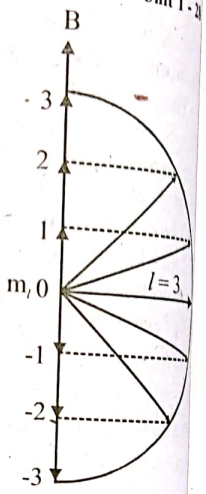
Figure

(5) Magnetic orbital quantum number (m_l):- The projection of the orbital quantum number l on the magnetic field direction is called the magnetic orbital

Visit us for more detail at : www.winsomepublication.in

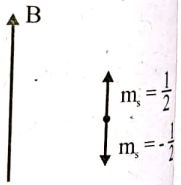
Scanned by CamScanner

quantum number m_l. The possible values of m_l are $l, l-1, l-2, \ldots, 0, -1, -2, \ldots -l$. i.e., there are $(2l+1)$ possible values of m_l. This is illustrated in Fig. for $l = 3$. The angle θ between l and B is given by $\cos\theta = m_l/l$. Conversely, the permitted orientations of the l vector relative to the field direction B is also $(2l+1)$. For example, if $l = 3$, the permitted orientations of l are 7, for with $m_l = 3, 2, 1, 0, -1, -2, -3$. Hence, the l vector can take only 7 directions shown in Fig. l cannot be inclined to B at any other angle. This is known as 'spatial quantisation'.



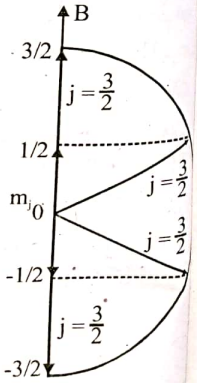
Figure

(6) Magnetic spin quantum number (m_s):- This is the projection of the spin vector s along the direction of the magnetic field. The spin angular momentum (s) can assume only two possible positions with respect to the magnetic field: it may be parallel to it or antiparallel. m_s can have only two values +1/2 or -1/2, as illustrated in figure.



Figure

(7) Magnetic total angular momentum quantum number (m_j):- This is the projection of total angular momentum vector j on the direction of the magnetic field. Since we are dealing with a single electron, j can have only odd half-integral values $\left(\because j = l \pm \frac{1}{2}\right)$. Hence, m_j must have only odd half-integral values. m_j can have only $(2j+1)$ values, from +j to -j zero excluded. Fig. shows the possible values of m_j for $j = 3/2$.



Figure

Notes:-

(1) The state of an electron in an atom is completely specified by four quantum numbers, n, l, m_l, and m_s.

(2) In spectroscopic notation, small letters, l, s, j and s, p, d, f, g etc., are used to describe the state of the electron and capital letters, L, S, J, and S, P, D, F, G, etc., are used to describe the state of the atoms as a whole.

Q.9. Write short note on characteristic X rays. (2010

Ans. Characteristic X-rays are emitted when outer-shell electrons fill a va

cancy in the inner shell of an atom, releasing X-rays in a pattern that is "characteristic" to each element. Characteristic X-rays were discovered by Charles Glover Barkla in 1909, who later won the Nobel Prize in Physics for his discovery in 1917.

Characteristic X-rays are produced when an element is bombarded with high-energy particles, which can be photons, electrons or ions (such as protons). When the incident particle strikes a bound electron (the target electron) in an atom, the target electron is ejected from the inner shell of the atom. After the electron has been ejected, the atom is left with a vacant energy level, also known as a core hole. Outer-shell electrons then fall into the inner shell, emitting quantized photons with an energy level equivalent to the energy difference between the higher and lower states. Each element has a unique set of energy levels, and thus the transition from higher to lower energy levels produces X-rays with frequencies that are characteristic to each element. When an electron falls from the L shell to the K shell, the X-ray emitted is called a K-alpha X-ray. Similarly, when an electron falls from the M shell to the K shell, the X-ray emitted is called a K-beta X-ray. Sometimes, however, instead of releasing the energy in the form of an X-ray, the energy can be transferred to another electron, which is then ejected from the atom. This is called the Auger effect.

Q.10. Discuss the Frank-Hertz experiment and explain the significance of result. How does it differentiate between ionization potential and resonance potential? (2009)

Ans. The Frank-Hertz Experiment:-

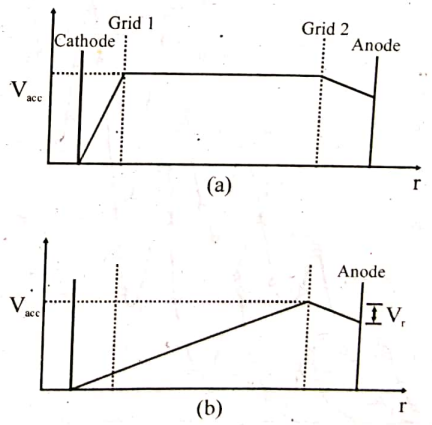


Figure - Different arrangements of the potential in Franck - Hertz tube

Scanned by CamScanner

We have seen that the existence of discrete frequencies in atomic radiation is proof of the existence of discrete energy levels in atoms, provided that Einstein's equation E = hν is accepted. But in 1914, James Frank and Gustav Hertz obtained direct evidence of the discreteness of atomic energy levels, in an experiment which displayed the effect of collisions between atoms and electrons. Even though radiation was not involved in the transfer of energy, the results showed that the electrons lost energy in discrete amounts in inelastic collisions with atoms. Thus it was clear that the process was governed by the existence of discrete energy levels in atoms. This is another of those classic experiments which are now performed in most undergraduate laboratories; it won the 1925 Nobel Prize for Frank and Hertz.

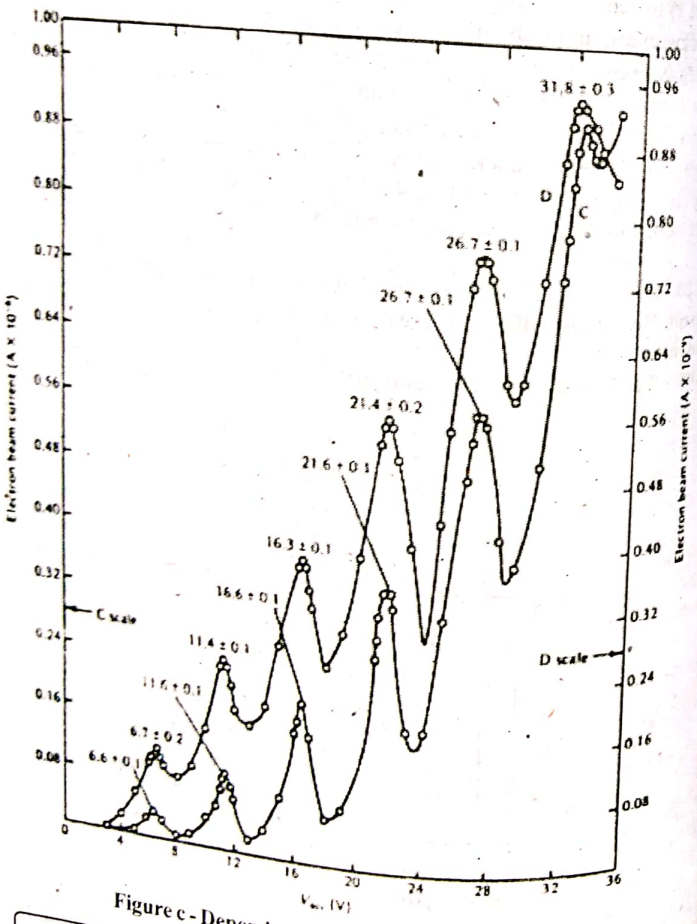


Figure c - Dependence of anode current on V_{acc}.

Visit us for more detail at : www.winsomepublication.in

The basic idea of Frank-Hertz experiment may be understood by reference to fig.(a). Electrons in a vapor-filled tube are accelerated by the grid potential V_{acc}. After passing Grid 2, the electrons are retarded by a reversed voltage v_r between grid and anode. A meter in the anode circuit measures the current produced by electrons which reach the anode. If there were no vapor present, an increase in grid voltage V_{acc} should always result in an increase in current, as it does in any vacuum tube. The same thing happens when the vapor is present, as long as V_{acc} is smaller than the energy needed to excite a vapor atom from its ground state to its first excited state. But as V_{acc} is increased, eventually the accelerating potential gives electrons a kinetic energy equal to this excitation energy. At this point, electrons are able to lose energy in inelastic collisions which excite atoms of the vapor. After such a collision an electron can not overcome the retarding potential V_r, so many electrons do not reach the anode, and the current falls. As V_{acc} is increased still further, the current increases again, as illustrated in fig., but it falls again when V_{acc} is large enough so that an electron can make two inelastic collisions with atoms of the vapor. Fig.(c) a whole series of drops in current, equally spaced at intervals of about 5V in V_{acc}, indicating that the first excited state of a vapor atom must be about 5eV above the ground state; this result is in reasonable agreement with other observations: the vapor atoms were mercury, for which there is spectroscopic evidence that the first excited state lies at 4.86eV.

The first drop in current does not occur at about 5V, but at 6.6 or 6.7V instead. The extra 1.7V must indicate the work function of the cathode (or more precisely, the contact potential between cathode and anode); the kinetic energy acquired by the electrons is reduced by the amount of energy needed to free an electron from the cathode. Second, you see that all of the drops in current can be attributed to excitation of the first excited state of mercury. Apparently when the electrons reached the energy needed to excite the first excited state, they collided to quickly with a mercury atom that they do not have time to acquire enough energy to excite any higher energy levels. This happens when the electrons are accelerated slowly, by grid potentials like those of Fig.(b). But with an arrangement of grid potentials like that shown in Fig. (a), the electrons gain energy faster, and one can see additional current drops which result from excitation of higher energy levels.

To make the connection between these observations and the line spectra doubly certain, Hertz in 1924 observed the spectrum of light emitted by mercury vapor in such a tube. He found that the appearance of certain spectral lines depended on the value of V_{acc}; a given line did not appear until the voltage V_{acc} was large enough to excite the state which was believed to give rise to that line. When the value of V_{acc} was sufficiently large the atom thus excited emitted the line in falling back to the ground state (or sometimes, to a lower excited state).

Excitation Potential (Radiation Potential or Resonance Potential) :-

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

Scanned by CamScanner

The energy in electron volts required to raise an atom from its normal state into an excited state is called excitation potential of the state.

Ionisation Potential: -

Ionisation potential of the atom may be defined as the energy required to remove an electron from a given orbit to an infinite distance from the nucleus.

Q.11. State and derive Moseley's law and show how it has been utilized in removing some of the defects of periodic classification of the elements. (2014)

Ans. Moseley's Law:-

The study of the characteristic X ray spectra of different elements was undertaken by Moseley in 1913-14. He used 38 different elements as the targets of an X ray tube. Using an X ray spectrometer with a crystal of potassium ferrocyanide, the reflected rays were recorded on a photographic plate instead of receiving them in an ionization chamber. The spectral lines emitted by these elements were observed to belong to two distinct series, a short wavelength group belonging to the K-series and a relatively large wavelength group belonging to the L-series.

Moseley found a striking regularity in the spectral lines of each series. The spectrum emitted by each element was found to be identical with that of the other except that the lines had different wavelengths. It was further found that the frequency of any particular line in a series varied regularly from one element to the next in the periodic table. On plotting a graph between the square root of the frequency of any particular line in a series (say the k_α line which is the most intense in the K-series) and the atomic number of the element emitting the line. Moseley obtained a straight line (fig.). He thus concluded that the square root of the frequency of any particular spectral line is proportional to the atomic number of the emitting element. The relation is known as Moseley's law and in mathematical form, it is expressed as

$$f = a(z - b)^2 \quad \text{.....(i)}$$

where a and b are constants. For the K_α line, the value of 'a' was found to be 3/4 Rc, where R is the Rydberg constant and c is the velocity of light, whereas 'b' was found to be nearly = 1. Thus eq.(i) reduces to

$$f = \frac{3}{4}Rc(z-1)^2$$

or

$$f = Rc(z-1)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) \quad \text{.....(ii)}$$

Eq. (ii) is similar to the formula for hydrogen spectral lines derived on the basis of the Bohr atom model. According to Bohr's theory, the above equation shows that the spectral line K_α is emitted when an electron jumps from an orbit of principal quantum number 2 to the orbit of principal quantum number 1. Further, there appears in Eq. (ii) a factor (z - 1) instead of z. This can easily be explained if we consider the origin of the K_α line. A fast moving

Visit us for more detail at : www.winsomepublication.in

incoming electron knocks out an electron from the inner-most K-shell, causing a vacancy there, which normally contains two electrons. The K-shell is thus left with one electron only. The electron jumping from the L-shell to fill the vacancy in the K-shell moves in an electric field of the positive nuclear charge and the negative charge of a single electron still left in the K-shell. The net charge is equivalent to a positive nuclear charge of magnitude (z - 1)e.

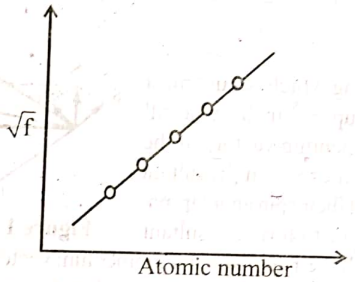


Figure - A graph between the square root of the frequency and the atomic number of the emitting elements

Importance of Moseley's Law:-

At the time Moseley performed his experiments, the elements in the periodic table were arranged according to their atomic weights. The atomic number of an element merely indicated its position amongst the elements arranged according to their atomic weights and had no physical significance. Moseley found that a straight line between $\sqrt{f}$ and z for various elements could be obtained if the order of nickel and cobalt as it existed in the periodic table at that time was reversed. Nickel with an atomic weight 58.69 was listed in the periodic table earlier than cobalt with atomic weight 58.94. Moseley found that these elements fitted his law if cobalt of higher atomic weight was given atomic number 27 and nickel of smaller atomic weight was given atomic number 28. Moseley thus concluded that there is in the elements a fundamental quantity which increases regularly from one element to the next in the periodic table. The quantity is the number of unit charges in the positive nucleus of the number of elements outside it. It has been named an atomic number and has a definite physical significance. Hence the importance of Moseley's law is

- Moseley's work thus led to the important conclusion that it is atomic number and not the atomic weight of an element which determines its characteristic physical and chemical properties. As a result, all the elements in the periodic table were rearranged according to their atomic number and the position of several elements corrected.
- Moseley's law also predicted the existence of elements undiscovered until then. For example, the regular sequence of points along the straight line showed a gap at Z = 43, showing the existence of the element technetium, which was discovered later.

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

Q.12. Discuss L-S and J-J coupling schemes in an atom. **(2018)**

Ans. Coupling Scheme: -

In an atom having two or more electrons, the orbital and spin angular momenta of all its electrons can be added together in two ways. The method of combination depends on the interaction or coupling between the orbital and spin angular momenta. Two types of schemes have been developed. They are (i) L-S coupling ir Russel-Saunders coupling and (ii) the j-j coupling.

(1) L-S coupling: -

The type of coupling which occurs most frequently is the L-S coupling. In this type, all the orbital angular momentum vectors of the various electrons combine to form a resultant L and independently, all their spin angular momentum vectors combine to form a resultant L and independently, all their spin angular momentum vectors combine to form a resultant S. These resultants L and S then combine to form the total angular momentum J of the atom. This scheme may be summarized as follows: $L = \sum l_i$; $S = \sum s_i$; $J = L + S$.L is always an integer including zero. S is an integer for an enven number of electrons, and odd multiple of 1/2 for an odd number of electrons (figure). Hence J must be an integer and J must be an odd multiple of 1/2 if S is an odd multiple of 1/2. It can be shown that, when L>S, J can have (2S + 1) values and when L<S, J can have (2L + 1) values. In particular, if L = 0, J can have only one value namely J = S.

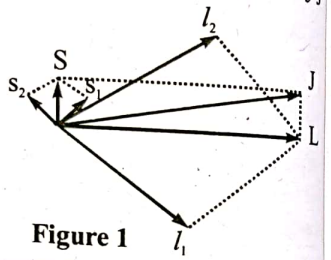


Figure 1

2 Electrons	3 Electrons	4 Electrons
S_2 S_1 S_1 S_2	S_3 S_2 S_1 S_2 S_1 S_3	S_4 S_3 S_2 S_1 S_3 S_2 S_1 S_4 S_2 S_1 S_3 S_4
S = 1, 0	3/2, 1/2	2, 1, 0

(2) The j-j coupling: -

This method is employed when the interaction between the spin and orbital vectors of each electron is stronger than the interaction between either the spin vectors or the orbital vectors of the different electrons. The orbital and spin angular momenta of each electron in the atom are added to obtain the resultant angular momentum of the electron.

Thus $\vec{J}_i = \vec{l}_i + \vec{s}_i$

The vector sum of all the individula j vectors gives the total angular momentum J of the atom. Thus $J = \sum j_i$. This type of coupling exists mainly in heavy atoms.

Visit us for more detail at : www.winsomepublication.in

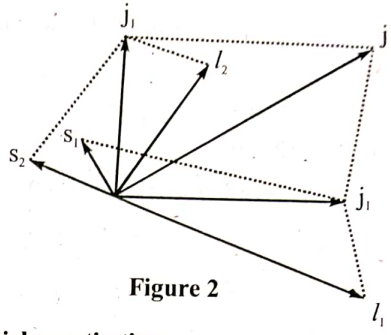


Figure 2

Application of Spatial quantisation: -

The resultant vectors **L, S** and **J** representing the atom, can be obtained by the above coupling schemes. According to quamtum theory **L, S** and **J** are qunatised in magnitude and direction. Hence the number of permitted orientations of **L, S** and **J** with respect to a given field direction are (2L + 1), (2S + 1) and (2J + 1) respectively. The corresponding magnetic quantum numbers $m_L = \sum m_l$, $m_S = \sum m_s$, and $m_j = \sum m_j$, can have only (2L + 1), (2S + 1) and (2J + 1) values respectively.

Numerical

Q.1. Find the wavelength of the Photon emitted when the hydrogen atom goes from n = 10 state to the ground state. **(2017)**

Ans. For any state corresponding to n in the hydrogen atom, you get

$$E_n = -\frac{13.6}{n^2}$$

Where - 13.6 eV is the approximate ground state energy of the hydrogen atom.

$$\text{So} \quad E_1 - E_{10} = -13.6\left(\frac{1}{n_1^2} - \frac{1}{n_{10}^2}\right) = -13.6\left(\frac{1}{1} - \frac{1}{100}\right) = -13.464$$

$$\text{So,} \quad \Delta E = 13.464 \times 1.6 \times 10^{-19} J = 21.54 \times 10^{-19} J$$

$$\text{In} \quad E = h\nu$$

$$\Rightarrow \quad \nu = \frac{21.54 \times 10^{-19} J}{6.63 \times 10^{-34} J.s} = 3.249 \times 10^{15} s^{-1}$$

but $c = \lambda\nu$ (c is the speed of light)

$$\text{so} \quad \lambda = \frac{3 \times 10^8 m/s}{3.249 \times 10^{15} s^{-1}} = 0.9233 \times 10^{-7} m = 923.3 \text{ Å}$$

Q.2. A X-ray tube operates at 12 KV. Find the maximum speed of the elec-

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

trons striking the anode (mass of electron = 9.1×10^{-31} kg, Charge of electron = 1.6×10^{-19} C). (2017)

Sol. $\lambda_{min} = \frac{12400}{V(\text{Volt})}\overset{0}{A} = \frac{12400}{12\times10^{3}}\overset{0}{A} = 1.03\overset{0}{A}$

Maximum velocity of electrons striking the anode is given by

$$\frac{1}{2}mv_{max}^2 = eV = 12\times10^{3}eV = 12\times10^{3}\times1.6\times10^{-19}J$$

$$\Rightarrow \quad v_{max} = \sqrt{\frac{12\times10^{3}\times1.6\times10^{-19}}{9.1\times10^{-31}}} = 4.59\times10^{7}\,m/s$$

Q.3. Find the Bohr's radius of hydrogen atom. (2014)

Ans. $\therefore \qquad r_n = \frac{n^2h^2}{4\pi^2mKZe^2}$

For hydrogen atom, radius of the first orbit is given by

$$r_H = \frac{(1)^2\left(6.63\times10^{-34}\right)^2}{4\pi^2\left(9.1\times10^{-31}\right)\left(9\times10^{9}\right)\times1\times\left(1.6\times10^{-19}\right)^2} = 0.53\overset{0}{A}$$

This is called Bohr's radius.

Q.4. An X-ray tube is operated at an anode potential of 10 KV and an anode current of 15 mA. Calculate the following -
(i) The number of electrons hitting the anode per second.
(ii) The minimum wavelength produced by the X-ray tube. (2011)

Sol. Given $V = 10\times10^{3}\,V$ and $I = 15\times10^{-3}\,A$

(i) For current = ne or $n = \frac{1}{e} = \frac{15\times10^{-3}}{1.6\times10^{-19}} = 9.38\times10^{16}$ electrons/sec

(ii) $\lambda_{min} = \frac{12400}{V}\overset{0}{A} = \frac{12400}{10\times10^{3}}\overset{0}{A} = 1.24\overset{0}{A}$

Q.5. Calculate the minimum wavelength of X-rays generated by the tube operating at 30 KV. (2009)

Ans. For maximum frequency, the kinetic energy (eV) should be converted into an X-ray photon.
Thus $h\nu = eV$

$$\nu = \frac{eV}{h} = \frac{30\times10^{3}eV}{4.14\times10^{-15}eV-s} = 7.2\times10^{18}\,Hz$$

Now for minumum wavelength $c = \nu\lambda$

then $\lambda = \frac{c}{\nu} = \frac{3\times10^{8}}{7.2\times10^{18}} = 0.4166\times10^{-10}\,m = 0.4166\overset{0}{A}$

Students preparing with WINSOME SERIES
get excellent marks in exam

Our Publication

WINSOME SERIES
for ALLAHABAD STATE UNIVERSITY
&
WINSOME SERIES
For Allahabad University

Scanned by CamScanner

Magnetic Properties Of Materials

Q.1. Distinguish between diamagnetic, paramagnetic and ferromagnetic substances. Give an account of Langevin's theory of diamagnetism. Show that the susceptibility of diamagnetic substance in independent of temperature. (2017)

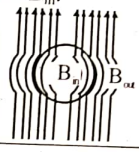
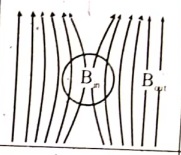
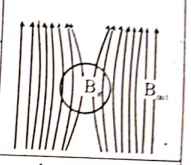
Other Related Questions -

Q. Show that the susceptibility of diamagnetic material is independent of temperature. (2016)

Q. Differentiate between paramagnetic and ferromagnetic substances. (2018)

Ans.

S.No.		Diamagnetic material	Paramagnetic material	Ferromagnetic material
1.	Magnetic moment	There is no permanent dipole moment (or) magnetic moment in each atom.	There is permanent dipole moment (or) magnetic moment in each atom.	There is enormous permanent dipole moment (or) magnetic moment in each atom.
2.	Spin alignment	No spin	All spins are randomly oriented.	Spin alignment is parallel in the same direction.
3.	Susceptibility and its temperature dependence	Susceptibility is always negative. It is independent of the temperature and strength of applied magnetic field.	It is always positive and small. It is inversely proportional to absolute temperature of the material. $\chi \propto \frac{1}{T}$ or $\chi = \frac{C}{T}$	It is always positive and very large. $\chi = \frac{C}{T-\theta}$ (Curie-Weiss law). (i) For $T > \theta$, paramagnetic behavior. (ii) For $T < \theta$, ferromagnetic behavior.

4.	Behavior of material in the presence of magnetic field.	When the material is placed in the magnetic field, the magnetic lines of force are repelled away from the material. $B_{out} > B_{in}$. 	The magnetic lines of force are attracted towards the centre of the material. $B_{in} > B_{out}$. 	The magnetic lines of force are highly attracted towards the centre of the material. $B_{in} >> B_{out}$. 
5.	Relative magnetic permeability (μ_r)	μ_r is slightly less than 1.	μ_r is slightly greater than 1.	μ_r is very much greater than 1. $\mu_r >> 1$
6.	Examples	Hydrogen, bismuth, antimony, gold and super conducting materials like Niobium.	Aluminium, Platinum, Sodium, Titanium, Zirconium and Chromium.	Iron, Nickel, Cobalt, Gadolinium.

Langevin's theory of diamagnetism:-

Consider an electron (mass = m, charge = e) rotating about the nucleus (charge = Ze) in a circular orbit of radius r. Let ω_0 be the angular velocity of the electron. Then

$$F_0 = m\omega_0^2 r = Ze^2 / \left(4\pi\varepsilon_0 r^2\right)$$

or $$\omega_0 = \sqrt{\frac{Ze^2}{4\pi\varepsilon_0 mr^3}} \quad \text{.....(i)}$$

The magnetic moment of the electron is

$$\vec{m} = \text{current} \times \text{area} = \frac{e\omega_0}{2\pi} \times \pi r^2 = \frac{e}{2}\omega_0 r^2 \quad \text{.....(ii)}$$

Let a magnetic field of induction B be now applied. B is normal to and into the page (Fig.)

