B.A./B.Sc. (Part III) Examination, 2010

Three-year Degree Course

MATHEMATICS

FIRST PAPER

Time : Three hours] [Maximum Marks : 60

Note : Attempt **all** the questions. Marks are indicated against each question.

1. Attempt **all** the parts :
   (a) Give an example of a group G and examples of a normal and a non normal subgroup of this group with brief justifications. 2
   (b) Show that the alternating group An is generated by the set of 3-cycles for all $n \geq 3$. 2
   (c) List the values of $\mu(n)$, $\forall\ 1 \leq n \leq 15$, where μ is the Mobius function. 2
   (d) Prove that the eigenvalues of a skew Hermitian matrix have zero real parts. 2
   (e) Prove that any set of pairwise orthogonal vectors in an inner product space is linearly independent. 2

2. (a) (i) Prove that any subgroup H of a group G containing the commutator subgroup G' of G, is a normal subgroup of G. 2
   (ii) Give an example of a group and a normal subgroup H of G for which H has a normal subgroup K which is not normal in G. Justify. 3

   OR

   (i) Give an example N of a normal subgroup of a non-abelian group G having an index > 2 in G. 2
   (ii) Prove that Sn has a trivial centre for all $n \geq 3$. 3

   (b) If $f : G \longrightarrow G'$ is a surjective group homomorphism, establish the order preserving correspondence between the set of all subgroups of G' and the set of all subgroups of G containing $K = \ker f$. Also show that $f^{-1}[f[H]] = H$ holds for every subgroup H of G which that $K \subseteq H$. 5

   OR

   (i) Show that if a normal subgroup H of an alternating group An contains a 3-cycle then it is equal to An, $n \neq 4$. 2
   (ii) Prove that the number of elements in any conjugacy class in a finite group always divides the order of the group. Hence or otherwise show that there are 3-cycles in A_4 which are not conjugate in A_4. 3

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3. (a) Prove that the groups $(\mathbb{Z}, +)$, $(\mathbb{Q}, +)$, S_3 and S_4 are indecomposable. 5

OR

(i) Consider the action $\lambda : G \to T(G)$ of a group G on itself by conjugation where T(G) is the group of all bijections of G. Prove that $I_m \lambda \subseteq$ Ant (G) and ker λ=Z(G) = centre of G.

What are the elements of $I_m \lambda$ called ? 3

(ii) If $i_m \lambda$ is cyclic, show that it must be singleton. 2

(b) If G is a finite group of order n and p is a prime dividing n, show that G must have a cyclic subgroup of order p. 5

OR

(i) Let 0(a) denote the order of an element in a group G. If the subset $S = \{a \in G \mid 0(a) = 2\}$ of G is a singleton {g}, show that $g \in Z(G)$. 2

(ii) Let Δ_G be the diagonal relation $\{(g, g) \mid g \in G)\}$ in a group G. Show that it is a subgroup of the product G×G and Δ_G is normal in G×G if and only if G is abelian. 3

4. (a) If $\varphi(n)$ denotes the number of elements in the set $\{m \in \mathbb{N} \mid 1 \le m \le n$ and god $(m,n)=1\}$ show that $\varphi(m_1 m_2) = \varphi(m_1)\,\varphi(m_2)$ holds whenever m_1, m_2 are relatively prime. Hence or otherwise derive a formula for $\varphi(n)$ in terms of prime factorization of n. 5

OR

Find the least positive integer which when divided by 4, 5 an d 9 leaves a remainder of 3, 2 and 7 respectively. What is the set of all such integers ? 5

(b) (i) Evaluate the Legendre symbol $\left(\frac{6}{17}\right)$ by the use of Gauss' lemma. 2

(ii) Evaluate the Legendre symbol $\left(\frac{1009}{229}\right)$. 3

OR

(i) Show that the set of all arithmetic functions with $f(1) \neq 0$ forms an abelion group under the operation of convolution product. 3

(ii) Prove that $\sum_{d/n} \frac{\mu(d)}{d} = \frac{\varphi(n)}{n}$ where μ is the Mobius function. 2

5. (a) If $\lambda_1, \ldots\ldots\lambda_n$ are distinct eigenvalues of a linear operator on a finite dimensional vector space V over F, and $W_i = \{\underline{v} \in V \mid T v = \lambda_i \underline{v}\}$ for $1 \le i \le r$, show that $\dim(w_1 + \ldots\ldots + w_r) = \sum_{i=1} \dim w_i$. 5

OR

(a) Prove that the eigenvectors of a linear operator T on a finite dimensional vector space V span V if and only if for each eigenvalue λ of T, multiplicity of λ in the characteristic equation of T + rank $(T-\lambda 1)$ = dim V holds. 5

(b) Find on orthonormal basis of the subspace of $\mathbb{R}^4$ spanned by the vectors {1,2,3,4), (–1,4,5,2), (–1,3,–1,7)}. 5

OR

Prove that a linear operator T on a finite dimensional inner product space V over $\mathbb{C}$, preserves inner products if and only if it carries an orthonormal basis of V to an orthonormal basis of V. Show that the above property holds if and only if $TT^* = T^* T = I$, where T^* is the adjoint of T.