An additional force F_L called Lorentz force, acts on the electron.

$$F_L = -e\left(v \times B\right) = -eBr\omega$$

The condition of stable motion is now given by

$$m r\omega^2 = \frac{Ze^2}{4\pi\varepsilon_0 r^2} - eBr\omega \qquad \text{.....(iii)}$$

or $$\omega^2 + \frac{eB}{m}\omega - \frac{Ze^2}{4\pi\varepsilon_0 mr^3} = 0$$

Solving the quadratic equation in ω,

$$\omega = \frac{\frac{-eB}{m} \pm \sqrt{\left(\frac{eB}{m}\right)^2 + 4\left(\frac{Ze^2}{4\pi\varepsilon_0 mr^3}\right)}}{2} = \pm\sqrt{\omega_0^2 + \left(\frac{eB}{2m}\right)^2} - \frac{eB}{2m}.$$

or $$\omega = \pm\omega_0 - \frac{eB}{2m} \qquad \left(\because \frac{eB}{2m} \ll \omega_0\right) \qquad \text{.....(iv)}$$

Thus the angular frequency is now different from ω_0. The result of establishing a field of flux density B is to set up a precessional motion of the electronic orbits with angular velocity - (e/2m)B. This is called Larmor theorem. Then

$$\left.\begin{array}{l}\text{change in frequency of}\\ \text{revolution of the electron}\end{array}\right\} = \delta n = -\frac{eB}{4\pi m}$$

The corresponding change in the magnetic moment of the electron is

$$\Delta m = \text{current} \times \text{area} = \left\{e \times \left(\frac{-eB}{4\pi m}\right)\right\} \times \pi r^2 = -\frac{Be^2 r^2}{4m} \qquad \text{.....(v)}$$

On summing overall electrons in the atom, the induced moment per atom becomes

$$\Delta m_{atom} = -\frac{Be^2 \sum r^2}{4m}$$

Let N be the number of atoms per unit volume. Then the magnetisation M is given by

$$M = -\frac{NBe^2 \sum r^2}{4m} \qquad \text{.....(vi)}$$

All the electron orbits are not oriented normal to the magnetic field. Hence r^2 in Eq.(vi) should be replaced by the average of the square of the projection of orbit radii for various electrons in a plane perpendicular to B. Hence we should replace r^2 in Eq.(vi) by 2/3 r^2.

$$\therefore \quad M = -\frac{NBe^2 \sum r^2}{6m}$$

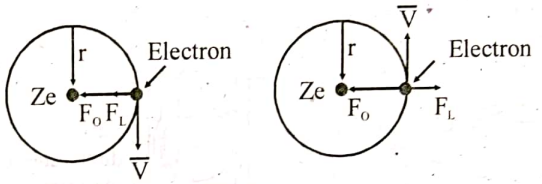
Volume susceptibility of the material

Visit us for more detail at : www.winsomepublication.in

$$\chi = \frac{M}{H} = -\frac{NBe^2 \sum r^2}{6mH} = -\frac{\mu_0 Ne^2 \sum r^2}{6m} \qquad (\because B = \mu_0 H)$$

$$\therefore \quad \chi = -\frac{\mu_0 Ne^2 \sum r^2}{6m} = -\frac{\mu_0 e^2}{6m} NZ < r^2 > \qquad \text{.....(vii)}$$

Eq. (vii) shows that χ is independent of the field strength and temperature. This is in accord with Curie's experimental results.



Figure

Q.2. Classify the materials on the basis of their magnetic properties obtain an expression for the magnetic susceptibility of paramagnetic materials. What is Curie temperature? (2015)

Other Related Questions -

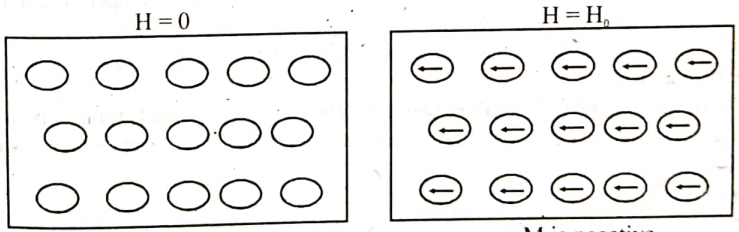
Q. Describe diamagnetism. (2016)

Q. Write short note on diamagnetism. (2013)

Q. Describe the basis of a classification of diamagnetic, paramagnetic and ferromagnetic materials. (2012)

Q. Discuss the properties of a paramagnetic material. (2019)

Ans. Diamagnetism:-



Figure

- The individual atoms of a diamagnetic material do not possess a permanent magnetic moment (Fig.a)
- When an external magnetic field H_0 is applied, the atoms acquire a small induced magnetic moment in a direction opposite to the direction of applied field (Fig.b).
- The strength of the induced magnetic moment is directly proportional to

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

Scanned by CamScanner

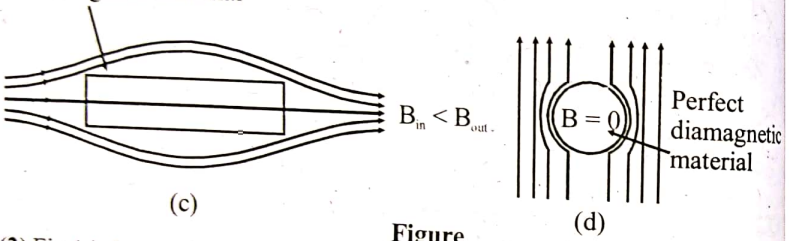
the applied field H_0.

- The induced dipoles and magnetization vanish as soon as the applied magnetic field is removed.
- Diamagnetism is a property of all atoms because of the influence of an applied magnetic field on the motion of electrons in their orbits.

Properties of diamagnetic materials:-

(1) Permanent dipoles are absent. There is no permanent dipole moment.

- Antimony, bismuth, mercury, gold and copper are some examples of diamagnetic substances.



(c) (d)

Figure

(2) Fig.(c) shows a bar of diamagnetic material placed in an external magnetic field. The magnetic lines of force are repelled or expelled. The field inside the material is reduced.

Fig.(d) shows the behaviour of a perfect diamagnetic material in the presence of the magnetic field.

- Diamagnetic materials repel the magnetic lines of force.
- The magnetic flux density B is less inside than outside.

(3) The magnetic susceptibility is negative ($\chi < 0$), i.e., magnetisation opposes the applied field. Magnetic susceptibility is independent of temperature and applied magnetic field strength.

(4) Relative permeability is slightly less than unity ($\mu_r < 1$).

Paramagnetism:-

(1) In the absence of external magnetic field - The individual atoms of paramagnetic material possess a permanent magnetic dipole moment of their own (Fig.e).

- Each atom possesses a permanent magnetic moment.
- When H = 0, all the magnetic moments are randomly oriented because of the ceaseless random thermal motion of the atom. So the net magnetization M = 0.

(2) When an external magnetic field is applied:- When an external magnetic field H_0 is applied, the magnetic dipoles tend to align themselves in the direction of the magnetic field (Fig.f). The individual atomic dipole moments point in the same direction. The material becomes magnetized. This effect is called paramagnetism.

- Magnetisation M and magnetic field H are in the same direction. Since $\chi = M/H$, the susceptibility χ is positive.

Visit us for more detail at : www.winsomepublication.in

- **Examples of paramagnetic materials:-** Platinum, aluminium, ferric oxide, ferrous sulphate, nickel sulphate, etc.

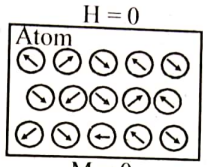


Figure (e)

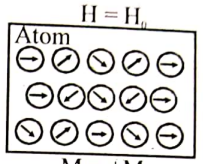


Figure (f)

Properties of paramagnetic materials:-

(1) Paramagnetic materials possess permanent magnetic dipoles.

(2) In the absence of an external applied field, the dipoles are randomly oriented. Hence the net magnetization in any given direction is zero.

(3) When placed inside a magnetic field, it attracts the magnetic lines of force (Fig.g). The field lines get concentrated inside the material, and the field inside is enhanced. This enhancement is slight, being one part in 10^5.

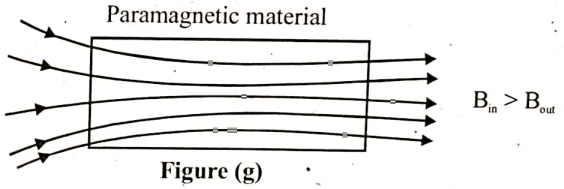


Figure (g)

(4) Paramagnetic susceptibility is positive and depends on temperature.

- $\chi = \frac{C}{T}$ is Curie's law.
- $\chi = \frac{C}{T - \theta}$ is called Curie-Weiss law.

Here, C is Curie constant and θ is a constant called paramagnetic Curie temperature.

(5) The value of the paramagnetic susceptibility is independent of the applied magnetic field strength.

(6) Spin alignment is random (Fig.h).

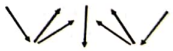


Figure (h)

Weiss theory of paramagnetism:-

Weiss introduced the concept of internal molecular field in order to explain the complicated type of dependence of susceptibility. In a real gas, the molecules are mutually influenced by their magnetic moments and consequently, there should exist within the gas a molecular field. This field produced at any point by all the neighbouring molecules, is proportional to and acting in the same sense as the intensity of magnetization (M). Let this internal molecu-

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

lar field be H_i. Now

$$H_i = \lambda M \qquad \text{.....(i)}$$

Here, λ is molecular field coefficient.

Therefore, the net effective field should be

$$H_e = (H + H_i) \qquad \text{.....(ii)}$$

Here, H is external applied field.

Following the Langevin theory along with this effective field,

$$M = \frac{Nm^2\mu_0 H_e}{3kT} = \frac{Nm^2\mu_0 (H + \lambda M)}{3kT}$$

Here, N = number of dipoles/unit volume,

m = magnetic moment of each atomic dipole,

k is Boltzmann's constant and T is the absolute temperature.

Rearranging the terms, we get

$$M\left(1 - \frac{Nm^2\mu_0\lambda}{3kT}\right) = \frac{Nm^2\mu_0 H}{3kT}$$

Let $C = \mu_0 Nm^2/3k$ and $\theta = C\lambda$. Then we get

$$M\left(1 - \frac{\theta}{T}\right) = \frac{CH}{T}$$

$$\therefore \quad \chi = \frac{M}{H} = \frac{C}{T\left(1 - \frac{\theta}{T}\right)} = \frac{C}{T - \theta} \qquad \text{.....(iii)}$$

Here C is called the Curie constant.

θ is called paramagnetic Curie point or the Curie temperature.

Eq. (iii) is called Curie-Weiss law.

Below Curie temperature ($T < \theta$), susceptibility becomes negative; i.e. paramagnetics would become diamagnetics. However, for most of the para magnetic substances, Curie temperature is quite low so that a situation fo which $T < \theta$ is rare.

Ferromagnetism:-

- Ferromagnetism is the existence of a spontaneous magnetization, even i zero applied field. Ferromagnetic materials have a small amount of magnetisatio even in the absence of an external field. This indicates that there is stron internal field within the material which makes the atomic magnetic moment align with each other.
- When placed in a magnetic field, ferromagnetic materials become strongl magnetized in the direction of the applied field. The direction of magnetizatio is the same as that of the external field.

Origin of ferromagnetism:-

Ferromagnetism arises due to permanent magnetic moments in th

Visit us for more detail at : www.winsomepublication.in

atoms or molecules of the material. When an external field is applied, the magnetic moments line up in the same direction as that of the applied field.

Examples of ferromagnetic materials:-

Iron (Fe), Cobalt (Co), Nickel (Ni), and Gadolinium (Gd).

Properties of ferromagnetic materials:-

(1) All the dipoles are aligned parallel to each other due to the magnetic interaction between any two dipoles. Fig.(i) shows the dipole alignment.

(2) Ferromagnetic materials have permanent dipole moment.

(3) When placed inside a magnetic field, a ferromagnetic material attracts the magnetic line of forces very strongly (Fig. j).

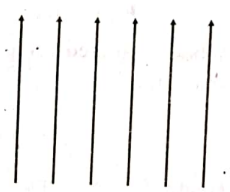
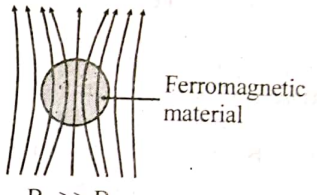


Figure (i)



$B_{in} >> B_{out}$

Figure (j)

(4) They exhibit magnetisation even in the absence of a magnetic field. This property of ferromagnetic materials is called spontaneous magnetisation.

(5) Ferromagnetic materials exhibit the phenomenon of hysteresis.

(6) On heating, they lose their magnetisation slowly.

(7) The magnetic susceptibility (χ) of ferromagnetic materials is very high and depends on temperature (T).

- As temperature increases, the value of susceptibility decreases.
- The ferromagnetic property depends on temperature. At high enough temperatures, a ferromagnet becomes a paramagnet.

The temperature of transition from ferromagnetism to paramagnetism is called the paramagnetic Curie temperature θ.

The susceptibility above the Curie temperature, i.e., in the paramagnetic phase is described by,

$$\chi = \frac{C}{T - \theta}(T > \theta).$$

Here, C is the Curie constant.

For $T > \theta$, paramagnetic behaviour.

For $T < \theta$, ferromagnetic behaviour.

Fig.(k) shows the variation of susceptibility with temperature for ferromagnetic materials.

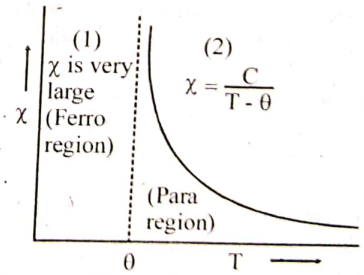


Figure (k)

(8) The relative permeability μ_r of ferromagnetic materials is very high. The

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

relative magnetic permeability is > 1000!.

Antiferromagnetism:-

Antiferromagnetic materials:-

• Antiferromagnetic materials are crystalline materials which possess a small positive susceptibility of the order of 10^{-3} to 10^{-5}.

• Consider an antiferromagnetic crystal containing two types of atoms A and B distributed over two interpenetrating lattices (Fig.*l*)

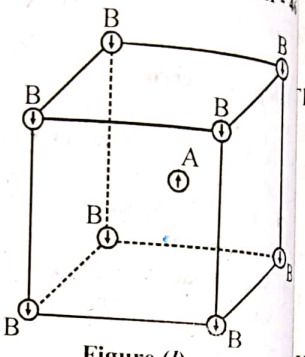


Figure (*l*)

B atom occupy the corner points of a body centered cubic lattice. A atoms occupy the body centres of these cubic lattices.

The atom at A-site has its spin up.

The atom at B-site has its spin down (Fig.m).

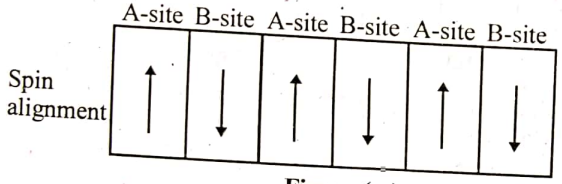


Figure (m)

The spin alignment of adjacent sites (A sites and B sites) is in an antiparallel manner. Hence their intensity of magnetisation is equal and opposite resulting in zero net magnetisation at T = 0K. Thus the susceptibility of antiferromagnetic materials at 0K is zero.

Above 0K, the alignment of spin magnetic moments in A sites and B sites are varying in a different manner. This will lead to a positive and small value of susceptibility for these materials.

Examples:- FeO (ferrous oxide), MnO (Manganese oxide), Cr_2O_3 (Chromium oxide) and salts of transition elements.

Definition of Antiferromagnetism:-

• If the A and B sublattice moments are exactly equal but opposite, the net moment is zero. This type of magnetic ordering is called antiferromagnetism.

In antiferromagnetism, the magnetic moments of sublattices in crystal cell are equal in magnitude but opposite in direction. So they cancel each other giving rise to net zero magnetization.

Variation of susceptibility with temperature of an antiferromagnetic material:-

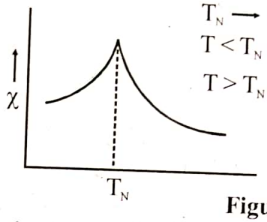
Fig.(n) shows the variation of susceptibility with temperature.

• The susceptibility increases with increasing temperature and it reaches a maximum at a certain temperature called Neel temperature, T_N.

Visit us for more detail at : www.winsomepublication.in

$\chi \propto T$ when $T < T_N$.

The material is antiferromagnetic below T_N.



$T_N \longrightarrow$ Neel temperature

$T < T_N$: Antiferromagnetic state $\chi \propto T$

$T > T_N$: Paramagnetic state $\chi = \frac{C}{T+\theta}$

Figure (n)

With further increase in temperature, the material reaches paramagnetic state. Above Neel temperature, the susceptibility decreases with temperature.

$$\chi = \frac{C}{T+\theta}.$$

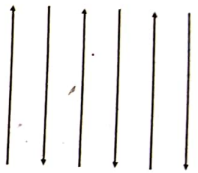


Figure (o)

where, C is the Curie constant and θ the paramagnetic Curie temperature.

Properties of Antiferromagnetic materials:-

) Electron spin of neighbouring atoms are aligned antiparallel; i.e., Spin alignment is antiparallel (fig.o).

) Antiferromagnetic susceptibility depends greatly on temperature.

Magnitude of Susceptibility	Temperature dependence	Examples
Small, positive	$\chi \propto T$ when $T < T_N$ $\chi = \frac{C}{T+\theta}$ when $T > T_N$	FeO (Ferrous oxide), MnO (Manganese oxide), Cr_2O_3 (Chromium oxide), Salts of transition elements

Ferrimagnetism:-

Ferrites and their applications:-

Ferrimagnetism is a special case of antiferromagnetism in which the opposite magnetic moments on each of the two sub-lattices are not exactly equal. Thus, when spontaneous anti-parallel arrangement of spins occurs, the material has a net paramagnetic moment and hence, a net magnetization too. This is called ferrimagnetism.

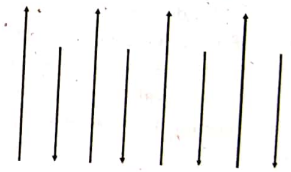


Figure (p)

In ferrimagnetic materials (also called ferrites) such as $MnFe_2O_4$, the magnetic moments of adjacent ions are antiparallel and of unequal strength (fig.p). So there is a finite net magnetisation. Ferrites exhibit spontaneous magnetization below the ferromagnetic Neel temperature T_N.

The general chemical formula of ferrites may be written as XY_2Z_4, where X is a divalent negative ion, Y is Fe^{3+}, and Z is mostly the divalent oxygen ion, O^{2-}. A

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

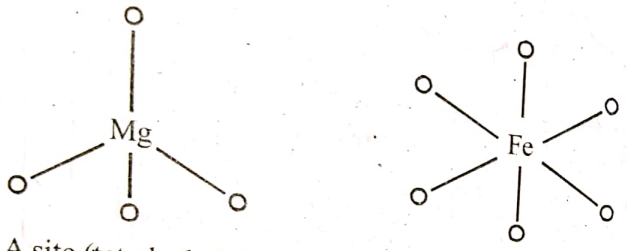
familiar example is Fe_3O_4 (or $Fe^{2+}Fe_2^{3+}O_4^{2-}$).

Structure of Ferrites:-

A ferrite crystal has the spinel structure.

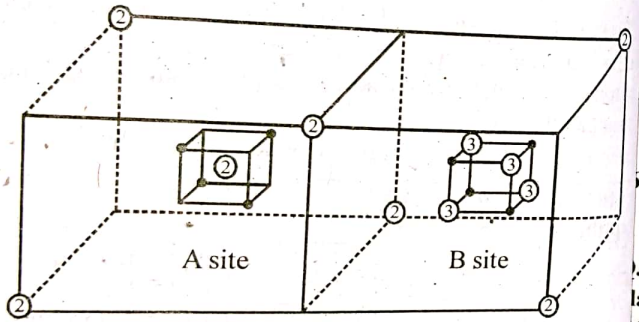
The mineral spinel is $MgAl_2O_4$.

Fig.(q and r) shows the regular spinel structure.



A site (tetrahedral site) B site (Octahedral site)

Figure (q)



② Divalent metal ion ③ Trivalent metal ion • Oxygen ion

Figure (r)

(1) Each divalent metal ion is surrounded by O^{2-} ions in a tetrahedral fas
For example, in $Mg^{2+}[Fe_2^{3+}]O_4^{2-}$, the structure of Mg^{2+} is given in F
It is called A site. There are 8 tetrahedral sites in the unit cell.

(2) Each trivalent metal ion (Fe^{3+}) is surrounded by $6O^{2-}$ ions in the octa
fashion as shown in fig. (q). It is called B site. There are 16 octahedra
in the unit cell.

- Thus in a regular spinel, each divalent metal ion (Mg^{2+}) exists in a te
dral site (A site) and each trivalent metal ion (Fe^{3+}) exists in an octah
site (B site).
- Hence, the sites A and B combine together to form a regular spinel
structure as shown in Fig. (r).

Properties:-

(1) These are metal oxides, but not metals.

(2) These materials exhibit hysteresis property.

high electrical resistivity ($\sim 10^{12}\,\Omega$ cm

a consequence, there will be no eddy current loss as usually noted with iron, at high frequencies.

4) High microwave dielectric constant (~ 10 to 12) (low dielectric loss).

5) A high magnetic permeability (500 - 1000) for the mixed Ni-Zn ferrites ($Ni_{0.36}Zn_{0.64}Fe_2O_4$) at low frequencies and ~ 10 at high frequencies ($\geq$ 300 MHz).

pplications of ferrites:-

1) Ferrites are used to produce low frequency ultrasonic waves by magnetostriction principle. Further these are used in the electromechanical transducers.

2) Ferrite rods are used in radio receivers (particularly in medium wave coil) to increase the sensitivity and selectivity of receiver.

3) Ferrites like Nickel Zinc ferrites are used as cores in audio and T.V. transformers.

4) Since for ferrites eddy current loss and hysteresis loss are small at microwave frequencies, these are widely used in non-reciprocal microwave devices like gyrator, circulator and isolator.

5) Ferrites are also used in digital computers and data processing circuits. Normally here ferrites with rectangular hysteresis loops are used as magnetic storage elements.

6) Based on nonlinear tensor permeability property, ferrites can be used in devices for power limiting and harmonic generation.

.3. Obtain an expression for the Larmor frequency of electron when laced in a magnetic field of intensity $\bar{B}$. (2015)

ther Related Questions -

. State and prove Larmor's theorem. (2014)

ns. Larmor precession:-

When an atom is subjected in external magnetic field, the electron orbit or orbital angular momentum precesses around about the field axis or direction. This precession is called Larmor precession, and the frequency of this precession is called Larmor frequency.

xplanation:-

A rotating electron in an orbit behaves as current loop thus atom behaves as atomic dipole whose magnetic moment and angular momentum are μ_L, and L respectively. Both μ_L and L operate in opposite direction perpendicular to plane of electron rotation as shown in Fig. When this atomic dipole is placed in magnetic field B, then a torque ($\tau = \mu_L \times B$) acts on it which is perpendicular to angular momentum. The acting torque causes a change in angular momentum because it is equal to rate of change of angular momentum i.e., $\tau = dL/dt$; dL is change in L in time dt). The change in angular momentum L acts perpendicular to L because it is in direction of τ which is perpendicular

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

to L. Due to this reason, the direction of angular momentum changes with t while its magnitude remains constant. As a result L traces a cone aroun This precession of angular momentum is called Larmor precession.

Suppose plane of electron orbit makes an angle θ with the directio magnetic field, hence angle between L or μ_L and B will be also L. Let L prec around B with angular velocity ω, and θ is the angle between μ_L and B. If L t around with an angle dϕ in time dt in plane perpendicular to B, then dϕ = ω

Since $\text{Angle} = \frac{\text{Arc}}{\text{Radius}}$

Thus, $d\phi = \frac{|dL|}{|L|\sin\theta} = \frac{dL}{L\sin\theta}$

$\omega dt = \frac{dL}{L\sin\theta}$

$\Rightarrow \quad \omega = \frac{dL}{dt} \times \frac{1}{L\sin\theta}$

$\omega = \frac{\tau}{L\sin\theta} = \frac{|\mu_L \times B|}{L\sin\theta}$

$\omega = \frac{\mu_L B\sin\theta}{L\sin\theta} = \frac{\mu_L}{L} B$

$\omega = \frac{e}{2m} B$ .....(i)

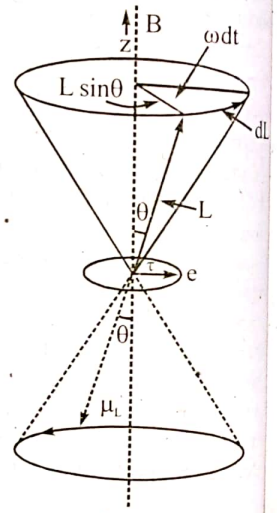


Figure - Precession of or angular momentum in influ of magnetic field.

Here ω is also called angular frequency of Larmor precession. If ν is La frequency, then ω = 2πν. So from Eq.(i), we have

$\nu = \frac{e}{4\pi m} B$ .....(ii)

Eq.(ii) shows that the Larmor frequency is independent of the angle betw or μ_L and B.

Q.4. Write short note on Larmour precession. **(2013**

Ans. Larmor precession (named after Joseph Larmor) is the precessi the magnetic moment of an object about an external magnetic field. Ob with a magnetic moment also have angular momentum and effective int electric current proportional to their angular momentum; these include trons, protons, other fermions, many atomic and nuclear systems, as we classical macroscopic systems. The external magnetic field exerts a torq the magnetic moment,

$\vec{\tau} = \vec{\mu} \times \vec{B} = \gamma \vec{J} \times \vec{B}$

where $\vec{\tau}$ is the torque, $\vec{\mu}$ is the magnetic dipole moment, $\vec{J}$ is the angular momentum vector, $\vec{B}$ is the external magnetic field, × symbolizes the cross product, and γ is the gyromagnetic ratio which gives the proportionality constant between the magnetic moment and the angular momentum. The phenomenon is similar to the precession of a tilted classical gyroscope in an external torque-exerting gravitational field.

Q.5. Give an account on Langevin theory of paramagnetism. (2012, 09)

Other Related Questions -

Q. Discuss the Langevin's theory of paramagnetism. What is Curie-Weiss law? (2018)

Q. Using a suitable theory (Langevin Theory) derive an expression for the temperature dependent paramagnetic susceptibility. What is Curie Law? (2019)

Ans. Langevin's theory of paramagnetism:-

He assumes that each atom has a permanent magnetic moment m. The only force acting on the atom is that due to the external field B. Let θ be the angle of inclination of the axis of the atomic dipole with the direction of the applied field B. Then magnetic potential energy of the atomic dipole is

$U = -mB\cos\theta$

Now, on classical statistics, the number of atoms making an angle between θ and θ + dθ is

$dn = Ce^{mB\cos\theta/kT} \sin\theta\, d\theta$

where k is Boltzmann's constant and T is the absolute temperature.

Put mB/kT = α. Then

$dn = Ce^{\alpha\cos\theta} \sin\theta\, d\theta$ .....(i)

Hence the total number of atomic magnets in unit volume of paramagnetic material

$n = \int_0^{\pi} dn = \int_0^{\pi} Ce^{\alpha\cos\theta} \sin\theta\, d\theta$ .....(ii)

Put cosθ = x. Then -sinθ dθ = dx.

$n = \int_{+1}^{-1} -Ce^{\alpha x} dx = C\int_{-1}^{+1} e^{\alpha x} dx$

$\therefore \quad C = \frac{n\alpha}{e^{\alpha} - e^{-\alpha}}$ .....(iii)

The component of each dipole moment parallel to B in m cos θ. The total

Scanned by CamScanner

magnetic moment of all the n atoms contained in unit volume of the gas is the magnetisation M. It is given by

$$M = \int_0^{\pi} m\cos\theta\, dn = \int_0^{\pi} m\cos\theta\, Ce^{\alpha\cos\theta}\sin\theta\, d\theta \quad \text{.....(iv)}$$

Put $\cos\theta = x$. Then, $-\sin\theta\, d\theta = dx$. Therefore, we get

$$M = \int_{-1}^{-1} -mxCe^{\alpha x}dx = Cm\int_{-1}^{+1} xe^{\alpha x}dx$$

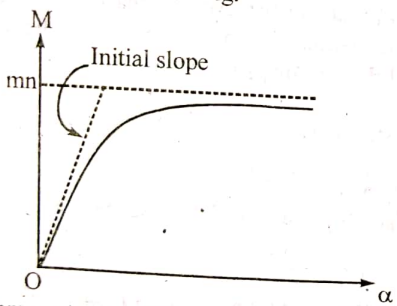
Evaluating this integral and substituting the value of C from (iii), we get

$$M = mn\left[\frac{e^{\alpha}+e^{-\alpha}}{e^{\alpha}-e^{-\alpha}} - \frac{1}{\alpha}\right]$$

$$M = mn\left[\coth\alpha - \frac{1}{\alpha}\right] = mn\,L(\alpha) \quad \text{.....(v)}$$

where $L(\alpha) = \left[\coth\alpha - \frac{1}{\alpha}\right]$ is called the Langevin function.

The variation of M with α is shown in Fig.



Case (i):- At low temperatures or large applied field, $L(\alpha) \to 1$. Hence, magnetisation M in this case will be

$$M = mn \quad \text{.....(vi)}$$

So saturation is reached when all the atomic dipoles are parallel to B.

Case (ii):- Under normal conditions α is very small. Then,

$$L(\alpha) = \coth\alpha - \frac{1}{\alpha} \approx \frac{\alpha}{3} \quad \text{.....(vii)}$$

$$\therefore \quad M = mn\frac{\alpha}{3} = \frac{nm^2B}{3kT} = \frac{nm^2\mu_0 H}{3kT} \quad \text{.....(viii)}$$

$$\chi = \frac{M}{H} = \frac{\mu_0 nm^2}{3kT} = \frac{C}{T} \quad \text{.....(ix)}$$

where $C = \mu_0 nm^2/3k$ is called the Curie constant.

Failure of Langevin Theory:-

(1) Langevin's theory was unable to explain a more complicated dependence of susceptibility upon temperature exhibited by several paramagnetics such as highly compressed and cooled gases, very concentrated solutions of salts etc.

(2) Langevin's theory could not account for the intimate relation between para-and ferro-magnetism.

Weiss Modification:-

Langevin's theory applies strictly only to gases, where the molecules are sufficiently far apart for their mutual interactions to be negligible. In liquids and solids such interactions may be large, and many substances obey the modified Curie-Weiss law

$$\chi = \frac{C}{(T-\theta)} \quad \text{.....(x)}$$

θ is called the Curie temperature and is characteristic of the substance. Eq.(x) holds only at temperatures where $T > |\theta|$. Eq.(x) is of the same form as Eq.(ix), except that the origin of temperature is shifted from 0 to θ.

Q.6. Show that susceptibility of a ferromagnetic material is variable.

Ans. The Susceptibility of a ferromagnetic material is

$$\chi = \frac{C}{T-\theta}\,(T > \theta)$$

The magnetic susceptibility (χ) of ferromagnetic materials is very high and depends on temperature (T). As temperature increases, the value of susceptibility decreases.

Q.7. Discuss in detail the origin of magnetism in materials. Obtain an expression for the magnetic moment of an electron rotating in a circular orbit. (2009)

Other Related Questions -

Q. What is Bhor magneton. (2019)

Q. Explain the origin of magnetic moment in an atom. Find the expression for the orbital, spin and total magnetic moment of the electron in any atom. (2019)

Ans. Origin of Magnetism: -

The magnetism of solid is originated from the electrons of atoms. Electrons are making orbital motions and spin rotations. Since an electron has a charge, magnetic moment appears just like magnetic field is generated when current flows in solenoid coils. The magnetic moment appears even from the spin orientation of electrons. The magnetic moment generated from

Scanned by CamScanner

the orbital or spin motion of a single electron is called **Bohr magneton**, which is the smallest unit of magnetic moment of solids.

Although many electrons are moving by orbital motions and spin orientations around a nuclei, only two electrons with up and down spins can occupy one orbit. Therefore, most of the magnetic moments originated from spins cancel each other and does not contribute to the magnetic moment of a solid. In the case of transition elements whose d-orbit is not fully occupied and rare earth elements whose p-orbit is not fully occupied, magnetic moment appear due to the spins. According to Hund's rule, electron are occupied so that the total spin becomes maximum. In other words, the number of up-spins increases from 1 to 5 according to the atomic number, there after, total spin amount decreases as up-spin and down-spin cancel each other when two electrons occupy one orbit. The reason why the actual metal never has such a high magnetization of 5 Bohr magneton is that the d-electrons are shared by many atoms in crystals as free electron. Thus, the maximum magnetization obtained from FeCo is 2.4 Bohr electron, which is less than half of the Bohr magnetron of an atom.

Spin and Orbital motion of an electron:-

The orbital motion of an electron around the nucleus is analogous to the current in a loop of wire.

The magnetic moment of a current carrying conductor is given by

$\alpha = I.A$ where I is the current in A and A is the area in m^2.

Oribital magnetic moment

The magnetic moment of an electron in orbit is given by

$\mu = \pi r^2 (ev/2\pi r) = evr/2$ ..... (1)

where r is the radius of orbit, e - charge and v is the velocity

The angular momentum of an electron must be an integral multiple of Planck's constant.

$mvr = nh/2\pi$ ..... (2)

where m is the mass and h is the Planck's constant.

If the electron revolves in the first orbit then n = 1

Therefore orbital magnetic moment of an electron is given by from (1) and (2)

$\alpha = eh/4\pi m$ .....(3)

which is known as Bohr magneton, the smallest possible orbital magnetic moment

Spin magnetic moment -

Similarly the smallest possible magnetic moment due to spin of the electron is

$\alpha = eh/4\pi m$

According to quantum theory the spin of electrons have only two possibilites +1/2 or -1/2.

Similar to eq.(3) we can write in the form

$\alpha = (e/2m)\, S$ .....(4)

Visit us for more detail at : www.

where S is the spin quantum number here given by $(1/2).(h/2\pi)$

In short,

$\alpha = g.(e/2m).S$ .....(5)

Here g is the term known as g- factor.

When g = 2, the spin contribution arises and when g = 1 the orbital contribution arises.

The mass of the nucleus is so large that the magnetic moment contribution can be neglected compared to the electronic magnetic moment.

The gyromagnetic ratio is proportional to the g-factor and 'g' arises due to the precession of the electrons similar to the precession of a top in a gravitational force. the value of g tells us whether the origin of magnetic moment is spin or orbital motion of electrons.

Q.8. Obtain the Curie Weiss Law. (2009)

Ans. Curie-Weiss Law:-

The Curie law is really just the magnetic analog of the ideal gas law, which is expressed in terms of the variables pVT (pressure, volume, temperature). For magnetic systems, one uses HMT (magnetic field, magnetization, temperature) and the thermodynamic relations derived for a perfect gas can be translated to a magnetic system by replacing p by H and V by -M.

When the pressure of a gas becomes too high or molecular interactions occur, deviations from ideal gas behaviour occur. Then, one turn to the Van der waals equation to describe the situation, or perhaps an even more elaborate correction is required. One source of the deviations can be the presence of an energy level whose population changes appreciably over the measured temperature interval; another source is the magnetic interaction which can occur between paramagnetic ions. To the simplest approximation, this behavior is expressed by a small modification of the Curie law, to the Curie-Weiss law,

$\chi = C/(T - \theta)$, ...(i)

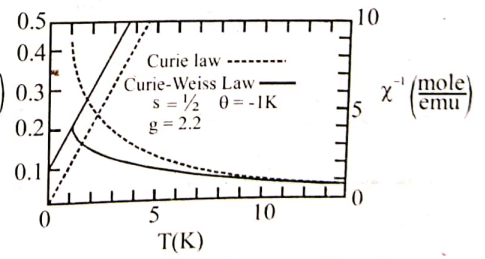


Figure - The Curie and Curie-Weiss law. The Curie constant C = 0.454emu-K/mol (y = ½, g = 2.2) has been used for both curves, while θ = - 1K has been used.

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

where the correction term, θ has the units of temperature. Negative values of θ are common, but this should not be confused with unphysical negative temperatures. The θ is obtained empirically from a plot of χ^{-1} vs. T, as for the Curie law, but now the intercept with the abscissa is not at the origin. When θ is negative it is called antiferromagnetic in sign; when θ is positive it is called ferromagnetic. The constant, θ, characteristic of any particular sample, is best evaluated when $T \geq 10\theta$, as curvature of χ^{-1} usually becomes apparent at smaller values of T.

The Curie and Curie-Weiss laws are compared for a hypothetical situation in fig.

Q.9. The magnetic moment of silver atom is only 1 Bohr magneton, although it has 47 electrons, Explain. (2016)

Ans. The magnetic moment for the silver atom is due to one unpaired electron.

Winsome Publication

Invite writers for publishing of their books.
Interested candidates send their resume as email
winsomepublication@rediffmail.com
or
Contact Us at - 9935951901

Quantum Concepts

Q.1. Describe Davisson and Germer experiment for the study of electron diffraction. What are the result of the experiment? (2017)

Other Related Questions -

Q. Describe Davisson and Germer experiment. (2015)

Q. Describe the Davisson and Germer's experiment and mention the important conclusion. (2009)

Ans. Davisson and Germer' s Experiment: -

Experimental arrangement: -

Figure (a) shows the experimental arrangement. Electrons are produced by heating a filament (f) by a low tension battery (L.T). The electrons are then accelerated through a potential difference V in the electron gun. This electron beam falls on a large single crystal of nickel. The electrons are scattered in all directions by the atoms in the crystal. The detector can be moved to any angle ϕ relative to the incident beam. The energy of the electrons in the primary beam, the angle at which they reach the target, and the position the detector could all be varied.

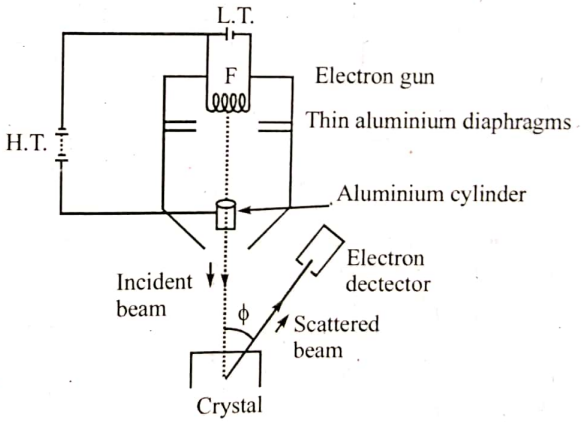


Figure (a)

Experimental procedure: -

The accelerating potential V is given a low value. The beam of electrons falls normally on the surface of the crystal. The detector is moved to various positions and the intensity of the diffracted beam at each position is noted. The intensity of diffracted beam is plotted against the angle between the incident beam and the beam entering the detector. The method of plotting is such that the intensity at any angle is proportional to the distance of the

Scanned by CamScanner

curve at that angle from the point of scattering. The observations are repeated for different accelerating voltages and a number of curves are drawn. Fig.(b) shows typical polar graphs of electron intensity.

(i) The graph remains fairly smooth, till the accelerating voltage becomes 44V when a **'bump'** appears on the curve.

(ii) As the accelerating voltage is increased, the length of the **'bump'** increases.

(iii) The **'bump'** becomes most prominent in the curve for 54 volt electrons at $\phi = 50^0$.

(iv) As the accelerating voltage is further increased, the bump decreases in length and finally disappears at 68 V.

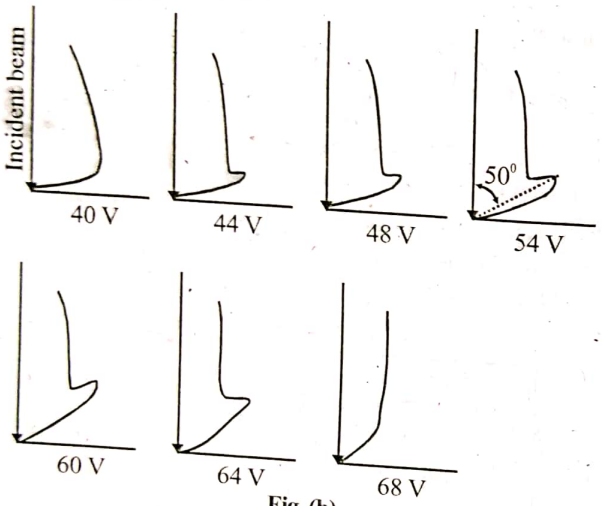


Fig. (b)

Q.2. Explain photoelectric effect. Give an account of Einstein explanation of Photoelectric effect on the basis of quantum theory. (2017)

Other Related Questions -

Q. Explain Photoelectric effect. List its various characteristics. (2012)

Q. What is photoelctric effect? Derive Einstein's photoelectric equation. (2019)

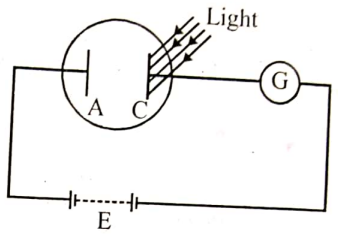
Ans. Photoelectric Effect: -

Whenever light or electromagnetic radiations (such as X-rays, Ultraviolet rays) fall on a metal surface, it emits electrons. This process of emission of electrons from a metal plate, when illuminated by light of suitable wavelength, is called the photoelectric effect. The electrons emitted are called the photoelectrons. In the case of alkali metals, photoelectric emission occurs even under the action of visible light. Zinc, cadmium etc., are sensitive to only ultraviolet light.

The nature of Photo-particles:-

Visit us for more detail at : www.winsomepublication.in

Experimental arrangement:-



The apparatus consists of two plates A and c placed in an evacuated quartz bulb (fig.).

The galvanometer (G) and battery (E) are connected as shown. When ultraviolet light is incident on the negative plate C, a current flows in the circuit as indicated by the galvanometer. But when light falls on the positive plate A, there is no current in the circuit. These observations show that photo-particle must be negatively charged.

Einstein's photoelectric equation:-

According to Einstein, light of frequency ν consists of corpuscles or photons each of energy $h\nu$. When a photon of light of frequency ν is incident on a metal, the energy is completely transferred to a free electron in the metal. A part of the energy acquired by the electron is used to pull out the electron from the surface of metal and the rest of it is utilised in imparting K.E. to the emitted electron. Let ϕ be the energy spent in extracting the electron from the emitter to which it is bound (photoelectric work function) and $1/2\ mv^2$ the K.E. of the photoelectron.

Then $$h\nu = \phi + \frac{1}{2}mv^2 \qquad \text{.....(i)}$$

This relation is called the Einstein's Photoelectric equation. If ν_0 is the threshold frequency which just ejects an electron from the metal without any velocity then, $\phi = h\nu_0$.

$$\therefore \quad h\nu = h\nu_0 + \frac{1}{2}mv_{max}^2 \qquad \text{.....(ii)}$$

Here, ν_{max} is the maximum velocity acquired by the electron.

or $$\frac{1}{2}mv_{max}^2 = h(\nu - \nu_0) \qquad \text{.....(iii)}$$

- The work function of a metal is defined as the energy which is just sufficient to liberate electrons from the metal surface with zero velocity.
- Eq.(iii) suggests that the energy of the emitted photoelectrons is independent of the intensity of the incident radiation but increases with the frequency.

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

Q.3. What are the postulates of wave mechanics? Derive Schrodinger's equation in one dimension. What is the significance of the wave function? Explain orthogonal and normalized wave function. (2017)

Other Related Questions -

Q. Write down the time dependent Schrodinger equation for a one-dimensional particle of mass m moving under a constant potential V_0. (2013)

Q. Establish the time dependent Schrodinger wave equation and reduce it to the time independent equation. (2015)

Q. Write the time dependent Schrodinger equation for a free particle of mass m and energy E and reduced it to time independent form. (2015)

Q. Obtain the time dependent one dimensional Schrodinger equation for a free particle of mass m. Discuss the physical significance of the wave function. (2009)

Q. Explain the requirement that are imposed on a physically acceptable wave function. What do you understand by the terms 'Eigen value' and 'Eigen function'? Find the eigen values and eigen functions for a particle in a one dimensional box. (2018)

Ans. Derivation of time-dependent form of Schrodinger Equation:-

The quantity that characterises the de Broglie waves is called the wave function. It is denoted by ψ. It may be a complex function. Let us assume that ψ is specified in the x direction by

$$\psi = Ae^{-i\omega(t - x/v)} \quad \text{.....(i)}$$

If ν is the frequency, then $\omega = 2\pi\nu$ and $\nu = \nu\lambda$.

$$\therefore \quad \psi = Ae^{-2\pi i(\nu t - x/\lambda)} \quad \text{.....(ii)}$$

Let E be the total energy and p the momentum of the particle.

Then, $\nu = \frac{E}{h}$ and $\lambda = \frac{h}{p}$.

Making these substitutions in Eq.(ii),

$$\psi = Ae^{-(2\pi i/h)(Et - px)}$$

$$\psi = Ae^{-(i/\hbar)(Et - px)} \quad \text{.....(iii)}$$

Eq. (iii) is a mathematical description of the wave equivalent of an unrestricted particle of total energy E and momentum p moving in the +x direction.

Differentiating Eq. (iii) twice with respect to x, we get

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{p^2}{\hbar^2}\psi$$

$$p^2\psi = -\hbar^2\frac{\partial^2\psi}{\partial x^2} \quad \text{.....(iv)}$$

Differentiating Eq.(iii) once with respect to t, we get

$$\frac{\partial\psi}{\partial t} = -\frac{iE}{\hbar}\psi$$

Visit us for more detail at : www.winsomepublication.in

$$E\psi = -\frac{\hbar}{i}\frac{\partial\psi}{\partial t} \quad \text{.....(v)}$$

At speeds small compared with that of light, the total energy E of a particle is the sum of its kinetic energy $p^2/2m$ and its potential energy V. V is in general a function of position x and time t.

$$\therefore \quad E = \frac{p^2}{2m} + V \quad \text{.....(vi)}$$

Multiplying both sides of Eq.(vi) by ψ, we get

$$E\psi = \frac{p^2\psi}{2m} + V\psi \quad \text{.....(vii)}$$

Substituting the expressions for $E\psi$ and $p^2\psi$ from Eqs.(v) and (iv) into Eq.(vii), we obtain

$$-\frac{\hbar}{i}\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi$$

or $$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi \quad \text{.....(viii)}$$

Eq.(viii) is the time-dependent form of Schrodinger's equation in one dimension.

In three dimensions, the time-dependent form of Schrodinger's equation is

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) + V\psi$$

Here, the particle's potential energy V is some function of x, y, z, and t.

Schrodinger's equation : Steady-state form - Time independent Schrodinger equation:-

In a great many situations, the potential energy of a particle does not depend upon time explicitly. The forces that act upon it, and hence V, vary with the position of the particle only. When this is true, Schrodinger's equation may be simplified by removing all reference to t.

The one-dimensional wave function ψ of an unrestricted particle may be written in the form

$$\psi = Ae^{-(i/\hbar)(Et - px)} = Ae^{-(iE/\hbar)t}.e^{+(ip/\hbar)x}$$

$$\therefore \quad \psi = \psi_0 e^{-(iE/\hbar)t} \quad \text{.....(i)}$$

Here, $\psi_0 = Ae^{+(ip/\hbar)x}$. That is, ψ is the product of a position dependent function ψ_0 and a time-dependent function $e^{-(iE/\hbar)t}$.

Differentiating Eq.(i) with respect to t, we get

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

$$\frac{\partial\psi}{\partial t} = -\frac{iE}{\hbar}\psi_0 e^{-(iE/\hbar)t} \qquad \text{.....(ii)}$$

Differentiating Eq.(i) twice with respect to x, we get

$$\frac{\partial^2\psi}{\partial x^2} = \frac{\partial^2\psi_0}{\partial x^2} e^{-(iE/\hbar)t} \qquad \text{.....(iii)}$$

We can substitute these values in the time-dependent form of Schrodinger's equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi$$

$$\therefore \quad E\psi_0 e^{-(iE/\hbar)t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi_0}{\partial x^2} e^{-(iE/\hbar)t} + V\psi_0 e^{-(iE/\hbar)t}$$

Dividing through by the commom exponential factor, we get

$$\frac{\partial^2\psi_0}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\psi_0 = 0 \qquad \text{.....(iv)}$$

Eq.(iv) is the steady-state form of Schrodinger's equation.
In three dimensions, it is

$$\nabla^2\psi_0 + \frac{2m}{\hbar^2}(E-V)\psi_0 = 0 \qquad \text{.....(v)}$$

Usually it is written in the form

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

Steady-state form of Schrodinger's equation in three dimensions is

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

Eigenvalues and Eigenfunctions:-

The values of energy E_n for which Schrodinger's steady-state equation can be solved are called eigenvalues. The corresponding wave functions ψ_n are called eigenfunctions.

Properties of the wave function:-

Physical significance of ψ:-

- By applying Schrodinger's equation to the motion of a particle, we get the wave function ψ. The future course of the particle's motion - like its initial state - is a matter of probabilities instead of certainties.
- The quantity whose variations make up matter waves is called the wave function, ψ. This function can be either real or complex.
  The wave function ψ itself, however, has no direct physical significance.

Visit us for more detail at : www.winsomepublication.in

- The only quantity having a physical meaning is the square of its magnitude $P = |\psi|^2 = \psi\psi^*$.
  Here, ψ* is the complex conjugate of ψ.
  The quantity P is the probability density.
- $|\psi|^2$, the square of the absolute value of the wave function, is called probability density.
  The probability of experimentally finding the body described by the wave function ψ at the point x, y,z at the time t is proportional to the value of $|\psi|^2$ there at t.
  A large value of $|\psi|^2$ means the strong possibility of the body's presence.
  A small value of $|\psi|^2$ means the slight possibility of its presence.
  This interpretation was made by Max Born.
- The probability of finding a particle in a volume dx dy dz is $|\psi|^2$ dx dy dz.
- The particle is certainly to be found somewhere in space.

$$\therefore \quad \iiint |\psi|^2 dx.dy.dz = 1.$$

The triple integral extends over all possible values of x, y, z.
A wave function (ψ) satisfying this relation is called a normalised wave function.

Orthogonal wave function:-

If the product of a function $\psi_1(x)$ and the complex conjugate $\psi_2^*(x)$ of a function $\psi_2(x)$ vanishes when integrated with respect to x over the interval $a \le x \le b$, that is, if

$$\int_a^b \psi_2^*(x)\psi_1(x)\,dx = 0,$$

then $\psi_1(x)$ and $\psi_2(x)$ are said to be orthogonal in the interval (a, b).

Normalised wave function:-

The probability of finding a particle in the volume elemer t V is given by ψψ* dV.
The total probability of finding the particle in the entire space is unity, i.e.,

$$\int \psi\psi^* dV = \int |\psi|^2 dV = 1.$$

Here, the integration extends over all space.
Any wave function satisfying the above equation is said to be normalised.
Very often ψ is not a normalised wave function. We know that it is possible to multiply ψ by a constant A, to give a new wave function, Aψ, which is also a solution of the wave equation. Now the problem is to choose the proper value of A such that the new wave function is a normalized function. In order that it

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

is a normalized function, it must meet the requirement

$$\int (A\psi)^* A\psi \, dx.dy.dz = 1 \quad \text{or,} \quad |A|^2 \int \psi\psi^* \, dx\, dy\, dz = 1$$

or $$|A|^2 = \frac{1}{\int \psi\psi^* \, dx\, dy\, dz}$$

$|A|$ is called normalizing constant.

Q.4. Consider a quantum particle of mass m moving in a one dimensional box of length L. Solve the Schrodinger wave equation and obtain the allowed energy states of the particle. When do these energy values approaches that of a free particle? (2014)

Q. An electron is confined to one-dimensional box of atomic size. Obtain expressions for the energy states and wavefunction of the electron assuming the potential energy to be zero inside the box and infinite at the walls. (2019)

Ans. The free particle:-

A free particle is defined as one which is subject to no forces of any kind, and so, moves in a region of constant potential. The particle motion is confined to the x-axis only. We shall consider the potential to be zero, $V = 0$. The time-independent Schrodinger equation becomes

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0 \qquad \text{.....(i)}$$

Define a quantity k, called the wave number, by

$$k = \sqrt{\frac{2mE}{\hbar^2}} \qquad \text{.....(ii)}$$

$$\therefore \quad \frac{d^2\psi}{dx^2} + k^2\psi = 0 \qquad \text{.....(iii)}$$

The solution of Eq.(iii) is

$$\psi = A\, e^{ikx} + B\, e^{-ikx}$$

Let us set $B = 0$. Then

$$\psi = A\, e^{ikx} \qquad \text{.....(iv)}$$

Eq.(v) describes a particle moving in the positive x-direction.

A problem appears in normalising the wave function. We ought to have

$$\int_{-\infty}^{\infty} \psi^* \psi dx = A^2 \int_{-\infty}^{\infty} dx = 1 \qquad \text{.....(v)}$$

$\int dx$ is infinite. So A must be zero. This difficulty arises since we are consid-

Visit us for more detail at : www.winsomepublication.in

ering an ideal case of infinite length. In practice, the particle is confined to a finite length so that normalisation is possible.

The probability density for the particle is $\psi^* \psi = A^2$, a constant independent of x. Consequently, the particle is equally likely to be found anywhere. So we have an infinite amount of uncertainty in its position, i.e., $\Delta x = \infty$. According to the expression $\Delta p \Delta x \geq \hbar/2$, Δp must be zero. Let us prove this.

$$< p > = \int_{-\infty}^{\infty} \psi^* \left(-i\hbar \frac{\partial}{\partial x}\right) \psi dx = \hbar k \int_{-\infty}^{\infty} \psi^* \psi dx = \hbar k$$

Notice that $< p >$ is positive, meaning that our particle has a momentum of exactly $\hbar k$ and is moving toward the right.

$$< p^2 > = \int_{-\infty}^{\infty} \psi^* \left(-\hbar^2\right) \frac{\partial^2}{\partial x^2} \psi dx = -\hbar^2 \left(-k^2\right) \int_{-\infty}^{\infty} \psi^* \psi dx = \hbar^2 k^2$$

$$\therefore \quad (\Delta p)^2 = < p^2 > - < p >^2 = 0.$$

Thus the momentum of the particle is precisely defined.

Note that for the particle in a square well, the energy is quantised. But for the free particle, there is no such restriction on the energy. Generally, bound systems (such as electrons trapped in an atom) will give rise to discrete energy levels. Unbound systems (such as two atoms that collide) will give rise to an energy continuum.

The particle in a box: Infinite square well potential:-

Consider a particle moving inside a box along the x-direction. The particle is bouncing back and forth between the walls of the box. The box has insurmountable potential barriers at $x = 0$ and $x = L$. i.e., the box is supposed to have walls of infinite height at $x = 0$ and $x = L$ (Fig.1). The particle has a mass m and its position x at any instant is given by $0 < x < L$.

The potential energy V of the particle is infinite on both sides of the box. The potential energy V of the particle can be assumed to be zero between $x = 0$ and $x = L$.

In terms of the boundary conditions imposed by the problem, the potential function is

$V = 0$ for $0 < x < L$

$V = \infty$ for $x \leq 0$

$V = \infty$ for $x \geq L$

The particle cannot exist outside the box and so its wave function ψ is 0 for $x \leq 0$ and $x \geq L$. Our task is to find what ψ is within the box, viz., between $x = 0$ and $x = L$.

Within the box, the Schrodinger's equation becomes

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} E\psi = 0 \qquad \text{.....(i)}$$

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

The general solution of this equation is

$$\psi = A\sin\frac{\sqrt{2mE}}{\hbar}x + B\cos\frac{\sqrt{2mE}}{\hbar}x \qquad \text{.....(ii)}$$

A and B are constants to be evaluated.

The boundary conditions are used to evaluate the constants A and B in Eq. (ii)

(a) $\psi = 0$ at $x = 0$. Hence $B = 0$.

(b) $\psi = 0$ at $x = L$.

ψ will be 0 at $x = L$ only when

$$\frac{\sqrt{2mE}}{\hbar}L = nx \qquad n = 1, 2, 3,..... \qquad \text{.....(iii)}$$

From Eq.(iii) it is clear that the energy of the particle can have only certain values, which are the eigen values.

These eigen values are found by solving Eq.(iii) for E_n.

$$\therefore \quad E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \qquad n = 1, 2, 3,..... \qquad \text{.....(iv)}$$

For each value of n, there is an energy level. Each value of E_n is called an eigenvalue. Thus inside the box, the particle can only have the discrete energy values specified by Eq. (iv). Note also that the particle cannot have zero energy

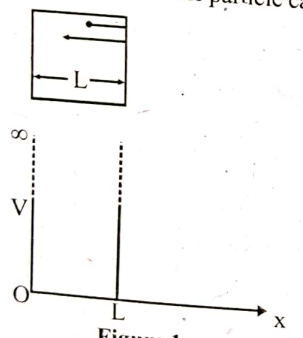


Figure 1

Wave functions:-

The wave functions of a particle in a box whose energies are E_n are from Eq. (ii) with $B = 0$.

$$\psi_n = A\sin\frac{\sqrt{2mE_n}}{\hbar}x \qquad \text{.....(v)}$$

Substituting Eq. (iv) for E_n gives

$$\psi_n = A\sin\frac{n\pi x}{L} \qquad \text{.....(vi)}$$

ψ_n are the eigenfunctions corresponding to the energy eigenvalues E_n.

Visit us for more detail at : www.winsomepublication.in

Evaluation of A and normalization of the wave function:-

It is certain that the particle is somewhere inside the box. Hence for a normalised wave function

$$\int_0^L \psi^*\psi\,dx = 1 \quad \text{i.e.,} \quad A^2\int_0^L \sin^2\left(\frac{n\pi x}{L}\right)dx = 1$$

$$\text{i.e.,} \quad A^2\int_0^L\left(\frac{1-\cos 2n\pi x/L}{2}\right)dx = 1 \quad \text{or} \quad A^2\frac{L}{2} = 1$$

$$\text{or} \quad A = \sqrt{\frac{2}{L}}$$

The normalized wave functions of the particle are

$$\psi_n = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L} \qquad n = 1, 2, 3,..... \qquad \text{.....(vii)}$$

The normalised wave functions ψ_1, ψ_2, and ψ_3 together with the probability densities $|\psi_1|^2$, $|\psi_2|^2$ and $|\psi_3|^2$ are plotted in Fig(2).

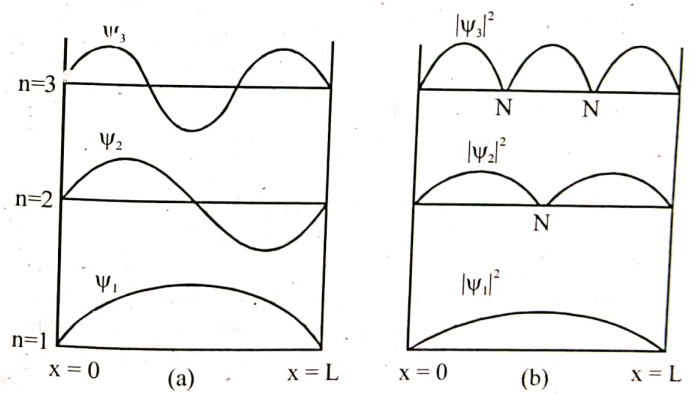


Figure (2)

Q.5. Write short note on Matter wave theory and its application. (2012)

Ans. Matter waves are a central part of the theory of quantum mechanics, being an example of wave–particle duality. All matter can exhibit wave-like behavior. For example, a beam of electrons can be diffracted just like a beam of light or a water wave. The concept that matter behaves like a wave was proposed by Louis de Broglie in 1924. It is also referred to as the de Broglie hypothesis. Matter waves are referred to as de Broglie waves.

The de Broglie wavelength is the wavelength, λ, associated with a

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

massive particle and is related to its momentum, p, through the Planck constant, h:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Wave-like behavior of matter was first experimentally demonstrated by George Paget Thomson's thin metal diffraction experiment, and independently in the Davisson–Germer experiment both using electrons, and it has also been confirmed for other elementary particles, neutral atoms and even molecules. Recently, it was also found that investigating the elementary process of diffusion gives the theoretical evidence of the relation of matter wave, regardless of the photon energy. It is thus revealed that the relation of matter wave is now not a hypothesis but an actual equation relevant to a characteristic of micro particle. The wave-like behavior of matter is crucial to the modern theory of atomic structure and particle physics.

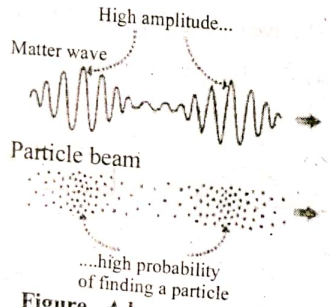


Figure - A beam of particles and its associated matter wave.

Q.6. What are phase and group velocities for waves. Consider the superimposition of a group of waves differing in frequencies and wave numbers by small amounts. Sketch the resultant wave. Explain the concept of wave packets. (2015)

Other Related Questions -

Q. **Differentiate between phase velocity and group of waves. Consider Superim-position of a group of simple harmonic waves having same amplitude but wave numbers and frequencies differing by small amount lying in the range Δk and Δv respectively. Sketch the pattern of the resultant wave and show that the group velocity is equal to the velocity of the particle. (2013)**

Q. **Define phase velocity and group veclocity. (2019)**

Ans. **Phase velocity (Wave velocity) of de Broglie Waves:-**

The phase velocity v_p of a monochromatic wave is the velocity with which a definite phase of the wave, such as its crest or trough, is

Visit us for more detail at : www.winsomepublication.in

propagated in a medium.

The equation of motion of a plane wave of frequency ν and and wavelength λ moving in the x-direction is

$$y = A\cos(\omega t - kx).$$

Here, A is the amplitude of the wave (Fig.a).

ω (= 2πν) is the angular frequency and k (= 2π/λ) is the propagation constant. This wave moves with a phase velocity

$$v_p = \nu\lambda = \omega/k$$

• A particle of mass m moving with velocity v has a wave associated with it whose wavelength is given by

$$\lambda = \frac{h}{mv}.$$

Let E be the total energy of the particle. Let ν be the frequency of the associated wave. We equate the quantum expression E = hν with the relativistic formula for total energy $E = mc^2$. So we get

$h\nu = mc^2$ or $\nu = mc^2/h$.

Let v_p be the de Broglie wave velocity. Then,

$$v_p = \nu\lambda = \left(\frac{mc^2}{h}\right)\left(\frac{h}{mv}\right) = \frac{c^2}{v}.$$

But the particle velocity v is always less than c (the velocity of light). Therefore, the de Broglie wave velocity v_p must be greater than c.

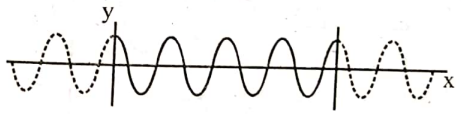


Figure (a)

Group velocity:-

It is possible to form a concentrated wave packet by taking a sum of a large number of plane waves with slightly different wavelengths and frequencies. When such a group travels in a dispersive medium, the phase velocities of its different components are different. The observed velocity is, however, the velocity with which the maximum amplitude of the group advances in the medium. This is called the "group velocity". Thus, the group velocity is the velocity with which the energy in the group is transmitted. The individual waves travel "inside" the group with their phase velocities.

• The velocity with which the centre of a wave group i.e., maximum amplitude, moves is called the group velocity of the wave group.

Expression for group velocity:-

Consider two waves that have the same amplitude A but differ by an amount Δω in angular frequency and an amount Δk in wave number. They can be represented by the equations

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

$y_1 = A\cos(\omega t - kx)$

$y_2 = A\cos[(\omega + \Delta\omega)t - (k + \Delta k)x]$

The superposition of the two waves will yield a single wave packet or wa group.

Let us find the velocity v_g with which the wave group travels.

The resultant displacement y at any time t and any position x is the sum of and y_2.

$$y = y_1 + y_2$$
$$= A\cos(\omega t - kx) + A\cos[(\omega + \Delta\omega)t - (k + \Delta k)x]$$
$$= 2A\cos\frac{1}{2}[(2\omega + \Delta\omega)t - (2k + \Delta k)x]\cos\frac{1}{2}(\Delta\omega t - \Delta kx)$$

$\Delta\omega$ and Δk are small compared with ω and k respectively. Therefore,

$$2\omega + \Delta\omega \approx 2\omega$$
$$2k + \Delta k \approx 2k.$$

$$\therefore \quad y = 2A\cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right)\cos(\omega t - kx) \qquad \text{.....(i)}$$

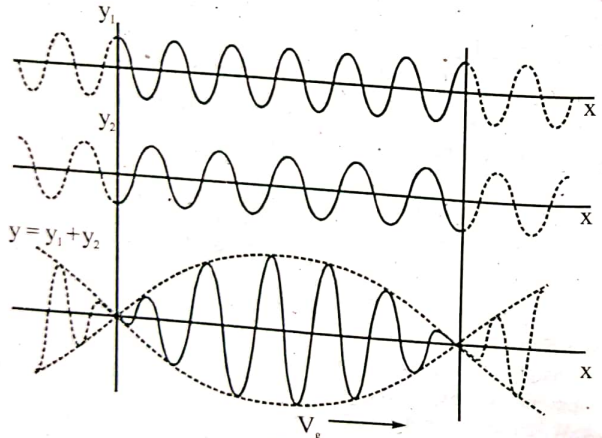


Figure (b)

This is the analytical expression for resultant wave (wave packet) due to superposition of the two waves. The second cosine function is the original wave. The coefficient of this cosine can be considered to be an amplitude that varies with x and t. This variation of amplitude is called the modulation of the wave.

Hence Eq.(i) represents a wave of angular frequency ω and wave number k that has superimposed upon it a modulation of angular frequency $\frac{1}{2}\Delta\omega$ (Fig.b).

The velocity v_g of the wave groups is

Visit us for more detail at : www.winsomepublication.in

$$v_g = \frac{\Delta\omega}{\Delta k} \qquad \text{.....(ii)}$$

When ω and k have continuous spreads, the group velocity is given by

$$v_g = \frac{d\omega}{dk} \qquad \text{.....(iii)}$$

This is the expression for the group velocity.

Wave Packet:-

A pure sine wave is completely unlocalized-it extends from $-\infty$ to $+\infty$. But a classical particle is completely localized. Our quantum description mixes particles and waves. The particles are approximately, but not completely, localized. An electron, for example, is bound to a specific atom. We know its position to within an uncertainty of the order of the diameter of the atom (10^{-10}m). But we don't know exactly where it is within that atom. The method used in physics to describe such a situation is that of a wave packet.

A particle in motion is represented by a wave that is confined to a small region of space around the particle. Such a wave is called a wave packet.

A wave packet is the superposition of a large number of waves. The waves interfere constructively in the vicinity of the particle, giving the resultant wave a large amplitude. The waves interfere destructively far from the particle. So the resultant wave has a small amplitude in regions where we don't expect to find the particle.

Fig.(c) shows a wave packet propagating along the x-axis. It consists of "ripples" contained within an "envelope".

The "envelope" travels at the group velocity (v_g).

The individual ripples travel at the phase velocity (v_p).

- The phase velocity of the wave is the speed with which a single component wave moves through the medium.
- The envelope moves at a speed different from that of the individual waves.

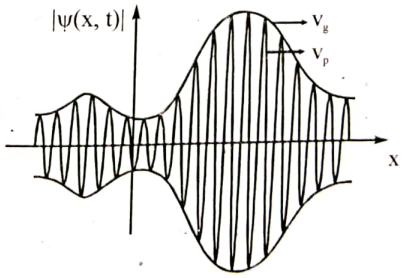


Figure (c)

Q.7. Find the relation between group velocity and phase velocity. (2019)

Ans. Relation between group velocity (v_g) and wave velocity or phase ve-

Visit us for more detail at : www.winsomepublicatio .n

Scanned by CamScanner

locity (v_p): -

We have relations,

Wave velocity, $v_p = \frac{\omega}{k}$ .....(i)

Group velocity, $v_g = \frac{d\omega}{dk}$ .....(ii)

The wave number is given by

$$k = \frac{2\pi}{\lambda}$$

$$\therefore \quad \frac{dk}{d\lambda} = -\frac{2\pi}{\lambda^2} \qquad .....(iii)$$

Also $\omega = 2\pi\nu = 2\pi\frac{v_p}{\lambda}$

$$\therefore \quad \frac{d\omega}{d\lambda} = 2\pi\left[-\frac{v_p}{\lambda^2} + \frac{1}{\lambda}\frac{dv_p}{d\lambda}\right]$$

or $$\frac{d\omega}{d\lambda} = -\frac{2\pi}{\lambda^2}\left[v_p - \lambda\frac{dv_p}{d\lambda}\right] \qquad .....(iv)$$

Dividing eq.(iv) by eq.(iii), we get

$$\frac{d\omega}{d\lambda}\cdot\frac{d\lambda}{dk} = \frac{-\frac{2\pi}{\lambda^2}\left[v_p - \lambda\frac{dv_p}{d\lambda}\right]}{-\frac{2\pi}{\lambda^2}}$$

or $$\frac{d\omega}{dk} = v_p - \lambda\frac{dv_p}{d\lambda}$$

$$\therefore \quad v_g = v_p - \lambda\frac{dv_p}{d\lambda} \qquad .....(v)$$

Eq.(v) gives the relationship between group velocity (v_g) and phase velocity (v_p).

From this equation the following two cases arises.

(i) For dispersive medium

$v_p = f(\lambda)$. Usually $dv_p/d\lambda$ is positive (normal dispersion),

$\therefore \quad v_g < v_p$.

Visit us for more detail at : www.winsomepublication.in

This is the case with de Broglie waves.

(ii) For non-dispersive medium

$$v_p \neq f(\lambda).\frac{dv_p}{d\lambda} = 0.$$

$\therefore \quad v_g = v_p$

This result is true for electromagnetic waves in vaccum.

If the phase velocity is same for all wavelengths, as is true for light waves in empty space, the group and phase velocities are the same.

Q.8. State Heisenberg's uncertainty principle. Describe some experiment which substantiates this principle. **(2015)**

Other Related Questions -

Q. State Heisenberg's uncertainty principle. Discuss the thought experiment of measurement of the position of the electron by a gamma ray spectroscope to sub-stantiate the uncertainty principle. **(2013)**

Q. Describe Heisenberg uncertainty principle and uncertainty relations. **(2019)**

Ans. Heisenberg's uncertainty principle:-

Statement:-

It is impossible to determine precisely and simultaneously the values of both the members of a pair of physical variables which describe the motion of an atomic system.

Such pairs of variables are called canonically conjugate variables.

Example (1):-

According to this principle, the position and momentum of a particle (say electron) cannot be determined simultaneously to any desired degree of accuracy.

Taking Δx as the error in determining its position and Δp the error in determining its momentum at the same instant, these quantities are related as follows:

$$\Delta x\,\Delta p \geq \hbar/2.$$

The product of the two errors is approximately of the order of Planck's constant $\hbar$.

- If Δx is small, Δp will be large and vice versa.

If one quantity is measured accurately, the other quantity becomes less accurate.

Example (2):-

Consider the wave group of Fig.

(a) A narrow de Broglie wave group. The narrower its wave group, the more precisely a particle's position can be specified (Fig.a).

But, the wavelength of the waves in a narrow packet is not well defined because there are not enough waves to measure λ accurately. Since

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

$\lambda = \frac{h}{mv}$, the particle's momentum mv is not a precise quantity. If we make

a series of momentum measurements, we will find a broad range of values

(b) A wide wave group. Now the wavelength λ can be precisely determined (Fig.b). The momentum mv is therefore a precise quantity. But the position of the particle is not well defined.

- It is impossible to know both the exact position and exact momentum of an object at the same time.

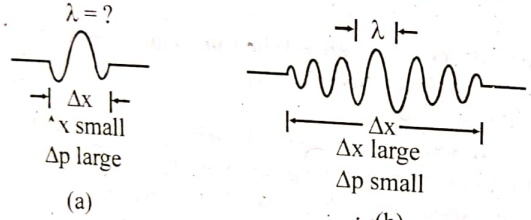


Illustration: Determination of position with γ-ray microscope:-

Suppose we try to measure the position and linear momentum of an electron using an imaginary microscope with a very high resolving power (Fig.).

Uncertainty in position measurement:-

The electron can be observed if atleast one photon is scattered by it into the microscope lens.

The resolving power of the microscope is given by the relation

$$\Delta x = \frac{\lambda}{2\sin\theta} \qquad \text{.....(i)}$$

Here, Δx is the distance between two points which can be just resolved by microscope. This is the range in which the electron would be visible when disturbed by the photon. Hence Δx is the uncertainty involved in the position measurement of the electron.

Uncertainty in momentum measurement:-

The incoming photon will interact with the electron through the Compton effect. To be able to see this electron, the scattered photon should enter the microscope within the angle 2θ. The momentum imparted by the photon to the electron during the impact is of the order of h/λ.

The component of this momentum along $OA = -\frac{h}{\lambda}\sin\theta$

The component of this momentum along $OB = \frac{h}{\lambda}\sin\theta$

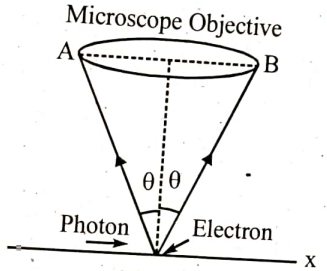
The uncertainty in the momentum measurement in the x-direction is

Visit us for more detail at : www.winsomepublication.in

$$\Delta p_x = \frac{h}{\lambda} - \left(-\frac{h}{\lambda}\sin\theta\right) = \frac{2h}{\lambda}\sin\theta \qquad \text{.....(ii)}$$

$$\therefore \quad \Delta x \times \Delta p_x = \frac{\lambda}{2\sin\theta} \times \frac{2h}{\lambda}\sin\theta = h$$

A more sophisticated approach will show that $\Delta x\,\Delta p_x \geq \hbar/2$.



Figure

Numerical

Q.1. Use Heisenberg's Uncertainty principle to show that electrons can not exist inside the nucleus. **(2013)**

Ans. Nuclear diameter is of the order of 10^{-15} m. For an electron to exist inside a nucleus, the uncertainty in its position must be at least of this order.

That is, $\Delta x \simeq 10^{-15}$ m.

The uncertainty in the electron momentum is

$$\Delta p \cong \frac{\hbar}{2(\Delta x)} \cong \frac{1.055\times10^{-34}}{2\times10^{-15}} = 0.527\times10^{-19}\ \text{kg ms}^{-1}$$

Energy of the electron, $E = \left(c^2p^2 + m_0^2c^4\right)^{1/2} = cp$

$$E = \left(3\times10^8\ \text{ms}^{-1}\right)\times\left(0.527\times10^{-19}\ \text{kg ms}^{-1}\right) = 1.581\times10^{-11}\ \text{J}$$

$$E = \frac{1.581\times10^{-11}}{1.6\times10^{-19}}\ \text{eV} = 98.8\ \text{MeV}$$

For an electron to be a constituent of the nucleus, it should have an energy of the order of 98.8 MeV. However, the energy of an electron emitted in a β-decay experiment is of the order of 3 MeV. Hence, we conclude that electrons do not reside in the nucleus.

Visit us for more detail at : www.winsor publication.in

Scanned by CamScanner

Q.2. How does the uncertainty principle prevent the electron to exist in side the nucleus. (2011

Ans. On the basis of Heisenberg's uncertainty principle, it can be shown a to why electron cannot exist within the atomic nucleus. The radius of th atomic nucleus is of the order of 10^{-15} m. Now, if the electron were to exist within the nucleus, then the maximum uncertainty in its position would have bee 10^{-15} m.

Now $$\Delta x \times \Delta p = \frac{h}{4\pi}$$

or $$\Delta x \times m\Delta v = \frac{h}{4\pi}$$

or $$\Delta v = \frac{h}{4\pi m \Delta x}$$

Mass of electron $m = 9.1 \times 10^{-31}$ kg, $\Delta x = 1 \times 10^{-15}$ m.

$$\therefore \quad \Delta v = \frac{6.6\times10^{-34}}{4\times3.14\times9.1\times10^{-31}\times1\times10^{-15}} = 5.77\times10^{10}\ m/s$$

The value of uncertainty in velocity, Δv is much higher than the velocity light (3.0×10^8 ms^{-1}) and therefore, it is not possible. Hence an electron cann be found within the atomic nucleus.

Q.3. Compare the uncertainties in the velocities of an electron and proto confind in a 10 Å box.

Sol. Let $\Delta x = 10$ Å. Then, (201

$$\Delta p \geq \frac{\hbar}{2\Delta x} = 5.27\times10^{-26}\ kg.m/s$$

For each case, we can treat as classical particles and so the uncertainties velocities are

$$v_e \geq \frac{\hbar}{2m_e\Delta x} = 5.79\times10^{4}\ m/s \qquad \text{for electron,}$$

$$\Delta v_p \geq \frac{\hbar}{2m_p\Delta x} = 3.15\times10^{1}\ m/s \qquad \text{for proton}$$

Q.4. Calculate the de Broglie Wavelength of an α particle accelerated a PD of 25000 Volts.

Sol. Given $m = 6.68 \times 10^{-27}$ kg; $V = 25000$ V; $q = 2e$ (201

Visit us for more detail at : www.winsomepublication.in

Now $$\lambda = \frac{h}{mv} = \frac{h}{(2meV)^{1/2}}$$

$$\lambda = \frac{6.626}{\sqrt{2\times6.68\times10^{-27}\times1.6\times10^{-19}\times25\times10^{3}}} = 6.4\times10^{-14}\ m$$

Q.5. A hydrogen like ion has the wavelength difference between the first lines of the Balmer and Lyman series equal to 16.58 nm. What ion is it?(2016)

Sol. Energy difference for the transitions in the two series $\Delta E_{11} - \Delta E_{32} = 1241/16.58 = 74.85$ eV.

$$13.6Z^2\left\{\left(\frac{1}{1^2}-\frac{1}{2^2}\right)-\left(\frac{1}{2^2}-\frac{1}{3^2}\right)\right\} = 74.85$$

Solving for Z, we get $Z = 3$

The ion is Li^{++}.

Q.6. Show that if an electron is accelerated through V volts then the deBroglie wavelength in angstrons is given by $\lambda = \frac{\sqrt{150}}{V}$. (2016)

Sol. We know that

$$\lambda = \frac{h}{P} = \frac{6.63\times10^{-34}\ J-s}{\left(2\times9.1\times10^{-31}\times1.6\times10^{-19}\right)^{1/2}} = 12.286\times10^{-10}\ m/V^{1/2}$$

$$= \left(\frac{151}{V}\right)^{1/2}$$

Q.7. An electron is confined to a box of length 10^{-10}m. Calculate minimum uncertainty in the measurement of its velocity. (2012)

Ans. From Heisenberg's uncertainty principle, we have

$$\Delta x.\Delta p \geq h \quad \text{or} \quad \Delta p = h/\Delta x$$

Let Δv be the uncertainty in the velocity of a particle of mass m. So we have,

$$\Delta p = m\Delta v \quad \text{or} \quad \Delta v = \Delta p/m$$

Now $\Delta x = 10^{-10}$m, $h = 1.05\times10^{-34}$ J-sec and $m = 9.0\times10^{-31}$ kg

Hence $$\Delta v = \frac{\hbar}{m\Delta x} = \frac{1.05\times10^{-34}}{9.0\times10^{-31}\times10^{-10}} = 1.166\times10^{6}\ m/sec.$$

Q.8. Calculate the maximum percentage change in the wavelength due to

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

Compton scattering of Photons of wavelength (i) 1 $\overset{0}{A}$ and (ii) 10 $\overset{0}{A}$. What conclusion do you draw from these results? **(2009)**

Ans. As we know

$$\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

$$(\Delta\lambda)_{max} = \frac{h}{m_0 c}(1-180^0) = \frac{2h}{m_0 c} = 0.48\,\overset{0}{A}$$

(a) For $\lambda = 1\,\overset{0}{A}$ $\quad \frac{(\Delta\lambda)_{max}}{\lambda} \times 100 = 4.8\%$

(a) For $\lambda = 10\,\overset{0}{A}$ $\quad \frac{(\Delta\lambda)_{max}}{\lambda} \times 100 = 0.48\%$

This shows that higer the wavelength of incident photon, smaller is maximum percentage change. This is due to the fact that in Compton scattering $\Delta\lambda$ is independent of λ.

Students preparing with WINSOME SERIES
get excellent marks in exam

Our Publication

WINSOME SERIES
for ALLAHABAD STATE UNIVERSITY
&
WINSOME SERIES
For Allahabad University

Visit us for more detail at : www.winsomepublication.in

Nuclear Physics

Q.1. Explain the terms, decay constant, half life and average life as applied to a radioactive substance. Find the relation between them. **(2017)**

Other Related Questions -

Q. Define half life of radioactive nucleus. **(2013)**

Q. Explain half-period of radio-active welement and derive an expression for it. **(2018)**

Ans. The Decay constant:-

If a radioactive sample contains N radioactive nuclei at some instant, the number of nuclei ΔN, that decay in a small time interval Δt is proportional to N; mathematically,

$$\frac{\Delta N}{\Delta t} \propto N \qquad \text{or} \qquad \Delta N = -\lambda N \Delta t \qquad \text{.....(i)}$$

where λ is a constant called the decay constant. The negative sign signifies that N decreases with time; that is, ΔN is negative. The value of λ for any isotope determines the rate at which that isotop will decay. The decay rate, or activity R, of a sample is defined as the number of decays per second. From Equation (i), we see the decay rate is

$$R = \left|\frac{\Delta N}{\Delta t}\right| = \lambda N \qquad \text{.....(ii)}$$

Isotopes with a large λ value decay rapidly; those with small λ decay slowly.

A general decay curve for a radioactive sample is shown in Active Fig. It can be shown from Eq.(i) (using calculus) that the number of nuclei present varies with time according to the equation

$$N = N_0 e^{-\lambda t} \qquad \text{.....(iii)}$$

where N is the number of radioactive nuclei present at time t, N_0 is the number present at time t = 0, and e = 2.718.... is Euler's constant. Processes that obey Eq.(iii) are sometimes said to undergo exponential decay.

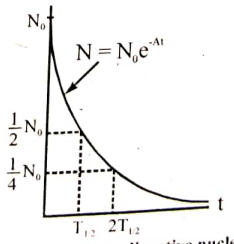


Figure - Plot of the exponential decay law for radioactive nuclei. The vertical axis represents the number of radioactive nuclei present at any time t, and the horizontal axis is time. The parameter $T_{1/2}$ is the half life of the sample.

Definition of half-life period:-

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

The half-life period of a radioactive substance is defined as the time required for one-half of the radioactive substance to disintegrate.

Value of half-life period:-

We know the relation $N = N_0 e^{-\lambda t}$.

If $T_{1/2}$ be the half-life period, then at

$t = T_{1/2}$, $N = N_0/2$.

$$\therefore \quad \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \text{ or } e^{\lambda T_{1/2}} = 2$$

$$\text{or} \quad \lambda T_{1/2} = \log_e 2 \quad \text{or} \quad T_{1/2} = \frac{\log_e 2}{\lambda}$$

$$\therefore \quad T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{0.6931}{\lambda}$$

The average life (The mean life):-

It is not possible to predict which atom of a radioactive substance will disintegrate at any instant. The atom which disintegrates first has zero life and that disintegrates last has infinite life. Thus the life of every atom is different and the actual lives of the various atoms range from zero to infinity.

$$\therefore \text{ The mean – life of a radioactive element} = \frac{\text{Sum of the lives of all the atoms}}{\text{Total number of atoms}}$$

Definition:-

The mean-life of a radioelement is defined as the total life time of all the radioactive atoms to the total number of such atoms in it.

Value of mean-life:- Let N_0 be the total number of radioactive atoms in the beginning. Let N be the number of atoms of that element after time t. Then $N = N_0 e^{-\lambda t}$. Let dN be the number of atoms disintegrating between time t and $t + \delta t$. These dN atoms have a life between t and $(t + \delta t)$. Since δt is very small, each of these atoms had a life of t.

$\therefore$ Total life of dN atoms = (dN)t.

The possible life of any of the total number N_0 radioactive atoms varies from 0 to ∞.

$$\therefore \quad \text{Total life time of all } N_0 \text{ atoms} = \int_0^\infty t\,dN$$

$$\text{Now, mean life} = \bar{T} = \frac{\text{total life – time}}{\text{total number of atoms}} = \frac{\int_0^\infty t\,dN}{N_0}$$

Now, $N = N_0 e^{-\lambda t}$

$$\therefore \quad \frac{dN}{dt} = -\lambda N_0 e^{-\lambda t} \quad \Rightarrow \quad dN = -\lambda N_0 e^{-\lambda t} dt$$

Visit us for more detail at : www.winsomepublication.in

$$dN = \lambda N_0 e^{-\lambda t} dt$$

(Leaving the negative sign which merely indicates the decrease in the number of atoms with time).

$$\text{Hence,} \quad \frac{\int_0^\infty t\lambda N_0 e^{-\lambda t} dt}{N_0} = \lambda \int_0^\infty t e^{-\lambda t} dt$$

Integrating by parts,

$$\bar{T} = \lambda \left[\frac{t\,e^{-\lambda t}}{-\lambda} - \int \frac{e^{-\lambda t} dt}{-\lambda} \right]_0^\infty$$

$$\bar{T} = \lambda \left[\frac{t\,e^{-\lambda t}}{-\lambda} - \frac{e^{-\lambda t}}{-\lambda^2} \right]_0^\infty = \lambda \left(\frac{1}{\lambda^2} \right) = \frac{1}{\lambda}$$

$$\therefore \quad \bar{T} = \frac{1}{\lambda}.$$

Thus the mean life $(\bar{T})$ of a radioactive substance is the reciprocal of the decay constant (λ).

Q.2. Write down the equation for radioactivity decay of nucleus. (2013)

Ans. Law of radioactive disintegration:-

Let N be the number of atoms present in a particular radioelement at a given instant t. Then the rate of decrease -dN/dt is proportional to N.

$$\therefore \quad -\frac{dN}{dt} = \lambda N \qquad \text{.....(i)}$$

Here λ is a constant known as the disintegration constant or decay constant of the radioactive element. It is defined as the ratio of the amount of the substance which disintegrates in a unit time to the amount of substance present.

Eq. (i) can be written as $\frac{dN}{N} = -\lambda dt$.

Integrating, $\log_e N = -\lambda t + C$ .....(ii)

Let the number of radioactive atoms initially present be N_0.

Then, when $t = 0$, $N = N_0$ $\therefore \log_e N_0 = C$.

Substituting for C in Eq. (ii), we get, $\log N = -\lambda t + \log N_0$

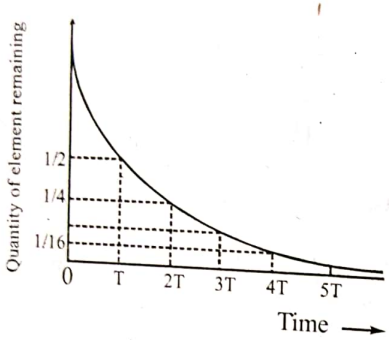
$$\text{or} \quad \log_e \frac{N}{N_0} = -\lambda t$$

$$\text{or} \quad N = N_0 e^{-\lambda t} \qquad \text{.....(iii)}$$

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

This equation shows that the number of atoms of a given radioactives substance decreases exponentially with time (Fig.).



Q.3. What is nuclear reactions? Explain with examples the principles involved in such reactions. What is Q value of a nuclear reaction? (2017)

Other Related Questions -

Q. Define Q value of a neuclear reaction. What do you understand by endoergic and exoergic nuclear reaction?

Q. (2017)

Ans. In nuclear physics and nuclear chemistry, a nuclear reaction is semantically considered to be the process in which two nuclei, or else a nucleus of an atom and a subatomic particle (such as a proton, neutron, or high energy electron) from outside the atom, collide to produce one or more nuclides that are different from the nuclide(s) that began the process. Thus, a nuclear reaction must cause a transformation of at least one nuclide to another. If a nucleus interacts with another nucleus or particle and they then separate without changing the nature of any nuclide, the process is simply referred to as a type of nuclear scattering, rather than a nuclear reaction.

In principle, a reaction can involve more than two particles colliding but because the probability of three or more nuclei to meet at the same time at the same place is much less than for two nuclei, such an event is exceptionally rare (see triple alpha process for an example very close to a three-body nuclear reaction). "Nuclear reaction" is a term implying a induced change in a nuclide and thus it does not apply to any type of radioactive decay (which by definition is a spontaneous process).

Natural nuclear reactions occur in the interaction between cosmic rays and matter, and nuclear reactions can be employed artificially to obtain nuclear energy, at an adjustable rate, on demand. Perhaps the most notable nuclear reactions are the nuclear chain reactions in fissionable materials that produce induced nuclear fission, and the various nuclear fusion reactions of light elements that power the energy production of the Sun and stars.

Nuclear reactions may be shown in a form similar to chemical equa-

Visit us for more detail at : www.winsomepublication.in

tions, for which invariant mass must balance for each side of the equation, and in which transformations of particles must follow certain conservation laws, such as conservation of charge and baryon number (total atomic mass number). An example of this notation follows:

$$^{6}_{3}\mathrm{Li} + {}^{2}_{1}\mathrm{H} \longrightarrow {}^{4}_{2}\mathrm{He} + ?$$

To balance the equation above for mass, charge and mass number, the second nucleus to the right must have atomic number 2 and mass number 4; it is therefore also helium-4. The complete equation therefore reads:

$$^{6}_{3}\mathrm{Li} + {}^{2}_{1}\mathrm{H} \longrightarrow {}^{4}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He}$$

or more simply:

$$^{6}_{3}\mathrm{Li} + {}^{2}_{1}\mathrm{H} \longrightarrow 2\,{}^{4}_{2}\mathrm{He}$$

Instead of using the full equations in the style above, in many situations a compact notation is used to describe nuclear reactions. This style of the form A(b,c)D is equivalent to A + b producing c + D. Common light particles are often abbreviated in this shorthand, typically p for proton, n for neutron, d for deuteron, α representing an alpha particle or helium-4, β for beta particle or electron, γ for gamma photon, etc. The reaction above would be written as Li-6(d,a)a.

In any nuclear reaction, certain quantities must be conserved. We shall merely list the various conservation laws that appear to be valid in ordinary nuclear interactions.

(1) Conservation of energy:-

The total energy of the products, including both mass energy and kinetic energy of the particles plus the energy involved must be equal to the mass energy of the initial ingredients plus the kinetic energy of the bombarding particle.

(2) Conservation of momentum:-

The total linear momentum of the products must be equal to the linear momentum of the bombarding particle (the target nucleus is ordinarily taken to be at rest).

(3) Conservation of angular momentum:-

The total angular momentum *I* comprising the vector sum of the intrinsic angular momentum s and relative orbital momentum *l* of the products must be equal to the total angular momentum of the initial particles.

(4) Conservation of charge:-

The total electric charge of the products must be equal to the total electric charge of initial particles.

(5) Conservation of nucleons:-

The law of conservation of nucleons state that the nucleons can neither be created nor can be destroyed so that the number of nucleons minus the number of anti-nucleons in the universe remain constant.

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

(6) Conservation of spin:-

The important conservation law asserts that the spin character of a closed system can not change, i.e., the statistics remains same as that existed before reaction.

(7) Conservation of parity:-

The parity of the system determined by the target nucleons and bombarding particle must be conserved throughout the reaction. The total parity of the system is the product of intrinsic-parities of the target nucleus and bombarding nucleus. No violation of parity has been observed in a nuclear reaction (strong nuclear forces). Although parity does not appear to be conserved in weak interactions.

(8) Conservation of isotopic spin:-

The invariance of the nuclear Hamiltonian function towards the charge character of the nucleons can be expressed analytically as an invarience towards rotational shifts of the axes in isotopic spin space and there should correspondingly exist a conservation law for the isotopic spin of a nuclear system.

Q value: -

In writing down the reaction equation, in a way analogous to a chemical equation, one may in addition give the reaction energy on the right side:

Target nucleus + projectile $\longrightarrow$ Final nucleus + ejectile + Q.

For the particular case, the reaction energy has already been calculated as Q = 22.2 MeV. Hence:

$$^{6}_{3}Li + {}^{2}_{1}H \longrightarrow 2\,{}^{4}_{2}He + 22.2MeV$$

The reaction energy (the "Q-value") is positive for exothermal reactions and negative for endothermal reactions. On the one hand, it is the difference between the sums of kinetic energies on the final side and on the initial side. But on the other hand, it is also the difference between the nuclear rest masses on the initial side and on the final side (in this way, we have calculated the Q-value above).

Nuclear reactions are divided energetically as

(1) Exoergic reactions:-

These reactions are possible even for $E_z = 0$. Thus for $E_z \to 0$, equation $\sqrt{E_v} = u \pm \sqrt{(u^2 + v)}$ .....(i)

where $u = \dfrac{\sqrt{(m_z m_v E_z)}}{M_Y + m_v}\cos\theta$ and $v = \dfrac{M_Y Q + E_z(M_Y - m_z)}{M_Y + m_v}$

gives $E_v = Q\,M_Y/(m_v + M_Y)$ $(Q > 0)$ .....(ii)

The kinetic energy E_v is the same for all angles θ. Physically, this is because the total momentum is effectively zero when $E_z \to 0$. In this case $Q = E_v + E_Y$ and $\theta + \phi = 180^0$.

Visit us for more detail at : www.winsomepublication.in

Commonly projectile x is lighter than the product nucleus Y. Thus v is +ve for all values of the bombarding energy and only one of the two solutions of eq.(i) will be positive. Thus E_v to be single valued $Q > 0$ and $M_Y > m_z$. E_v depends on $\cos\theta$ and is smallest in the backward direction $\theta = 180^0$.

(2) Endoergic reactions:-

All endoergic reactions have negative Q-values. When $E_z \to 0$, eq.(i) gives $u^2 + v$ = -ve and hence $\sqrt{E_v}$ is imaginary. It means that these reactions are not possible. The smallest value of E_z at which reactions can take place is called the threshold energy. The reaction first becomes possible when E_z is large enough to make $u^2 + v = 0$, hence

$$E_z = -Q\left[\frac{m_v + M_Y}{m_v + M_Y - m_z - (m_z m_v / M_Y)\sin^2\theta}\right]$$

At $\theta = 0$, E_z is minimum and is the threshold energy.

$\therefore \quad (E_z)_{th} = -Q\,(M_Y + m_v)/(M_Y + m_v - m_z)$ .....(iii)

By using relation $(M_x + m_z - M_Y - m_v)c^2 = Q$ and an excellent approximation $M_x >> Q/c^2$, we have

$$(E_z)_{th} = -Q\,(m_z + M_x)/M_x \qquad \text{.....(iv)}$$

Thus we see that at the threshold of the reaction particles first appear in the $\theta = 0$ direction with the K.E.

$$E_v = u^2 = (E_z)_{th}\left(\frac{m_z m_v}{m_v + M_Y}\right)^2 \qquad \text{.....(v)}$$

As the bombarding energy is raised, particle y begins to appear at $\theta > 0$.

Q.4. Give Soddy-Fajan's law of radioactivity. **(2018)**

Ans. Soddy Fajan's displacement law: -

1. In all known radioactive transformation either an α or a β-particle (i.e. never both or more than one of each kind) is emitted by the atom.
2. When a radioactive atom emits an α particle, a new atom is formed whose mass number is less by four units and atomic number less by two units than those of the parent atom.

$$_{92}U^{238} \xrightarrow{-\alpha} {}_{90}Th^{234}$$

3. When a radioactive atom emits a β-particle, the new atom formed has the same mass number but the atomic number is increased by one unit.

$$_{90}Th^{234} \xrightarrow{-\beta^0} {}_{91}pa^{234}$$

Q.5. Define the unit "1 curie" and "1 Rutherford". **(2019)**

Ans. Generally we express the activity of a radioactive substance in terms of curie and submultiples milicurie (mCi) and micro curie (μCi). The curie is

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

defined as the quantity of a radioactive substance which give 3.70×10^{10} diintegrations/second. This is the number of disintegrations per second per gram of radium. Anothre unit rutherford (Rd) is also used. It is defined as the quantity of a radioactive substance which gives 10^6 disintegrations/sec. It is to be noted that for a substance with very short half-life period, very little of the substance is required for 1 curie of activity. On the other hand, for the substance with very long half-life period, a very large quantity of the substance is required for 1 curie of activity.

Q.6. Discuss Bohr's theory of nuclear disintegration and find Q-value equatio for a nuclear reaction. **(2018)**

Ans. Bohr's theory of nuclear disintegration: -

Bohr assumed that the nucleus is somewhat like a liquid drop. When a projectile strikes the nucleus, the projectile is captured, is called a compound nucleus. The energy of the projectile is quickly distributed among the nucleons inside the compound nucleus. Therefore, the compound nucleus resembles a heatedup liquid drop and may be considered to exist in a variety of quasi-stationary excitation states. The compound nucleus persists in its excited state untli a particular nucleon momentarily happens to have a large fraction of the excitation energy to escape from the compound nucleus, thus, leading to a disintegration. This process is similar to the slow evaporation of particles from the surface of a liquid drop.

A nuclear reaction may be represented by

$$a + X \rightarrow Y + b$$

This formula signifies that the particle a interacts with the nucleus X to yiled the nucleus Y and particle b. The above equation is usually abbreviated and is written as

$$X(a, b)Y.$$

For example, ${}_6C^{12}(d, n)_7N^{13}$ stands for a reaction between an incident deuteron ($d = {}_1H^2$) and ${}_6C^{12}$ nucleus to produce a 7N13 nucleus with the emission of a neutron.

The Q-value equation for a nuclear reaction (Neuclear reaction kinematics):-

Let a particle of mass M_1 moving with velocity v_1 collide with a target nucleus of mass M_0 at rest (figure). After collision, the particle O of mass M_3 is emitted with velocity v_3 at an angle θ and therecoiling nucleus P of mass M_2 is emitted with a velocity v_2 at an angle ϕ. The conservation of linear momentum in the plane of the paper yields the equations

$$M_1v_1 = M_3v_3\cos\theta + M_2v_2\cos\phi \quad \text{.....(i)}$$

$$0 = M_3v_3\sin\theta - M_2v_2\sin\phi \quad \text{.....(ii)}$$

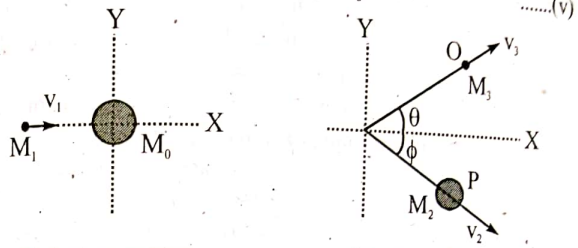
$\therefore$ $M_2v_2\cos\phi = M_1v_1 - M_3v_3\cos\theta$ .....(iii)

$M_2v_2\sin\phi = M_3v_3\sin\theta$ .....(iv)

Squaring and adding, we get

Visit us for more detail at : www.winsomepublication.in

$$M_2^2v_2^2 = M_1^2v_1^2 + M_3^2v_3^2 - 2M_1M_3v_1v_3\cos\theta \quad \text{.....(v)}$$



(a) before collision (b) after collision

Using the kinetic energy relations,

$$E_{k_1} = \frac{1}{2}M_1v_1^2,\ E_{k_2} = \frac{1}{2}M_2v_2^2 \text{ and } E_{k_3} = \frac{1}{2}M_3v_3^2$$

Substituting these values in eq.(v), we have

$$2M_2E_{k_2} = 2M_1E_{k_1} + 2M_3E_{k_3} - 2\left(M_1M_3E_{k_1}E_{k_3}\right)^{1/2}2\cos\theta$$

or $$E_{k_2} = \frac{M_1E_{k_1}}{M_2} + \frac{M_3E_{k_3}}{M_2} - \frac{2}{M_2}\left(M_1M_3E_{k_1}E_{k_3}\right)^{1/2}2\cos\theta \quad \text{.....(vi)}$$

Let $$Q = \left(E_{k_2} + E_{k_3}\right) - E_{k_1} \quad \text{.....(vii)}$$

Substituting the value of E_{k_2} from eq.(vi) in eq.(vii), we have

$$Q = \frac{M_1E_{k_1}}{M_2} + \frac{M_3}{M_2}E_{k_3} - \frac{2}{M_2}\left(M_1M_3E_{k_1}E_{k_3}\right)^{1/2}\cos\theta + E_{k_3} - E_{k_1}$$

or $$Q = E_{k_3}\left(1 + \frac{M_3}{M_2}\right) - E_{k_1}\left(1 + \frac{M_1}{M_2}\right) - \frac{2}{M_2}\left(M_1M_3E_{k_1}E_{k_3}\right)\cos\theta$$

If $\theta = 90^0$, $\cos90^0 = 0$, Hence eq.(viii) reduces to

$$Q = E_{k_3}\left(1 + \frac{M_3}{M_2}\right) - E_{k_1}\left(1 - \frac{M_1}{M_2}\right)$$

Q.7. Discuss the Yukawa Meson theory and explain how does it explain nuclear forces. **(2016)**

Other Related Questions -

Q. Describe the main features of nuclear forces. **(2016)**

Q. Write some of the important properties of nuclear forces. **(2009)**

Ans. Nuclear forces:-

Since stable nuclei exist, it follows that there must be certain forces acting between their nucleons that bind them into the nucleus. These

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

are called nuclear forces. The nuclear force must be strongly attractive, in order to overcome the electrostatic repulsion between protons. Only three kinds of attractive forces can be conceived in the nucleus, viz, neutron-neutron (n-n), neutron-proton (n-p) and proton-proton (p-p) interactions.

Yukawa attributed the following characteristics to the nuclear forces.

(1) Nuclear forces are effective only at short ranges:-

Nuclear forces are appreciable only when the distance between nucleons is of the order of 10^{-15} m or less. The force vanishes for all practical purposes at distances greater than a few times 10^{-15} m. These distances are called the action radii or range of the nuclear forces. In the up-to-date version of the exchange theory of nuclear forces, it is supposed that interaction between nucleons is accomplished by the exchange of π-mesons.

Rest energy of the π-meson = $\Delta E = mc^2$. According to Heisenberg's uncertainty principle, the time required for nucleons to exchange π-mesons cannot exceed Δt, for which $\Delta E \Delta t \geq \hbar$. The distance that a π-meson can travel away from a nucleon in the nucleus during the time Δt, even at a velocity $\approx c$, is

$$R_0 \approx \frac{\hbar}{mc} \approx 1.2 \times 10^{-15}\,m$$

This approximately coincides with the value of the nuclear radius and is of the order of magnitude of the nuclear force range.

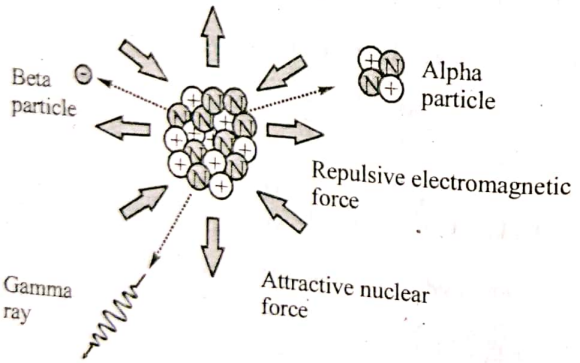


Figure - Nuclear forces

(2) Nuclear forces are charge independent:-

The nuclear forces acting between two protons, or between two neutrons, or between a proton and a neutron, are the same. It follows that nuclear forces are of a non-electric nature.

(3) Nuclear forces are the strongest known forces in nature.

(4) Nuclear forces have saturation property:-

Nuclear forces are limited in range. As a result, each nucleon interacts

Visit us for more detail at : www.winsomepublication.in

with only a limited number of nucleons nearest to it. This effect is referred to as the saturation of nuclear forces.

Meson Theory of Nuclear Forces:-

According to the meson theory of nuclear forces, all nucleons consist of identical cores surrounded by a "cloud" of one or more mesons. Mesons may be neutral or may have a positive or negative charge. The sole difference between neutrons and protons is supposed to lie in the composition of their respective meson clouds. Yukawa assumed that π meson is exchanged between the nucleons and that this exchange is responsible for the nuclear binding forces. The forces that act, between one neutron and another, and between one proton and another, are the result of the exchange of neutral mesons (π^0) between them. The force between a neutron and a proton is the result of the exchange of charged mesons (π^+ and π^-) between them.

Thus a neutron emits a π^- meson and is converted into a proton:

$$n \rightarrow p + \pi^-$$

The absorption of the π^- meson by the proton (with which the neutron was interacting) converts it into a neutron:

$$p + \pi^- \rightarrow n$$

In the reverse process, a proton emits a π^+ meson, becoming a neutron and the neutron, on receiving this π^+ meson, becomes a proton:

$$p \rightarrow n + \pi^+$$

$$n + \pi^+ \rightarrow p$$

Thus in the nucleus of an atom, attractive forces exist between (1) proton and proton (2) proton and neutron and (3) neutron and neutron. These forces of attraction are much larger than the electrostatic force of repulsion between the protons, thus giving a stability to the nucleus.

Just as a photon is a quantum of electromagnetic field, a meson is a quantum of nuclear field. Yukawa considered the equation for particle of mass m as,

$$\left(\nabla^2 - \frac{m^2c^2}{\hbar^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi = 0 \qquad \text{.....(i)}$$

This is a relativistic equation valid only for spinless particles.

Separating the time dependent part, the equation for the radial part is

$$\left(\nabla^2 - \mu^2\right)\phi(r) = 0 \qquad \text{.....(ii)}$$

where $\mu = mc/\hbar$

The solution of Eq. (ii) is $\phi(r) = -g\frac{e^{-\mu r}}{r}$ .....(iii)

...insomepublication.in

Scanned by CamScanner

Here g is a constant, which plays the same role as the charge q in electromagnetic theory. In analogy with electromagnetism, the potential between two nucleons is then given by

$$V(r) = -g^2 \frac{e^{-\mu r}}{r} \qquad \text{.....(iv)}$$

Here g^2 is called the 'Coupling constant'. This argument made Yukawa predict the existence of pion as a quantum of nuclear force field.

The range of the pion field is $\frac{\hbar}{m_\pi c} \approx 1.4\,\text{fm}$.

The form of V(r) given by Eq.(iv) is known as the one-pion-exchange potential (OPEP).

Mass of the meson:-

On the basis of the range of nuclear force and the uncertainty principle, it is possible to estimate mass of the meson. According to uncertainty principle, $\Delta E \times \Delta t = \hbar$.

Here, ΔE and Δt are the uncertainties in energy and time.

The range of nuclear force is $R \approx 1.4 \times 10^{-15}$ m.

Assume that the meson travels between nuclei at approximately the speed of light c. Let Δt be the time interval between the emission of meson from one nucleon and the absorption by the other nucleon.

Then $\Delta t = R/c$. $\therefore \Delta E = \frac{\hbar}{\Delta t} = \frac{\hbar c}{R}$

$\therefore$ The minimum meson mass is specified by $m \geq \frac{\hbar}{Rc}$.

In terms of the electronic mass m_e, the mass of the meson is

$$\frac{m}{m_e} = \frac{\hbar}{m_e Rc} = \frac{1.054 \times 10^{-34}}{(9.108 \times 10^{-31})(1.4 \times 10^{-15})(3 \times 10^8)} = 275$$

i.e. mass of the meson $\approx 275\times$ mass of electron.

In 1947, Powell discovered π meson of mass about 273 m_e. This particle showed strong interaction with nucleons and was recognized as the Yukawa particle.

Merits:-

The discovery of the meson of mass of about 273 electron mass and the existence of positive, negative and neutral meson, lends some support to this theory.

- The experimental values of the magnetic moments of a free proton and of a free neutron also lend some support to the "Yukawa's meson field theory" of

Visit us for more

nuclear forces.

A free proton is, for a part of its life-time, a neutron with a closely bound meson. Hence the magnetic moment of a free proton can be the resultant of the true magnetic moment of the proton and the magnetic moment of the meson. Thus the net magnetic moment of a free proton will exceed that given by the simple theory.

Similarly, a neutron is for a fraction of its life-time dissociated into a proton and a negative meson. This combination will have a negative magnetic moment. It follows that, though uncharged, a neutron will have a negative magnetic moment.

Q.8. Describe the basic principle of gas detectors used for the detection of nuclear radiation. **(2016, 2009)**

Other related questions:-

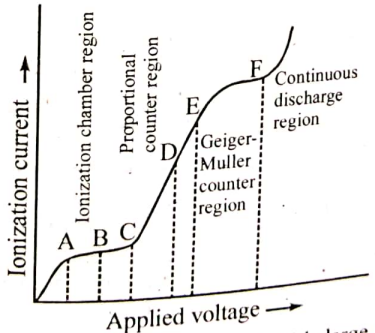
Q. Describe in brief the principle of proportional counter.

Ans. Principle:- When the potential difference across the two electrodes of an ionisation chamber is increased beyond the saturation region, the ions gain sufficient energies to produce secondary ionisation due to collision with gas atoms in the chamber. This results in greater ionisation current. In a certain range of applied voltages (Fig.), the pulse size is proportional to the original number of ion pairs, and the device is called a proportional counter.

The main regions used for measurements are:

(1) The ionization chamber region AB.
(2) The proportional counter region CD.
(3) The Geiger-Muller region EF.

After the point F, the tube becomes a simple discharge tube in which the current is produced even after the ionization event has ceased.



- Since the central wire is very thin and the p.d. fairly large, the electric field E = dV/dr at a distance r from the centre is very high. If b is the radius of the cylinder and a the radius of the wire, the radial field E at a distance r from the centre is giv

...tion.in

Scanned by CamScanner

$$E = \frac{V}{r \log_e (b/a)}.$$

Here V is the positive voltage of the central wire relative to the outer cylinder. Thus, in a proportional counter, the field strength near the wire is very great. Hence electrons travelling towards the wire are rapidly accelerated when near it, and produce additional electrons in that region due to the phenomena of ionization by collision. This process is called the gas multiplication.

Q.9. Write the short note on GM counter. **(2013, 17)**

Other Related Questions -

Q. Discuss briefly the construction and working of a GM detector can it measure the entry of radiation. **(2009)**

Ans. Geiger-Muller counter:-

Construction:- It consists of a metal chamber C containing air or some other gas at a pressure of about 10 cm of Hg (Fig.a).

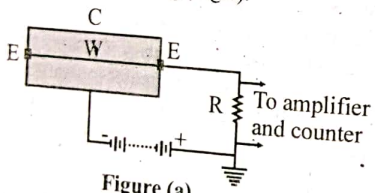


Figure (a)

- A fine tungsten wire (W) is stretched along the axis of the tube and is insulated from it by ebonite plugs EE.
- The wire is connected to the positive terminal of a high tension battery (about 1000 to 3000 volts) through a high resistance R (about 100 megohms) and the negative terminal is connected to the chamber C. The D.C. Voltage is kept slightly less than that which will cause a discharge between the electrodes.

Working:-

When an ionizing particle (say an α particle) enters the counter, ionisation takes place and a few ions are produced.

- If the applied P.D. is strong enough, these ions are multiplied by further collisions. An avalanche of electrons moves towards the central wire. This is equivalent to a small current impulse which flows through the resistance R. The critical potential is lowered momentarily, causing a sudden discharge through the resistance R. The P.D. thus developed across R is amplified by vacuum tube circuits and is made to operate a mechanical counter. In this way, single particles can be registered.
- The sudden pulse of discharge sweeps away the ions from the chamber and the counter is ready to register the arrival of the next particle.

Fig.(b) shows the voltage characteristics of a G.M counter. This is a plot of

Visit us for more detail at : www.winsomepublication.in

the counting rate against the counter potential with a radioactive source placed near the counter.

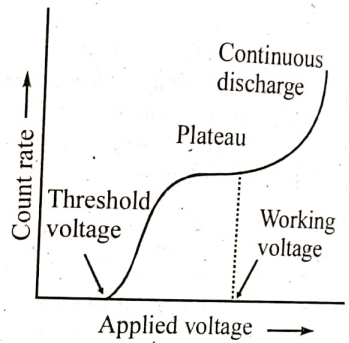


Figure (b)

- There is a threshold below which the tube does not work. This can be several hundred volts.
- As the applied potential is increased, the counting begins and rises rapidly to a flat portion of the curve called the plateau. This is the region of the counter operation. Here the counting rate is, more or less, independent of small changes in p.d. across the tube.
- Beyond the plateau, the applied electric field is so high that a continuous discharge takes place in the tube. The count rate increases very rapidly. It does not require any ionizing event for this to happen. So the tube must not be used in this region.

Efficiency of the counter:-

The efficiency of the counter is defined as the ratio of the observed counts/sec. to the number of ionizing particles entering the counter per second. Counting efficiency is defined as the ability of its counting, if at least one ion pair is produced in it.

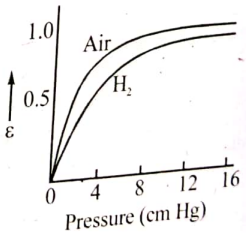


Figure (c)

Counting efficiency $= \varepsilon = 1 - e^{-sp}$

- s = specific ionization at one atmosphere;
- p = pressure in atmospheres;
- l = path length of the ionization particle in the counter.

Fig.(c) shows the efficiency ε of a GM counter, as a function of pressure for air and hydrogen.

Quenching in a GM counter:-

The process of removing all the ions from the chamber due to continuous discharge and making it ready for fresh event is called quenching. This is achieved by using quenching agents like alcohol or halogens.

Quenching Process:-

When a G.M. tube operates in Geiger region, the secondary electrons

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

increase the current pulse by further ionisation of gas molecules. The object of counter is to produce a single pulse due to entry of a single particle. The tube should not then give any succeeding spurious pulses but should recover as quickly as possible to be in the state to record the next entering particle. Therefore, it is desirable to prohibit the production of spurious pulses following the main required pulse due to a single particle entry. The process of prohibiting the undesirable secondary pulses is called the quenching.

Self quenching of GM-counter:-

A typical GM-counter contains 90% Ar and 10% ethyl alcohol by weight. The ionisation potential of alcohol (11.3 eV) is lower than that of argon (15.7 volt). As a result, the inert gas ions get neutralized by transferring their charge to alcohol ions during their long path towards cathode. The alcohol ions produced, capture electrons from the cathode and are neutralized. Hence there is no multiple pulsing and the discharge is quenched soon (fraction of a milli-second) after the initial ionisation.

Resolving time of a GM-counter:-

Fig.(d) gives the three time-intervals- dead time, recovery time and resolving time- of operation of a GM-counter.

(i) Dead time:-

The time, during which the counter is incapable of responding to a second ionizing event, is known as dead time (~200 μ sec). It is called dead time because during this period the counter is insensitive (dead) to further pulses. After the initial ionisation, the cloud of slow moving massive positive ions formed over the anode wire effectively lowers its potential relative to cathode. Thus the G.M.counter is dead for about 200 μs after each ionising event.

(ii) Recovery time:-

As the positive ion-sheath moves further toward the cathode, the counter sensitivity gradually returns to its original value and the second pulse can now be recorded. This time, during which pulses of reduced size are produced, is known as recovery time.

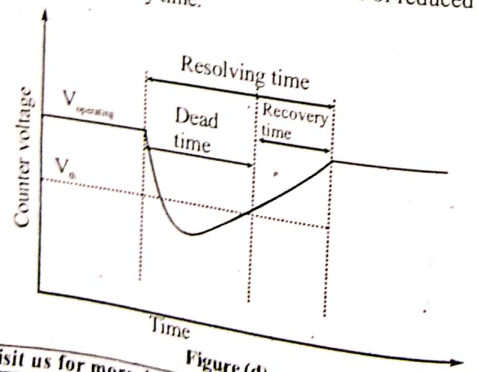


Figure (d)

Visit us for more detail at : www.winsomepublication.in

(iii) Resolving time:-

The dead time plus recovery time corresponds to the time which positive ion sheath takes to reach the cathode, and is known as the resolving time of the counter. This is the time between just recorded pulses.

Expression for resolving time:-

Assume that a counter with a resolving time τ responds at a rate n counts per unit time when exposed to N particles per unit time. In unit time, the total insensitive time is nτ. The number of counts missed is Nnτ.

The number of counts missed is the error in counting.

$$N n\tau = N - n \qquad \text{.....(1)}$$

$$\therefore \quad N = n/(1 - n\tau) \qquad \text{.....(2)}$$

From Eq. (2), we can find the true counting rate N from the observed counting rate n, if τ is known.

Determination of τ :-

By measuring separately the counting rates n_1' and n_2' with two radioactive sources 1 and 2 of nearly equal strength, τ can be determined. Let n_t' denote the total counting rate when both sources are placed in the same position as before and n_b be the background rate (i.e. with no source).

$$\therefore \quad n_1 = n_1' - n_b; \qquad n_2 = n_2' - n_b; \qquad n_t = n_t' - n_b \qquad \text{.....(3)}$$

Here n_1, n_2, n_t are the above counting rates corrected for background. If N_1, N_2 and N_t be the true rates of arrival of particles in the counter in the above three situations, we have, using Eq. (2),

$$N_1 = \frac{n_1}{1 - n_1\tau}; \qquad N_2 = \frac{n_2}{1 - n_2\tau}; \qquad N_t = \frac{n_t}{1 - n_t\tau} \qquad \text{.....(4)}$$

Also, $N_t = N_1 + N_2$ .....(5)

Assuming the values of terms involving τ^2 to be negligible, we have, solving for τ

$$\tau = \frac{n_1 + n_2 - n_t}{2n_1 n_2} \qquad \text{.....(6)}$$

Advantages:-

(1) The counter set-up is portable (with the transistorised electronics) and serves best for mineral prospecting, apart from its several other applications in cosmic ray work.

(2) The pulse height is constant over a range of applied voltages, as in Fig.b. So the power supply does not have to be precisely regulated as it does for a proportional counter. Also, the pulses are several volts in height, which makes amplifiers unnecessary.

Disadvantages:-

(1) It is insensitive for a period of 200 to 400 μs following each pulse. This prevents its use at very high counting rates.

(2) It cannot provide information about the particle or photon causing a pulse.

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

Q.10. Explain the principle of scintillation counter. Mention its supermacy over the other counters. (2014)

Other Related Questions -

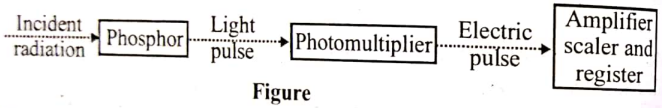
Q. Discuss the principle and working of a Scintillation counter. What are the advantages of this counter in comparison with other counter. (2011)

Q. Write short note on Scintillation counter. (2018)

Ans. Scintillation counter:-

Principle:-

When high energy radiation falls on a fluorescent material, it produces flashes of light called 'scintillations'. The energy of the particle is converted to light.



Figure

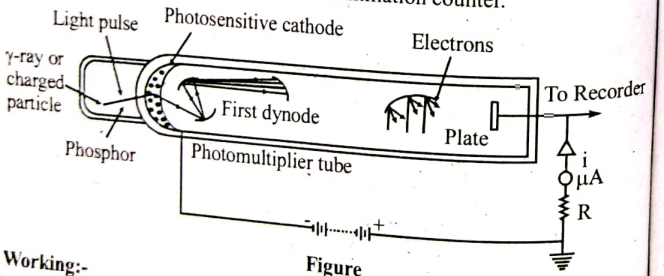
Fig.(a) shows the main parts of scintillation counter.

The atoms of the phosphor are excited or ionised by the energy loss of an impinging α, β or γ ray. When the atoms return to their ground states, photons are emitted, in the blue and ultraviolet regions of the optical spectrum.

The phosphor is optically coupled to the envelope of a photomultiplier tube. The output pulse from the photomultiplier is fed to a pulse amplifier followed by a scaler circuit.

Construction:-

Fig.(b) shows the diagram of a scintillation counter.



Figure

Working:-

(i) The incident radiation is absorbed by the 'phosphor' (or scintillator). The atoms (or molecules) are excited.

(ii) When the atoms (or molecules) return to their ground states, photons are emitted.

(iii) The phosphor is optically coupled to the envelope of a photomultiplier tube. The photons strike the photocathode, causing the ejection of photoelectrons.

Visit us for more detail at : www.winsomepublication.in

(iv) As these photo-electrons leave the photocathode, they are directed by a focusing electrode to the first multiplier electrode or dynode. This electrode has the property of emitting three, four or five electrons for every single electron which strikes its surface. There may be from 10 to 14 such multiplier stages in a given tube. Hence from the emission of one single electron from the cathode, a burst of one million electrons may impinge on the final stage in the tube (the anode).

If m is the electron multiplication factor of a single dynode stage and there are n stages, then total amplification is m^n.

(v) The output pulse from the photomultiplier is fed to a pulse amplifier followed by a scaler circuit.

The following scintillators are generally used for the detection of particles and photons.

Sodium Iodide (Thallium Activated). This is the most commonly used phosphor and is largely used in the study of γ-rays.

Sodium iodide crystals are usually doped with a small amount of thallium and denoted by NaI(Tl). The Tl atoms act as luminescence centres. NaI(Tl) crystals are dense enough to have good efficiency of γ-ray detection.

Table:- Common scintillators and their uses-

		Name	Form	Wavelength of Maximum emission	General remarks
1		**Inorganic scintillators**			
	(a)	NaI (Tl)	Crystal	4100 Å	Mainly used to detect γ-rays, hygroscopic for γ - rays
	(b)	Cs(Tl)	Crystal	White	For γ -rays nonhygroscopic
	(c)	ZnS(Ag)	Powder	4500 Å	For α-rays
	(d)	LiI(Eu)	Crystal	-	Used for neutron detection
2		**Organic crystal**			
	(a)	Anthracene	Crystal	4470 Å	For β detection
	(b)	Stibene	Crystal	4100 Å	For β detection because it gives higher yield of photons [Anthracene is better]

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

3		Organic sulphide			
	(a)	p-terphenyl in toluence	Solution	4500 Å	For β -particle
	(b)	p-terphenyl in polysterene	Plastic	4450 Å	For β -particle
4		Xe – gas	Gas	Ultraviolet	For γ -rays

Advantages: -

(1) The height of the pulse is proportional to the energy of the incident γ-radiations.

(2) In a solid scintillator, the number of atoms per unit volume is more. Therefore a stronger pulse is observed in comparison to a gas scintillator.

(3) The dead time of the counter is very small and hence a high counting rate is possible.

(4) The pulse duration is extremely short and so the resolving power is high.

(5) If a large scintillator is used, the scattered γ-rays are also taken into account thereby giving rise to an increased photo peak efficiency.

Disadvantages: -

(1) The light output decreases with increase in energy of radiation. It also varies with the type of radiation.

(2) The overall efficiency is low because of the complex nature of phenomena-namely the conversion of radiation into light and then re-conversion into electrons.

Q.11. Write short note on elementary particles. (2017)

Other Related Questions -

Q. Write short note on elementary particles and their classification. (2013)

Q. Write short note on elementary particles. (2012, 09)

Ans. The typical photo-type of the elementary particle was the electron. The problem of the dualistic nature of matter was resolved by the quantum theory of fields: the elementary particles nothing but the quanta of a corresponding field.

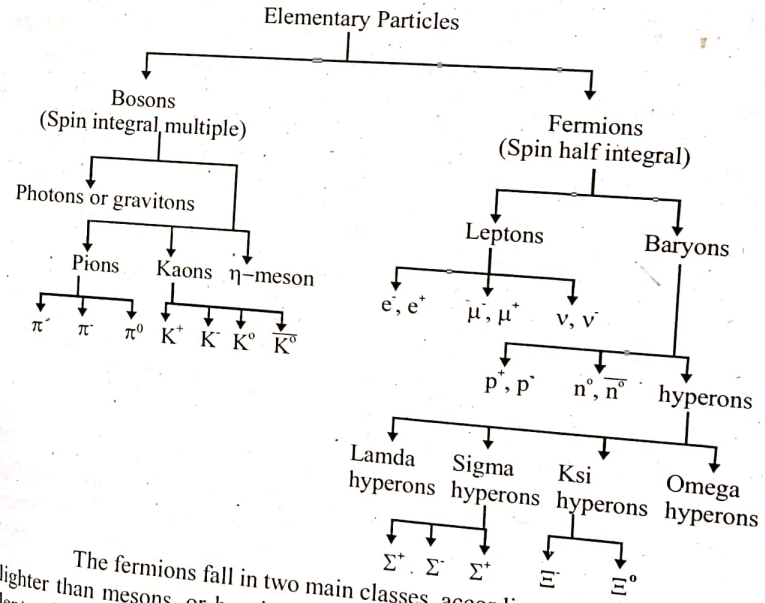
Classification of elementary particles:-

The elementary particles are separated into two general groups, called bosons and fermions. These two groups have different types of spin and their behaviour is controlled respectively by a different kind of statistic (i.e. the Bose statistic or the Fermi statistic, hence the names). Bosons are particles with intrinsic angular momentum equal to an integral multiple of $\hbar$. Fermions are all those particles in which the spin is half integral. The most important difference between the two classes of particles is that there is no conservation

Visit us for more detail at : www.winsomepublication.in

law controlling the total number of bosons in the Universe, whereas the total number of fermions is strictly conserved.

Boson is a term, which not only includes material particles but also includes those quanta and photons which arise from interactions. Thus in the case of the simple electromagnetic field the bosons are merely the light photons or the X-ray photons. The photon has a mass of zero and a spin of unity and is consequently described as a massless boson. A massless boson, called a graviton with a probable spin of two units has been postulated as a field particle for gravity. These bosons, created by the electromagnetic field, are essentially of one kind, while the bosons formed in the strong interaction are of two distinct kinds. First there are those which are konown as pions or π-mesons (π^+, π^- and π^0). The second group of bosons are much heavier than that of pions, and are known as kaons or K-mesons (K^+, K^-, K^0).



The fermions fall in two main classes, according to whether they are lighter than mesons, or heavier. Those in the lighter group are often called leptons (after the Greek word meaning light in weight), whilst those in the heavier group are called baryons (after the Greek word for heavy). The leptons are the electrons, muons and neutrions and their anti-particles. These are all with masses less than the pions and with spin half. Leptons interact weakly with other particles. The total number of leptons minus the total number of anti-leptons remains unchanged in all reactions and decay processes involving leptons and anti-leptons. The baryons consist of the two nucleons with

Visit us for more detail

Scanned by CamScanner

their anti-particles (n^0, $\overline{n^0}$; p^+, p^-) and the hyperons. Hyperons are the extremely unstable somewhat heavier particles and can be divided into four subgroups, Λ^0- particle (a neutral particle of mass about 180 m_e), the Σ-particles (Σ^-, Σ^0 and Σ^+ with masses in the range 2320 to 2340 m_e), the Ξ-particles (Ξ^- and Ξ^0 with masses near 2580 m_e) and the Ω^--particle (of mass about 3284 m_e). There is no reason to doubt the existence of the anti-particles of these fermions. The total number of baryons minus the total number of anti baryons is absolutely conserved in all interactions.

The kaons and pions together with the baryons are placed into a group of strongly interacting particles, called hadrons.

Q.12. Name different types of interaction between elementary particles. Write the conservation laws for strong interaction. (2015)

Other Related Question -

Q. Write short note on conservation laws. (2012)

Q. State the conservation laws of elementary particles. (2019)

Ans. The fundamental interaction:-

Four kinds of interaction between elementary particles can account for all known processes in the physical universe on all scales of size. In Table the four basic interactions are summarised.

Interaction	Particles Affected	Range	Relative Strength	Particles Exchanged
Strong	Hadrons	$\sim 10^{-15}$ m	1	Mesons
Electromagnetic	Charged Particles	∞	$\sim 10^{-2}$	Photons
Weak	Hadrons and leptons	$\sim 10^{-17}$ m	$\sim 10^{-13}$	Intermediate Bosons
Gravitational	All	∞	$\sim 10^{-40}$	Gravitons

(1) Strong interaction:-

A familiar example of strong interaction is the forces which hold nucleons together (nuclear forces) in the atomic nucleus. The strong nuclear interaction is independent of the electric charge. The range of these interactions is about 10^{-15} m. Time interval of such an interaction is roughly 10^{-23} s.

(2) Electromagnetic interaction:-

It operates on all charged particles. Thus electromagnetic interactions are charge dependent (attractive as well as repulsive). The range is infinite and the interaction works through the photon. The formation of electron-positron pair from γ-ray is an example of electromagnetic interaction.

(3) Weak interaction:-

All strong interactions take place in times of about 10^{-23} s (characteristic time). Yet it has been observed that some of the resulting particles, although energetically unstable, suffer no decay until a time 10^{13} times greater

Visit us for more detail at : www.winsomepublication.in

than 10^{-23} is reached. That is, their decay takes place in time of about 10^{-10} s. For example, β-decay of radioactive nuclei does not take place until a time 10^{13} times greater than that involved in strong interactions has approached. Had there been strong nuclear or electromagnetic interactions, there would have been no such delay in the decay process. Therefore, this delay in the decay process suggests that either these particles are not subjected to strong interacting forces or there is some new conservation law or prohibition which forbids the decay. But since most of the particles involved are subjected to either nuclear force or have electric charge or both, there must be some rule which stops the process. But eventually the decays do happen, there must be some other type of interaction as predicted by Fermi to explain β-decay. Since particles take long time to respond to such an interaction, force involved must be very weak compared with strong nuclear forces. The range of such an interaction is less than 10^{-17} m. The characteristic time of this interaction is $\approx 10^{-10}$ s. Yukawa, in 1938, suggested that there should be a field quantum for the weak interaction, corresponding to the photon and pion. This so called intermediate vector boson has not been experimentally detected as a free particle. The weak interaction is responsible for the decay of strange and non-strange particles and for non-leptonic decays of strange particles.

(4) Gravitational interaction:-

It is the weakest of the four types of interactions. It has infinite range. Although gravity has a measurable influence on macroscopic bodies, its interaction with subnuclear particles is very small. Gravitation can be explained in terms of the interactions of 'gravitons'. Their mass must be zero, and therefore, their velocity must be that of light. As the gravitational field is extremely weak, the gravitons can not be detected in the laboratory.

Of the four basic forces, only gravitational force is universal. Weak forces affect every particle other than photon. Electromagnetic force is confined to charged particles. Strong forces of the most selective and they serve as the criteria for classifying all known particles other than photons into two broad categories, the leptons and the hadrons. Leptons and photons are light particles and do not feel the strong forces. But hadrons feel the strong forces and participate in strong interactions besides taking part in other types of interactions as well. Thus the proton is a strongly interacting nuclear constituent and is therefore, a hadron. At the same time, because of its charge and mass, it must also experience electromagnetic and gravitational forces. The fact that it can be created by β-decay of neutron shows that the proton is involved in weak interactions as well.

Conservation laws and symmetry:-

A very important set of conservation laws is related to symmetries involving parity (P), charge conjugation (C), and time reversal (T).

Conservation of parity:-

Parity relates to the symmetry of the wave function that represents

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

the system. If the wave function is unchanged, when the coordinates (x, y, z) are replaced by (-x, -y, -z), then the system has a parity of +1. If the wave function has its sign changed, when the coordinates are reversed, then the system has a parity of -1. If we write $\psi(x, y, z) = P\psi(-x, -y, -z)$, we can regard P as a quantum number characterizing ψ whose possible values are +1 and -1. During a reaction in which parity is conserved, the total parity number does not change.

Changing the coordinates (x, y, z) into (-x, -y, -z) converts a right-handed coordinate system into a left-handed coordinate system. In terms of symmetry, the meaning of conservation of parity is that in any situation where parity is conserved, the description of the reaction will not be changed if the word "left" is changed to the word "right" and vice versa. This means that such reactions can provide no clue that will distinguish between the directions right and left. Prior to 1956 it was believed that all reactions in nature obeyed the law of conservation of parity. However, Yang and Lee pointed out that in reactions involving the weak interaction, parity was not conserved, and that experiments could be devised that would absolutely distinguish between right and left. Indeed parity conservation is found to hold true only in the strong and electromagnetic interactions.

Charge conjugation symmetry:-

Charge conjugation is the act of symmetry operation in which every particle in a system is replaced by its antiparticle. If the anti-system, or antimatter counterpart exhibits the same physical phenomena, then charge parity (C) is conserved. For example, if in a hydrogen atom, the proton is replaced by an antiproton and the electron is replaced by a positron, then this antimatter atom will behave exactly like an ordinary atom, if observed by people also made of antimatter. In fact C is not conserved in the weak interaction.

Time reversal symmetry :-

Time parity T describes the behaviour of a wave function when t is replaced by -t. The symmetry operation that corresponds to the conservation of time parity is time reversal. Time reversal symmetry implies that the direction of time is not significant, so that the reverse of any process that can occur is also a process that can occur. In other words, if symmetry under time reversal holds, it is impossible to establish by viewing it whether a motion picture of an event is being run forward or backward. Prior to 1964, time parity T was considered to be conserved in every interaction. It was discovered in 1964 that one form of the K^0 kaon can decay into $\pi^+ + \pi^-$, which violates the conservation of T. The symmetry of phenomena under time reversal thus does not seem to be universal.

Combined inversion of CPT:-

The combined symmetry operation in which the anti-matter mirror-image of a system is run in reverse allows a test of CPT invariance. All the evidence supports the conservation of CPT. The conservation of CPT means that for every process there is an antimattter mirror-image counterpart that

Visit us for more detail at : www.winsomepublication.in

takes place in reverse. This particular symmetry seems to hold for all interactions, even though its component symmetries sometimes fail individually.

Q.13. Write short note on liquid drop model. **(2017)**

Ans. A nuclear model usually associated with the semi-empirical mass formula was suggested by Bohr in 1937. In this model the finer features of nuclear forces are ignored but the strong internucleon attraction is stressed. The essential assumptions are ; (1) The nucleus consists of incompressible matter so that $R = R_0 A^{1/3}$; (2) The nuclear force is identical for every nucleons ; (3) The nuclear force saturates. Thus one might inquire whether a nucleus can be represented as a crystalline aggregate of nucleons. But it gives that the zero point vibrations of the nucleons about their mean rest positions would be too voilent for stability. Hence the individual nucleons must be able to move about within the nucleus much as does an atom of a liquid and one might, therefore, think of a nucleus as being like a small drop of liquid. Such a model is thus known as liquid drop model.

The idea that the molecules in the drop of liquid correspond to the nucleons in the nucleus is confirmed due to following similarities: (1) The nuclear forces are analogous to the surface tension force of a liquid ; (2) The nucleons behave in a manner similar to that of molecules in a liquid ; (3) The fact that the density of nuclear matter is almost independent of A shows resemblance to liquid drop where the density of a liquid is independent of the size of the drop ; (4) The constant binding energy per nucleon is analogous to the latent heat of vaporisation ; (5) The disintegration of nuclei by the emission of particles is analogous to the evaporisation of molecules from the surface of liquid ; (6) The energy of nuclei corresponds to internal thermal vibrations of drop molecules ; (7) The formation of compound nucleus and absorption of bombarding particles are correspond to the condensation of drops.

Inspite of these similarities, there are following differences : (1) Molecules attract one another at distances larger than the dimensions of the electron shells and repel strongly when the distance is smaller than the size of the electron orbits. Nuclear forces are attractive within the smaller range, the range of nuclear forces. (2) The average K.E. of the molecules in the liquid is of the order of 0.1 eV, the corresponding de Broglie wavelength is 5×10^{-11}m which is very much smaller than the intermolecular distances. The average K.E. of nucleons in nuclei is of the order of 10 MeV, the corresponding $\lambda \simeq 6 \times 10^{-15}$m, which is of the order of inter-nucleon distances. Hence the motion of the molecules in the liquid is of classical character whereas in nuclei the motion of the nucleons is of quantum character.

Semi-empirical formula gives no information about any other properties of nuclei than their energies and the Z/A ratio. In estimating the properties of the excited states of nuclei, one has to consider deformation of a spherical drop giving rise to periodic oscillations of the surface.

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

A spherical drop (a nucleus) of radius R_0 is deformed. If $R(\theta, \phi)$ be the distance of the deformed surface from the centre at an angle θ, ϕ, the difference can be expressed as

$$q(\theta,\phi) = R(\theta,\phi) - R_0 = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} q_{lm} Y_{lm}(\theta,\phi) \quad \text{.....(i)}$$

To make the problem simpler, let us consider cylindrically symmetric deformations for which $m = 0$ and $q_{l,0}$ oscillate harmonically in time as

$$q_{l,0} = q_l \cos(\omega_l t) \quad \text{.....(ii)}$$

The characteristic frequency ω_l is determined by the dynamics of the vibration. The surface tension opposes the surface deformation. Since the liquid is incompressible, the waves at the surface implies motion within the liquid. The wavelength of surface vibrations of a liquid drop is given by the relation * = R/l. The mass μ participating in the vibration is equal to the mass of the outer shell of thickness *, given by

$$\mu = M(3*/R) \quad \text{.....(iii)}$$

The kinetic energy in the vibration $T = 1/2\mu q_l^2$. .....(iv)

Due to the wave of amplitude q_l and wavelength *, the plane surface S increases by an amount

$$\Delta S = 1/2\,(q_l/*)^2\, S \quad \text{.....(v)}$$

The change in surface energy of the drop $\Delta S\alpha$ is equal to the potential energy. Hence we get

$$\Delta E_s = 1/2\,(q_l/*)^2\, S\alpha = 1/2\, Kq_l^2 \quad \text{.....(vi)}$$

From relation (iv) and (vi), we get the frequency

$$\omega_l = (K/\mu)^{1/2} = (4\pi\alpha l^3/3M)^{1/2} \quad \text{.....(vii)}$$

The specific properties of the spherical surface change the l^3 into $l(l-1)(l+2)$. The surface tension coefficient α can be calculated in terms of surface energy $E_s = a_s A^{2/3}$ as

$$4\pi R^2\alpha = E_s = a_s A^{2/3} \text{ or } \alpha = a_s/4\pi R_0^2 \quad \text{.....(viii)}$$

Hence relation (vii) becomes

$$\omega_l = [l(l-1)(l+2)(a_s/3R_0^2 MA)]^{1/2} \quad \text{.....(ix)}$$

This gives us too high value of the excitation energy. The frequency ω_l is reduced by the Coulomb effect, as

$$\omega_l = \left[l(l-1)\left\{(l+2) - \frac{10\gamma}{2l+1}\right\}\frac{a_s}{3r_0^2 MA}\right]^{1/2} \quad \text{.....(x)}$$

where γ is the ratio between the Coulomb energy $E_c = \frac{3}{5}\frac{Z^2e^2}{4\pi\varepsilon_0 R}$ and the surface tension energy $E_s = 4\pi\alpha R^2 = a_s A^{2/3}$. This equation leads to somewhat smaller frequencies for heavier nuclei, but is insufficient to represent the actual level

Visit us for more detail at : www.winsomepublication.in

distances.

Relation (x) shows that the frequency becomes imaginary when γ is larger than a certain limiting value γ_c. Its smallest value is 2 for $l = 2$. Hence condition for stability against surface deformation is

$$\gamma_c (= E_c/E_s = 0.0474\, Z^2/A) < 2 \text{ or } Z^2/A < 42.2 \quad \text{.....(xi)}$$

We therefore, expect in this model that the nuclei near the limit of Z^2/A split into two parts by the additional supply of small amounts of energy.

Results:-

The liquid drop model not only gives atomic masses and binding energies accurately, but also predicts α and β emission properties. The binding energy formula does not include closed shell effects, but it can be used to provide a base line from which shell effects can be calculated. This model is able to explain certain features of nuclear fission, but is not very successful in describing the actual excited states as it gives too large level distances. This model is the forerunner of the collective model of nuclear structure. It forms the basis of Bohr's theory of the compound nucleus formation in nuclear reactions.

Q.14. Define the radioactive tracers. **(2016)**

Other Related Questions -

Q. What do you understand by radioactive tracers? Describe some of its applications.

Ans. A radioactive tracer or radioactive label, is a chemical compound in which one or more atoms have been replaced by a radionuclide so by virtue of its radioactive decay it can be used to explore the mechanism of chemical reactions by tracing the path that the radioisotope follows from reactants to products. Radiolabeling is thus the radioactive form of isotopic labeling.

Radioisotopes of hydrogen, carbon, phosphorus, sulphur, and iodine have been used extensively to trace the path of biochemical reactions. A radioactive tracer can also be used to track the distribution of a substance within a natural system such as a cell or tissue, or as a flow tracer to track fluid flow. Radioactive tracers are also used to determine the location of fractures created by hydraulic fracturing in natural gas production. Radioactive tracers form the basis of a variety of imaging systems, such as, PET scans, SPECT scans and technetium scans. Radiocarbon dating uses the naturally occurring carbon-14 isotope as an isotopic label.

Q.15. Define activity of a radioactive substance. Find activity law. **(2016)**

Other Related Questions -

Q. Define activity of the radioactive nucleus. **(2013)**

Q. Define activity of a radioactive substance. Find the radioactivity decay law. **(2019)**

Ans. The activity of a radioactive substance is defined as the average number of atoms disintegrating per unit time.

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

An activity of one decay per second is one Becquerel (1 Bq)
Activity A is directly proportional to the number of parent nuclei N present at that instant:

$$A \propto N \qquad \Rightarrow \qquad A = -\frac{dN}{dt} = \lambda N$$

where N is number of parent nuclei, t is the time, λ is the decay constant.

Q.16. Write three important properties of nuclear force? (2014)

Ans. Since nuclei contain charged particles, protons, the forces acting between the protons should be forces of repulsion. If there were no other forces acting between the particles of nuclei, the latter would not be able to retain a stable state, because the repelling protons would fly out in all directions. To have a stable state of a nucleus we have to assume that some other forces act on the protons. These attractive forces exceed the forces of repulsion. Even aside from the sign of the force, electrostatic forces are much too weak to account for the main effects, gravitational forces are weaker still and offer no assistance in the problem. It is therefore necessary to postulate an entirely new type of interaction, known as nuclear force. Let us collect following information on nuclear forces from study of complex nuclei.

(a) Saturation property:-

We know that the nuclei have the same density ($R = R_0 A^{1/3}$) and the binding energy per nucleon for nuclei with $A > 40$ is constant. These facts imply that the nuclear force saturates. Nucleons attract each other strongly only if they are in the same orbital state. According to the Pauli principle only two neutrons and two protons will be found in the same orbital state. Therefore it is possible to find four nucleons strongly bound or α-particle structure, also confirmed by binding energy curve.

(b) Charge independence:-

We know that the atomic mass number A is approximately equal to twice the atomic number Z for the light and intermediate nuclei. It shows that light nuclei prefer to add nucleons in n-p pairs, i.e. there is a strong interaction between neutrons and protons. The neutron excess in heavy nuclei confirms that n-n force is attractive, but it is sufficiently attractive to lead to a stable dineutron. For a good approximation one can write n-n $\simeq$ p-p $\simeq$ n-p or the forces as charge independent.

(c) Shape of the potential:-

As the nuclear interactions do not extend to very large distances beyond the nuclear radius, and $1/r^n$ ($r >> 1$) character is not useful to solve the problem. To avoid this difficulty, the idea of cut-off in the potential is introduced. The parameters are taken to be the strength V_0 (the value of potential at the origin) and the range α (the distance beyond which potential goes to zero rapidly). It is less than nuclear dimensions.

Visit us for more detail at : www.winsomepublication.in

Q.17. What are magic nuclei? Why are they most stable. (2015)

Ans. Magic nuclei are nuclei in which either the number of protons or the number of neutrons corresponds to a filled shell. The properties of magic nuclei are rather different from those of other nuclei.

It is now generally believed that the number 50, 82, 126 and 152 are magic numbers. Other magic numbers which are encountered in the literature refer either to the region of extremely light nuclei, in which these numbers have little significance, or to numbers whose existence is open to doubt.

The best example of a magic nucleus is the lead isotope of atomic weight 208. This nucleus, so to speak, is a doubly magic nucleus since it contains a magic number of protons (82) and a magic number of neutrons (126). Pb^{208} exhibits properties which are characteristic of a closed system. Whereas the first excited levels in ordinary heavy nuclei are at energies below 200 kev, the excitation of Pb^{208} requires an energy of 2.6 Mev. Furthermore, an anomalously small energy is required for adding a nucleon. Neutron capture is usually accompanied by the excitation of the resulting nucleus to about 7 Mev; neutron capture in Pb^{208} leads to an excitation of only 4 Mev.

Probably the best way of determining the magic nuclei is an investigation of the energy required for adding nuclei to α-particles (i.e., the energy given off in capture of α-particles by a nucleus). The dependence of the affinity energy for α-particles on the number of nucleons in the nucleus is found to be a more reliable characteristic than the energy required for adding a neutron or proton, which depends on the number of particles in an irregular way, changing discontinuously in going from even to odd elements. Studies of the systematics of α-active elements and reactions involving the emission of α-particles are extremely useful in this connection.

It is unfortunate that, having established the magic number, there is still very little we can say regarding the nucleon states. Many more experiments will be required before the pattern of states for heavy nuclei becomes in any way complete. The level scheme and the order of filling which frequently appears in the literature is based on a simplified model of the potential well and an idealized filling system which, as is apparent from the light-nucleus examples, does not correspond to reality.

Moreover, experiment indicates that in heavy nuclei the model of a particle moving in a central field requires essential modification because the original assumption as to spherical symmetry is no longer valid.

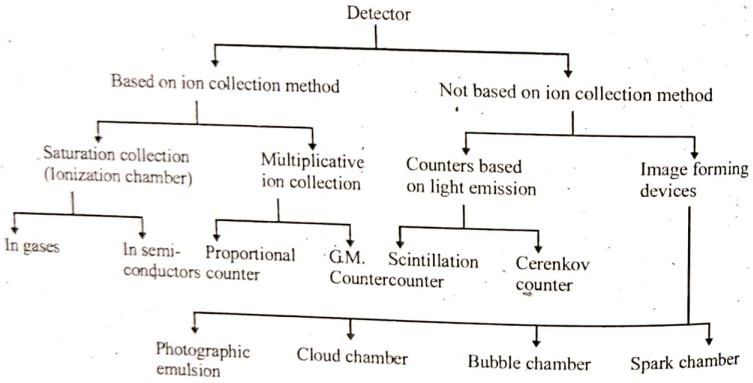
It is undoubtedly true that magic nuclei are spherical in shape. However, this nuclear shape is extremely unstable, especially for heavy nuclei; the Coulomb forces, which tend to elongate the nucleus and the surface tension forces, which tend to reduce the surface in heavy nuclei, almost completely cancel each other. Hence, even under the effect of the relatively weak surface forces the nucleus becomes a sphere.

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

Q.18. What are the basic principle of detection of nuclear radiation? Name different types of nuclear detector and explain the working of any one of them. (2015)

Ans. All methods for detection of radioactivity are based on interactions of the charged particles. As a result of interaction ions are produced and reduction in energy is observed. In the case of neutrons and γ-rays, the ionizations produced are caused by secondary charged particles. In research, detectors are used to detect and record the number of particles emitted in various experiments involved in the study of nuclear radiation, disintegration and transmutation. In the study of petroleum, geology, cosmic rays, reaction rate, molecular structure, surface properties, equilibrium measurements and catalysis the nuclear detectors have found a use. The detectors are classified as:



Detectors are also classified as electric (Ionisation chamber, Proportional counter, G.M. counter, semiconductor counter), and optical (scintillation counter, Cerenkov counter, Photographic emulsion, Cloud chamber, Bubble chamber, Spark chamber). The same detector can be used in studies of various radiations. Interactions of the radiations in the detector may depend on different phenomena.

Ionization chamber:-

Normally the gas in a container, in which are fixed two parallel electrodes, does not conduct electricity and no current will be observed until the voltage becomes very large to permit a spark to pass between the electrodes. The gas becomes ionized when charged particles pass through it. If no voltage is applied to the container there will be no directing force acting the ions and no current will flow through the gas. If a small potential difference is applied to the electrodes, the -ve ions (or electrons) will be attracted to the anode and the +ve ions will go to the cathode. At the surface of the anode there will be a high concentration of negative ions. Recombination in this region will be a small

Visit us for more detail at : www.winsomepublication.in

because the rate of recombination is proportional to the product $n^+ n^-$ and here n^+ is very small. All positive ions produced in this region move out promptly towards the cathode. An analogous situation will occur at cathode. Therefore the recombination will be maximum at the center of the space. A small current will flow through a resistor connecting the electrodes externally. The size of this current pulse will depend upon the number of initially produced ions but will be less than if the recombination is zero. As the voltage between the electrodes is increased, the ions will move faster, the number of recombination will be minimum and the pulse size will thus be increased. The pulse size increases with the voltage. until it reaches to a value corresponding to the no recombination of ions. This value of current is called the saturation current and the chamber is said to reach in saturation. The further increase in voltage will not change the pulse size. The voltage range over which the pulse size is constant depends on many factors, such as the nature and pressure of gas, shape and size of the electrodes. The pulse size depends on the nature of the ionising event.

The ionisation chamber works in the region of constant pulse size. In this region the applied voltage is high enough to prevent recombination of ions and low enough to prevent multiplication.

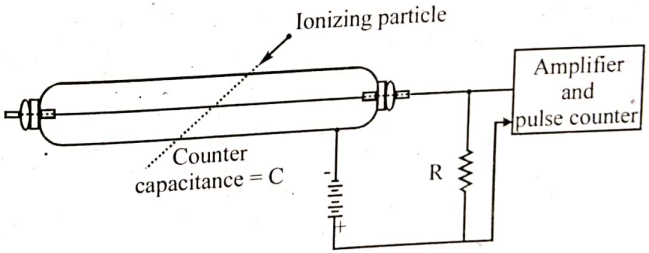


Figure - Ionization chamber

Ionization chamber, in its simple form, consists of a cylindrical conducting chamber containing a central conducting electrode insulated from it. The gases used in it are usually air or hydrogen at atmospheric pressure, or at greater pressures for γ-ray detection. For the detection of thermal neutrons boron is introduced in the form of BF_3.

The schematic diagram of a cylindrical ionization chamber is shown in Fig. One electrode of the chamber is connected to the high-voltage supply so that its potential is V_0 and the other is connected to ground through a resistance R.

In the initial stages, the gold-leaf electroscopes of great sensitivities or the quadrant electrometers were used. The next step in the development was a modification of the electroscope, designed by Lauritsen, where the gold leaf was replaced by a tiny metal coated quartz fibre. It could not give quick re-

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

sponse to particles that follow one another in rapid succession. This difficulty was overcome to a great extent by a device known as linear or dc amplifier, which increases, without distorting, the feeble ionisation current from a single particle to a value which can easily register on an oscillograph or an electrical counter. dc amplifiers are more susceptible to disturbance and drift and more difficult to arrange with several successive stages of amplification than those designed for amplifying alternating currents. An important development is the use of vibrating reed electrometer. In it the vibrating reed converts the IR voltage to an approximately sinusoidal potential, which is then amplified in a highly stable audiofrequency amplifier.

Ionization chamber instruments may be grouped into two categories, depending upon the value of the time constant RC of the system relative to the frequency of arrival of the ionizing events. If RC is long compared to time between ionizing events a steady state is reached and a direct current may be measured. On the other hand, if RC is small each pulse produced by an ionizing event may be detected separately. These two types are also known as intergrating and non-integrating respectively and the instrument as current and pulse ionization chambers.

Let us consider the theory in a simple case when a track of n ion pairs is formed parallel to and at a distance x_0 from the axial wire. The electrodes are parallel and separated by a distance d. A voltage V_0 is applied to the electrodes, so that electrons and positive ions drift in opposite directions in a uniform electric field V_0/d. Let us first assume that the time constant RC (where C = capacitance of the chamber + input capacitance of the amplifier) is much greater than the collection time of ions. So that the current through R may begin as soon as the electrons and +ve ions move apart in the chamber.

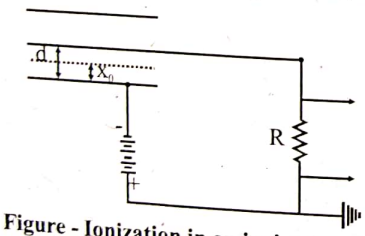


Figure - Ionization in an ionization chamber

The change of potential at the positive electrode at the time t, due to a track of n ion pairs formed at t = 0 is given by

$$V(t) = n[q_e(t) + q_p(t)]/C \qquad \text{.....(i)}$$

where $q_e(t)$, the charge induced on the positive electrode at time t by a single electron, is given by

$$q_e(t) = -e\frac{\text{P.D. between the electron at time t and the } -\text{ ve electrode}}{\text{P.D. between the electrodes}}$$

Visit us for more detail at : www.winsomepublication.in

$$q_e(t) = -e\frac{d - x_e(t)}{d} = -e\frac{\left[d - \left(x_0 - v_e t\right)\right]}{d} \qquad \text{.....(ii)}$$

and $q_p(t)$, the charge induced on the positive electrode at time t by a single positive ion, is given by

$$q_p(t) = +e\frac{\text{P.D. between the + ve ion at time t and } -\text{ ve electrode}}{\text{P.D. between the electrodes}}$$

$$q_p(t) = e\frac{d - x_p(t)}{d} = e\frac{\left[d - \left(x_0 + v_p t\right)\right]}{d} \qquad \text{.....(iii)}$$

The relations (ii) and (iii) are only true for parallel plate geometry in which potential gradient is uniform. Substituting eq.(ii) and (iii) in eq. (i) we get

$$V(t) = -ne(v_e t + v_p t)/Cd \qquad \text{.....(iv)}$$

As the electron drift velocities v_e are about a thousand times greater than the heavy ion velocities v_p, hence for the time between 0 and t_e where t_e is the electron collection time ($x_0 = v_e t_e$), the above relation reduces to

$$V(t) = -nev_e t/Cd \qquad \text{.....(v)}$$

For $t > t_e$, there is an abrupt decrease in the rate of rise of potential, the new rate being v_p/v_e times the original rate. The change of potential becomes -ne/C when $t = t_p$, the time in which positive ions reach the negative electrode. This limit is proportional to n and is independent of where the ions were formed in the chamber. The shape of the collector potential curve does depend upon where the ions are formed.

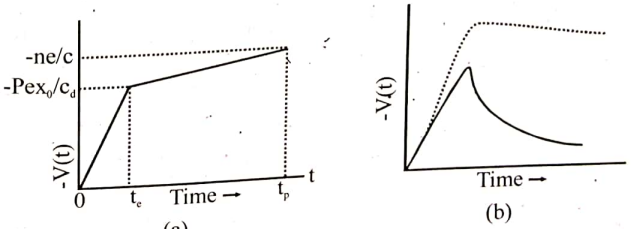


Figure - (a) Shape of potential pulse (b) Pulse clipping

With sufficient amplification, a large pulse will appear at the amplifier output terminal. To have the height of the output pulse proportional to the amount of ionization produced by the particle in the chamber, two kinds of proportional or linear amplifiers can be used. One known as slow amplifiers, in which the shortest time constant is chosen long compared with the drift time of the +ve ions, have several disadvantages. The others known as fast amplifiers, in which time constant RC is made short enough so that the induced potentials by the +ve ions are not of interest and clip the pulses. Thus the slow linear rise,

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

after the collection of electrons, is removed and the height of the recorded pulses depends on $V(t_c)$ and not on $V(t_p)$. This is shown in fig.(b). Unfortunately such amplifiers have the disadvantage that the pulse height depends on $V(t_c)$ and hence on the position of formation of the ionization track. This defect can be overcome by inserting a grid between the electrodes. A grid of many fine wires (spaced 2 mm apart and 0.2 mm diameter) is mounted at a distance d/3 from the +ve electrode and is maintained at a potential $-V_0/2$, so that the field between the grid and the negative electrode is weaker than the field between the grid and the positive electrode. If the complete range of an ionizing particle is confined to the region between the negative electrode and grid, the grid screens the +ve electrode from the electrons and +ve ions. Electrons, while drifting through the grid, induce a change of potential at the +ve electrode. As positive ions are screened from the +ve electrode by the grid, hence have no effect. Thus the pulse height increases due to the induction effect of the moving electrons. When all the electrons are collected it attains a maximum value given by

$$V_{max} = -ne/C \qquad \text{.....(vi)}$$

It is also independent of the position and orientation of the ionization track. The height of the amplified pulses from the ionization chamber

$$V = AEe/WC \qquad \text{.....(vii)}$$

where A is the voltage gain of the amplifier, E the energy of the ionizing particle, e the electronic charge and W the mean energy required to produce an ion pair in the gas filled in the chamber.

Ionization chambers can measure accurately the energies of α-particles and other ions with discrete spectra. These may be used to count fissions. Low fission rate can be measured in the presence of high α-particles. Unfortunately, β-particles and γ-rays do not produce enough ionization to give pulse detectable above the background noise and hence can not be detected by ionization chambers.

Q.19. Give examples of Isotopes and isotones. **(2015)**

Ans. Isotopes: -

Isotopes are variants of a particular chemical element which differ in neutron number. All isotopes of a given element have the same number of protons in each atom. The term isotope is formed from the Greek roots isos (equal) and topos (place), meaning "the same place"; thus, the meaning behind the name is that different isotopes of a single element occupy the same position on the periodic table.

The number of protons within the atom's nucleus is called atomic number and is equal to the number of electrons in the neutral (non-ionized) atom. Each atomic number identifies a specific element, but not the isotope; an atom of a given element may have a wide range in its number of neutrons. The number of nucleons (both protons and neutrons) in the nucleus is the atom's

Visit us for more detail at : www.winsomepublication.in

mass number, and each isotope of a given element has a different mass number.

For example, carbon-12, carbon-13 and carbon-14 are three isotopes of the element carbon with mass numbers 12, 13 and 14 respectively. The atomic number of carbon is 6, which means that every carbon atom has 6 protons, so that the neutron numbers of these isotopes are 6, 7 and 8 respectively.

Isotones: -

Two nuclides are isotones if they have the same neutron number N, but different proton number Z. For example, boron-12 and carbon-13 nuclei both contain 7 neutrons, and so are isotones. Similarly, ^{36}S, ^{37}Cl, ^{38}Ar, ^{39}K, and ^{40}Ca nuclei are all isotones of 20 because they all contain 20 neutrons.

Q.20. Give examples of mirror nuclei. **(2015)**

Ans. There exists many light unstable nuclei in which Z differs from N by unity. Among these there are pairs which consist of the same odd number of nucleons, say $(2Z + 1) = A$, but one of them has Z protons and $(Z + 1)$ neutrons, while the other $(Z + 1)$ protons and Z neutrons. Such nuclei are called mirror nuclei. The few examples are

$$({}_1H^3, {}_2He^3); ({}_3Li^7, {}_4Be^7); ({}_6C^{13}, {}_7N^{13}); ({}_{19}K^{39}, {}_{20}Ca^{39}).$$

Q.21. Write (i) three isotopes and (iii) three isobars of $_8O^{16}$ nucleus. **(2013)**

Ans. Isotopes - ${}_8O^{16}, {}_8O^{17}, {}_8O^{18}$

Isobars - ${}_7N^{16}$,

Q.22. Write short note on Shell Model. **(2009)**

Ans. In nuclear physics and nuclear chemistry, the nuclear shell model is a model of the atomic nucleus which uses the Pauli exclusion principle to describe the structure of the nucleus in terms of energy levels. The first shell model was proposed by Dmitry Ivanenko (together with E. Gapon) in 1932. The model was developed in 1949 following independent work by several physicists, most notably Eugene Paul Wigner, Maria Goeppert Mayer and J. Hans D. Jensen, who shared the 1963 Nobel Prize in Physics for their contributions.

The shell model is partly analogous to the atomic shell model which describes the arrangement of electrons in an atom, in that a filled shell results in greater stability. When adding nucleons (protons or neutrons) to a nucleus, there are certain points where the binding energy of the next nucleon is significantly less than the last one. This observation, that there are certain magic numbers of nucleons: 2, 8, 20, 28, 50, 82, 126 which are more tightly bound than the next higher number, is the origin of the shell model.

The shells for protons and for neutrons are independent of each other. Therefore, one can have "magic nuclei" where one nucleon type or the other is at a magic number, and "doubly magic nuclei", where both are. Due to some variations in orbital filling, the upper magic numbers are 126 and, speculatively, 184 for neutrons but only 114 for protons, playing a role in the search

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

for the so-called island of stability. Some semimagic numbers have been found, notably Z = 40 giving nuclear shell filling for the various elements; 16 may also be a magic number.

In order to get these numbers, the nuclear shell model starts from an average potential with a shape something between the square well and the harmonic oscillator. To this potential a spin orbit term is added. Even so, the total perturbation does not coincide with experiment, and an empirical spin orbit coupling must be added with at least two or three different values of its coupling constant, depending on the nuclei being studied.

Nevertheless, the magic numbers of nucleons, as well as other properties, can be arrived at by approximating the model with a three-dimensional harmonic oscillator plus a spin-orbit interaction. A more realistic but also complicated potential is known as Woods–Saxon potential.

Q.23. How was neutrons discovered? Write three basic properties of neutron. **(2015)**

Ans. The discovery of the neutron and its properties was central to the extraordinary developments in atomic physics that occurred in the first half of the 20th century. Early in the century, Ernest Rutherford developed a crude model for the atom, based on the gold foil experiment of Hans Geiger and Ernest Marsden. In this model, atoms had their mass and positive electric charge concentrated in a very small nucleus. By 1920 chemical isotopes had been discovered, the atomic masses had been determined to be integer multiples of the mass of the hydrogen atom, and the atomic number had been identified as the charge on the nucleus. Throughout the 1920s, the nucleus was viewed as composed of combinations of protons and electrons, the two elementary particles known at the time, but that model presented several experimental and theoretical contradictions.

The essential nature of the atomic nucleus was established with the discovery of the neutron by James Chadwick in 1932 and the determination that it was a new elementary particle, distinct from the proton.

The uncharged neutron was immediately exploited as a new means to probe nuclear structure, leading to such discoveries as the creation of new radioactive elements by neutron irradiation (1934) and the fission of uranium atoms by neutrons (1938). The discovery of fission led to the creation of both nuclear power and weapons by the end of World War II. Both the proton and neutron were presumed to be elementary particles until the 1960s when they were determined to be composite particles built from quarks.

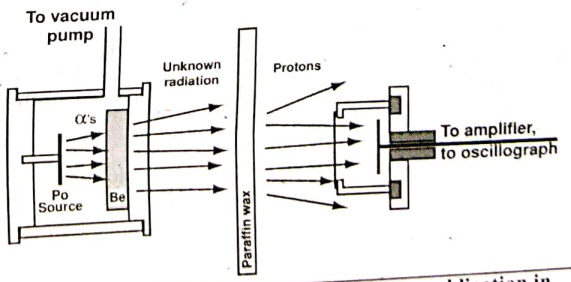
In 1930, Walther Bothe and Herbert Becker in Giessen, Germany found that if the energetic alpha particles emitted from polonium fell on certain light elements, specifically beryllium $\left({}^{9}_{4}\mathrm{Be}\right)$, boron $\left({}^{11}_{5}\mathrm{B}\right)$, or lithium $\left({}^{7}_{3}\mathrm{Li}\right)$, an unusually penetrating radiation was produced. Beryllium produced the most

Visit us for more detail at : www.winsomepublication.in

intense such radiation. Polonium is highly radioactive, producing energetic alpha radiation. Polonium was commonly used for such scattering experiments at the time. Since the observed radiation was not influenced by an electric field (neutrons have no charge), it was thought to be gamma radiation. The radiation was more penetrating than any gamma rays known, and the details of experimental results were difficult to interpret.

Two years later Irene Joliot-Curie and Frederic Joliot in Paris showed that if this unknown radiation fell on paraffin wax, or any other hydrogen-containing compound, it ejected protons of very high energy (5 MeV). This observation was not in itself inconsistent with the assumed gamma ray nature of the new radiation, but that interpretation (Compton scattering) had a logical problem. From energy and momentum considerations, a gamma ray would have to have impossibly high energy (50 MeV) to scatter a massive proton. In Rome, the young physicist Ettore Majorana declared that the manner in which the new radiation interacted with protons required a new neutral particle.

On hearing of the Paris results, neither Rutherford nor James Chadwick at the Cavendish Laboratory believed the gamma ray hypothesis. Assisted by Norman Feather, Chadwick quickly performed a series of experiments showing that the gamma ray hypothesis was untenable. The previous year, Chadwick, J.E.R. Constable, and E.C. Pollard had already conducted experiments on disintegrating light elements using alpha radiation from polonium. They had also developed more accurate and efficient methods for detecting, counting, and recording the ejected protons. Chadwick repeated the creation of the radiation using beryllium and aimed the radiation at paraffin following the Paris experiment. Paraffin wax is a hydrocarbon high in hydrogen content, hence offers a target dense with protons; since neutrons and protons have almost equal mass, protons scatter energetically from neutrons. Chadwick measured the range of these protons, and also measured how the new radiation impacted the atoms of various gases. He found that the new radiation consisted of not gamma rays, but uncharged particles with about the same mass as the proton. These particles were neutrons. Chadwick won the Nobel Prize in Physics for this discovery in 1935.



Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

The year 1932 was later referred to as the "annus mirabilis" for nuclear physics in the Cavendish Laboratory, with discoveries of the neutron, artificial nuclear disintegration by the Cockcroft–Walton particle accelerator, and the positron.

Basic properties of the neutron:-

(1) Neutron mass:-

Since the neutron is not a charged particle, its mass could not be determined from deflection measurements in electric and magnetic fields. Chadwick gave a rough estimation of the mass of a neutron from the information obtained in collisions of neutrons with protons on the one hand and with nitrogen nuclei on the other. From the equations for the conservation of energy and for the conservation of momentum for a head on collision, and from the lengths of accompanying tracks in the cloud chamber, Chadwick found that neutron mass $M_n = 1.15$ amu, roughly the same as that of the proton.

A more precise value for the mass of the neutron was obtained by Chadwick from the study of the nuclear reaction

$$_5B^{11} + {}_2He^4 \rightarrow {}_7N^{14} + {}_0n^1 + Q.$$

The Q value was found from the kinetic energies of B^{11}, N^{14}, $_0n^1$ and α-particles. Substituting all the known values in terms of atomic mass units in the above equation Chadwick obtained 1.0067 mu. for the mass of the neutron.

(2) Neutron Charge:-

The neutron is usually assumed to have no net charge. Dee investigated the ionization produced in air in a cloud chamber irradiated by fast neutrons and found one ion pair per 3 metre of the path and thus concluded that the charge of the one neutron is less than 1/700 of the proton charge. Rabi and his coworkers have found the neutron charges less than 10^{-12} electron charge. Fermi and Marshall set an upper limit of $\sim 10^{-18}$ electron charge for the charge on the neutron.

(3) Neutron Statistics and spin:-

Since the deuteron is a loose combination of proton and neutron and is supposed to have a spin 1. From the study of band spectra of deuterium and the proton spin (1/2) the neutron spin comes out to 1/2. Neutrons have been found to obey Fermi statistics. The experimental fact that the magnetic moment of the deuteron is nearly equal to the algebraic sum of the magnetic moments of the neutron, and proton strongly supports this value. This value of 1/2 for the spin of the neutron has been confirmed by reflection of neutrons from magnetised mirrors by Hughes and Burgy.

(4) Wave Properties:-

For non-relativistic velocities ($v \ll c$), the wavelength associated with neutrons is given by

$$\lambda = \frac{h}{p} = \frac{h}{(2ME)^{1/2}} = \frac{2.86\times10^{-11}}{E^{1/2}} \text{ metre}$$

Visit us for more detail at : www.winsomepublication.in

where E is in eV. Thus for thermal neutrons $\lambda = 1.82\ \overset{0}{A}$ which is comparable with atomic dimensions. For E = 1 MeV, $\lambda = 2.86\times10^{-14}$ metre approaching nuclear dimensions. The relativistic correction is important above energy of 100 MeV. The wave properties are of primary importance in determining the nature of the interaction between neutrons and nuclei.

Q.24. Explain the construction, working and necessary mathematical formula of cyclotron. Why is it not suitable for accelerating electrons? (2014)

Other Related Questions -

Q. Write notes on Cyclotron and its application. (2012)

Q. Using necessary diagram, describe construction, working and theory of Cyclotron. (2019)

Ans. The Cyclotron:-

Principle:-

The cyclotron is a "magnetic resonance accelerator".

- The positive ions acquire energy from r.f. field when they cross the gap between two hollow metallic electrodes connected to the field. The time of transit of ions between electrodes is such that r.f. field is always in a phase to accelerate the ions when they cross the gap. Thus ions gain energy from r.f. field in steps every time they cross the gap between the two electrodes and eventually acquire energy much higher than the equivalent peak accelerating voltage.

The electric field provides the energy and magnetic field guides the motion.

- The value of the magnetic field strength depends upon the frequency of the oscillating electric field applied between the dees. It is so chosen as to give resonance between the arrival of the charged particle in the gap and the reversal of the voltage between the dees. This is done by adjusting the time taken by the charged particle to describe a semi-circular path equal to half the time period of oscillation of the applied high frequency electric field i.e.,

$$\frac{\pi}{\omega} = \frac{T}{2}$$

The frequency

$$f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{Be}{2\pi m}.$$

This gives the cyclotron resonance condition for a charged particle of a given value of e/m.

Construction:-

The cyclotron (fig.) consists of two hollow semicircular metal boxes, D_1, D_2 called "dees". A source of ions is located near the mid-point of the gap between the "dees". The "dees" are insulated from each other and are en-

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

closed in another vacuum chamber. The "dees" are connected to a powerful radio-frequency oscillator. The whole apparatus is placed between the pole-pieces of a strong electromagnet. The magnetic field is perpendicular to the plane of the "dees".

Theory:-

Suppose a positive ion leaves the ion source at the centre of the chamber at the instant when the "dees" D_1 and D_2 are at the maximum negative and positive A.C. potentials respectively. The positive ion will be accelerated towards the negative dee D_1 before entering it.

• The ions enter the space inside the dee with a velocity v given by

$$Ve = \frac{1}{2}mv^2$$

Here, V is the applied voltage.

e and m are the charge and mass of the ion respectively.

• When the ion is inside the "dee", it is not accelerated since this space is field free. Inside the dee, under the action of the applied magnetic field, the ions travel in a circular path of radius r given by

$$Bev = mv^2/r \quad \text{.....(i)}$$

Here, B = the flux density of the magnetic field.

or $r = mv/Be$ .....(ii)

The angular velocity of the ion in its circular path $= \omega = \frac{v}{r} = \frac{Be}{m}$ .....(iii)

The time taken by the ion to travel the semicircular path $= t = \frac{\pi}{\omega} = \frac{\pi m}{Be}$ .....(iv)

Suppose the strength of the field (B) or the frequency of the oscillator (f) are so adjusted that by the time the ion has described a semicircular path and just enters the space between D_1 and D_2, D_2 has become negative with respect to D_1. The ion is then accelerated towards D_2 and enters the space inside it with a greater velocity. Since the ion is now moving with greater velocity, it will describe a semicircle of greater radius in the second "dee". But from the equation $t = \pi m/Be$ it is clear that the time taken by the ion to describe a semicircle is independent of both the radius of the path (r) and the velocity of the ion (v). Hence the ion describes all semicircles, whatever be their radii, in exactly the same time. This process continues until the ion reaches the periphery of the dees. The ion thus spirals round in circles of increasing radius and acquires high energy. The ion will finally come out of the dees in the direction indicated, through the window.

Energy of an ion:-

Let r_{max} be the radius of the outermost orbit described by the ion and v_{max} the maximum velocity gained by the ion in its final orbit. Then the equation for the motion of the ion in a magnetic field is

Visit us for more detail at : www.winsomepublication.in

$$Bev_{max} = \frac{mv_{max}^2}{r_{max}}$$

or $$v_{max} = B\frac{e}{m}r_{max} \quad \text{.....(v)}$$

∴ The energy of the ion

$$E = \frac{1}{2}mv_{max}^2 = \frac{B^2r_{max}^2}{2}\left[\frac{e^2}{m}\right] \quad \text{.....(vi)}$$

The condition for acceleration of the ion in the inter-dee gap is that

The time taken by the ion to travel the semicircular path

= Half the time period of oscillation of the applied high frequency voltage

i.e., $$\frac{\pi m}{Be} = \frac{T}{2} \text{ or } T = \frac{2\pi m}{Be}$$

∴ Frequency of the oscillator

$$f = \frac{Be}{2\pi m} \quad \text{.....(vii)}$$

Hence the energy of the ion is given by

$$E = 2\pi^2 r_{max}^2 f^2 m \quad \text{.....(viii)}$$

The particles are ejected out of the cyclotron not continuously but as pulsed streams.

Limitations of the cyclotron:-

The energies to which particles can be accelerated in a cyclotron are limited by the relativistic increase of mass with velocity. The mass of a particle, when moving with a velocity v is given by $m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$ where m_0 is the rest mass and c the velocity of light.

According to eq.(iv),

The time taken by the ion to travel the semicircular path = $t = \frac{\pi m}{Be} = \frac{T}{2}$

∴ Frequency of ion = $n = \frac{1}{T} = \frac{Be}{2\pi m}$ or $n = \frac{Be\sqrt{1 - v^2/c^2}}{2\pi m_0}$

Therefore, the frequency of rotation of the ion decreases with increase in velocity. The ions take longer time to describe their semicircular paths than the fixed period of the oscillating electric field. Thus, the ions lag behind the applied potential and finally they are not accelerated further. Due to this reason, the energy of the ions produced by the cyclotron is limited. This limitation can be overcome in the following two ways.

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

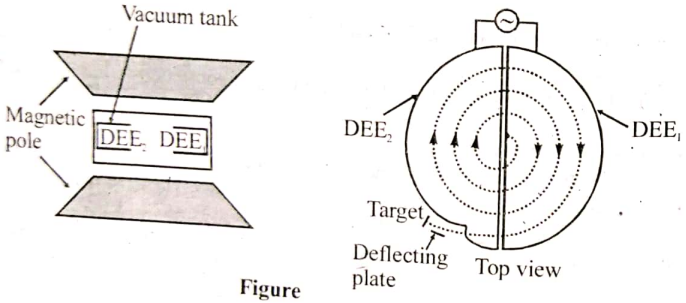
Now, the frequency of the ion = $Be\frac{\sqrt{1-v^2/c^2}}{2\pi m_0}$

(i) Field variation:-

The frequency of the ion can be kept constant by increasing the magnetic field (B) at such a rate that the product $B\sqrt{1-v^2/c^2}$ remains constant. For this purpose, the value of the magnetic field B should increase, as velocity of the ion increases, so that the product $B\sqrt{1-v^2/c^2}$ remains unchanged. This type of machine in which the frequency of electric field is kept constant and magnetic field is varied is called synchrotron.

(ii) Frequency modulation:-

In another form of apparatus, the frequency of the applied A.C. is varied so that it is always equal to the frequency of rotation of the ion. This type of machine in which magnetic field is kept constant and the frequency of the applied electric field is varied is called a frequency modulated cyclotron or synchro-cyclotron.



Figure

Q.25. Write short note on Thermonuclear energy. **(2009)**

Ans. The energy obtained after nuclear fusion or fission of atomic nuclei at very high temperature is termed as thermonuclear energy. The energy emitted is a rich source of power and is termed as atomic energy. A common example is hydrogen bomb.

Q.26. Sketch nucleon binding energy per particle vs mass number curve and explain fission and fusion process of nuclei. **(2014)**

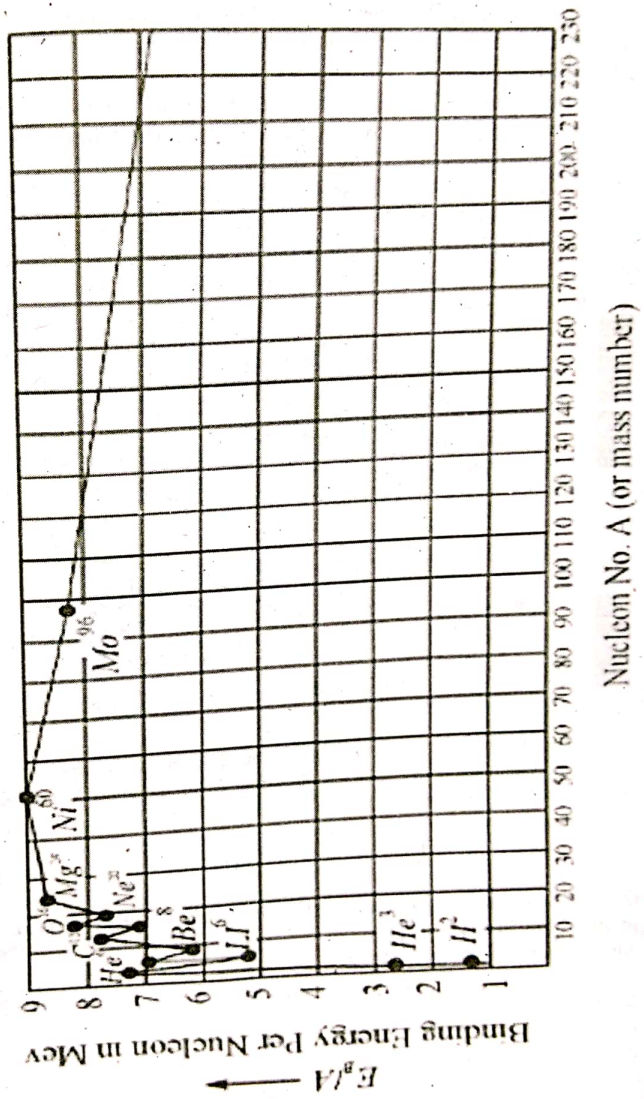
Other Related Questions -

Q. Sketch nuclear binding energy per particle vs mass number curve and explain why energy is released in fission and fusion? **(2012)**

Q. With the help of this curve explain how energy is released in the process of fission and fussion. **(2009)**

Visit us for more detail at : www.winsomepublication.in

Ans. Nuclear binding energy curve: -



Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

Nuclear fission:-

The process of breaking up of the nucleus of a heavy atom into two, more or less equal fragments with the release of a large amount of energy is called fission.

When uranium is bombarded with neutron, a uranium nucleus captures a slow neutron, forming an unstable compound nucleus. The compound nucleus splits into two nearly equal parts. Some neutrons are also released in this process. The schematic equation for the fission process is -

$$_{92}U^{235} + {}_0n^1 \rightarrow {}_{92}U^{236*} \rightarrow X + Y + \text{neutrons} \quad \text{.....(i)}$$

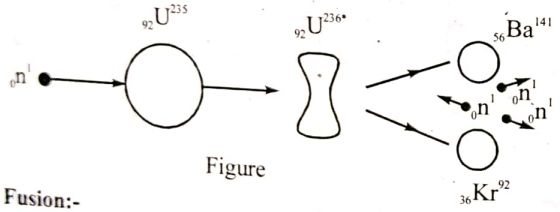
$_{92}U^{236*}$ is a highly unstable isotope, and X and Y are the fission fragments. The fragments are not uniquely determined, because there are various combinations of fragments possible and a number of neutrons are given off. Typical fission reactions are

$$_{92}U^{235} + {}_0n^1 \rightarrow {}_{92}U^{236*} \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_0n^1 + Q \quad \text{.....(ii)}$$

$$_{92}U^{235} + {}_0n^1 \rightarrow {}_{92}U^{236*} \rightarrow {}_{54}Xe^{140} + {}_{38}Sr^{94} + 2\,{}_0n^1 + Q \quad \text{.....(iii)}$$

Here, Q is the energy released in the reaction.

According to Eq. (ii), when $_{92}U^{235}$ is bombarded by a slow moving neutron, the nucleus becomes unstable ($_{92}U^{236*}$) and splits into $_{56}Ba^{141}$ and $_{36}Kr^{92}$ releasing 3 neutrons and energy Q (fig.).



Figure

Nuclear Fusion:-

In this process, two or more light nuclei combine together to form a single heavy nucleus. For example, when four hydrogen nuclei are fused together, a helium nucleus is formed. The mass of the single nucleus formed is always less than the sum of the masses of the individual light nuclei. The difference in mass is converted into energy according to Einstein's equation $E = mc^2$.

Q.27. Describe the semi-empirical binding energy mass formula and explain its significance. (2019)

Ans. Semi empirical formula: -

The liquid-drop model can be used to obtain an expression for the binding energy of the nucleus. Weizacker proposed the semi-empirical nuclear binding energy formula for a nucleus of mass number A, containing Z protons and N neutrons. It is written as

$$B.E. = aA - bA^{2/3} - \frac{cZ(Z-1)}{A^{1/3}} - \frac{d(N-Z)^2}{A} \pm \frac{\delta}{A^{3/4}}$$

Visit us for more detail at : www.winsomepublication.in

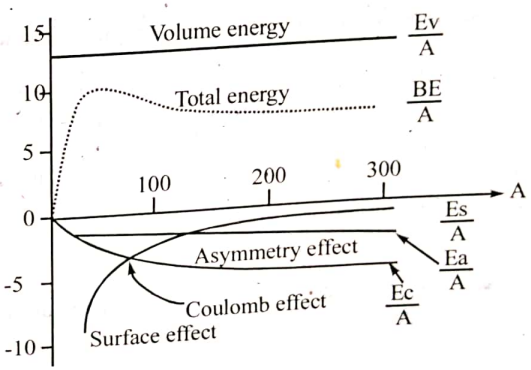
where a, b, c, d and δ are constants.

Explanation of the terms: -

1. The first term is called the volume energy of a nucleus ($E_v = aA$). The larger the total number of nucleons A, the more difficult it will be to remove the individual protons and neutrons from the nucleus. The B.E. is directly proportional to the total number of nucleons A.

2. The nucleons, at the surface of the nucleus, are not completely surrounded by other nucleons. Hence energy of the nucleon on the surface is less than that in the interior. The number of surface nuclons depends upon the surface area of the nucleus. A nucleus of radius R has an area of $4\pi R^2 = 4\pi r_0^2 A^{2/3}$. Hence the surface effect reduces the B.E. by $E_s = bA^{2/3}$. The negative enrgy E_s is called the surface energy of a nucleus. It is most significant for the lighter nuclei, since a greater fraction of their nucleons are on the surface.

3. The electrostatic repulsion between each pair of protons in a nucleus also contributes towards decreasing its B.E. The Coulomb energy E_c of a nucleus is the work that must be done to bring together Z protons from infinity into a volume equal to that of the nucleus. Hence $E_c \propto Z(Z-1)/2$ (the number of proton pairs in a nulceus containing Z protons) and E_c is inversely proportional to the nulcear radius $R = r_0 A^{1/3}$. E_c is negative because it arises from a force that oppose nuclear stability.



4. The fourth term $E_a = \frac{d(N-Z)^2}{A}$ originates from the lack of symmetry between the number of protons (Z) and the number of neutrons (N) in the nucleus. The maximum stability of a nucleus occurs when N = Z. Any departure from this introduces an asymmetry N-Z, which results in a decrease in stability. The decrease in the B.E. arising from this is called the asymmetric energy (E_a). This is also negative.

5. The final correction term δ allows for the fact that even-even nuclei are more stable than odd-odd nuclei. δ is positive for even-even nuclei, is negative for

Visit us for more detail at : www.winsomepublication.in

Scanned by CamScanner

odd-odd nuclei and $\delta = 0$ for an odd A.

The best values of the constant, expressed in MeV are a = 15.760; b = 17.810, c = 0.711, d = 23.702, δ = 34.

The contributions of the various effects in Weizacker's empirical formula are represented schematically in the graph of figure.

Q.28. Define mass defect. **(2009)**

Ans. The nucleons are bound together in a nucleus and the energy has to be supplied in order to break apart the constituents into free nucleons. The energy with which nucleons are bounded together in a nucleus is called as Binding Energy (B.E.). In order to free nucleons from a bounded nucleus, this much of energy (= B.E.) is to be supplied.

It is observed that the mass of a nucleus is always less than the mass of constituent (free) nucleons. This difference in mass is called as mass defect and is denoted as Δm.

If m_n is mass of a neutron; m_p is mass of a proton and

$m(Z, A)$ = mass of bounded nucleus, then

$\Delta m = Z.m_p + (A - Z).m_n - m(Z, A)$

This mass-defect is in form of energy and is responsible for binding the nucleons together.

Q.29. Explain why energy is released in fusion of light nuclei and fission of heavy nuclei. **(2013)**

Ans. Evidently, nuclei of intermediate mass are the most stable, since the greatest amount of energy must be supplied to liberate each of their nucleons. This fact suggests that a large amount of energy will be liberated if heavier nuclei can somehow be split into lighter ones or if light nuclei can somehow be joined to form heavier ones. The former process is known as nuclear fission and the latter as nuclear fusion. Both the processes indeed occur under proper circumstances and do evolve energy as predicted.

Q.30. The half life of uranium is 5000 millions years. In how many years will 1 gm of the element be reduced to 1 centigram. **(2017)**

Same As -

Q. The half life of $_{11}Na^{24}$ is 15 hour. How long 80% of the sample of this nuclei of decay? **(2014)**

Sol. Here N = 0.01 gram, N_0 = 1 gram, $T_{1/2}$ = 5000 million years, t = ?

$\therefore$ Radioactive constant $= \lambda = \dfrac{0.6931}{T_{1/2}} = \dfrac{0.6931}{5000}$

Now $t = \dfrac{1}{\lambda}\log_e\left(\dfrac{N_0}{N}\right) = \dfrac{5000}{0.6931}\log_e\left(\dfrac{1}{0.01}\right) = 33221$ million years

Visit us for more detail at : www.winsomepublication.in