6. (a) Prove that every real symmetric matrix is congruent to a diagonal matrix with set of diagonal elements $\subseteq \{0, 1, -1\}$. What is the number of nonzero terms on the diagonal ? Reduce $\begin{bmatrix} 1 & -1 & 2 \\ -1 & 3 & -3 \\ 2 & -3 & 5 \end{bmatrix}$ to diagonal form by congruent reduction. 5

OR

Transform the equation $3x^2+2y^2+3z^2-2xz+2=0$ by an orthogonal change of variables and show that it represents an ellipsoid revolution. 5

(b) (i) Prove that the eigenvectors corresponding to distinct eigenvalues of a unitary matrix are orthogonal. 3

(ii) If A is a 2×2 real symmetric matrix, show that either A has distinct eigenvalues or it is a scalar times the identity matrix. 2

OR

Find the matrix of the standard inner product form on IR^3 with respect to the basis. {(1,2,3), (0,4,5), (0,0,6). 5

MATHEMATICS (SECOND PAPER)-2010

Time : Three hours] [Maximum Marks : 60

Note : Attempt **all** questions, symbols used have their usual meaning.

1. Prove or disprove :

(a) The series 2

$$\sum_{n=1}^{\infty} (1-x)x^{n-1}$$

is uniformly convergent in [0, 1].

(b) $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$ is divergent. 2

(c) For a linear map $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ and $a \in R^2$, f' (a) = f. 2

(d) The origin is a saddle point for the function f(x, y) = xy. 2

(e) *Cl* Int $\mathbb{Q}$ = Int *Cl* $\mathbb{Q}$. 2

2. (a) Let $\Sigma u_n(x)$ be a series of continuous functions defined on an interval I of $\mathbb{R}$. If it converges uniformly to s(x), then show that s(x) is a continuous function. 5

(b) Show that the series $\Sigma u_n(x)$ for which the n-th partial sum $S_n(X)$ is given by $S_n(x) = \frac{nx}{1+n^2x^2}$ defined on [0, 1] is not uniformly convergent. Further, show that, its sum is a continuous function. 5

OR

(a) State Weierstrass M-test. Use it to show that for $a \in \mathbb{R} - \mathbb{Z}$, the series $\frac{1}{4} - \frac{\sin x}{a^2-1} + \frac{\sin 2x}{a^2-4} - \frac{\sin 3x}{a^2-9} + \ldots\ldots\ldots$ converges uniformly on every finite interval of $\mathbb{R}$. 5

(b) Define the map 'exp' and prove that exp (x + y) = exp x. exp y. 5

3. (a) Show that : $\int_0^\infty \frac{1}{\sqrt{x}+x^2}dx$ converges

(b) Show that $\int_0^\infty x^{m-1}e^{-x}dx$ Converges if and only if m > 0. 5

OR

(a) Test the convergence of $\int_0^\infty \cos x^2 dx$ 5

(b) Show that $\int_0^\infty \frac{\sin x}{x}$ converges, but not absolutely. 5

4. (a) Show that $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} \frac{x^2y}{x^4+y^2}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

has all directional derivatives at (0,0). Is f differentialble at (0,0) ? Support your answer. 5

(b) Let $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be function possessing partial derivatives D_1f and D_2f with D_1f continuous at $(a,b) \in \mathbb{R}^2$. Show that f is differentiable at (a,b). 5

OR

(a) Let $F : \mathbb{R}^m \longrightarrow \mathbb{R}^n$ be differentiable at. $a \in \mathbb{R}^m$. Prove that $\|f'(a)(\bar{v})\| \leq M\|v\|$, $v \in \mathbb{R}^m$, for some $M > 0$. 5

(b) Let $f : S \longrightarrow \mathbb{R}$ be a differentiable map, where S is an open set of $\mathbb{R}^n$. If for each $x \in S$, $x.\nabla f(x) = pf(x)$, then show that f is homogeneous of degree p. 5

5. (a) State and prove the 'Implicit function theorem. 6

(b) If $f : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ be defined by f(x,y,z) = (x+y+z, x−y−xz), where $(x,y,z) \in \mathbb{R}^3$, then show that there is an open interval of $\mathbb{R}$ containing 0 and a C^1–map $g : G \longrightarrow \mathbb{R}^2$ such that for $x \in G$, f(x, g(x) = 0. 4

OR

(a) Approximate log (1+x+2y) by a polynomial of degree one. 4

(b) Let $f(x, y) = x^3+y^3-3x-y-1$. Find critical points of f. Also, find points of local minima and local maxima. 6

6. (a) Prove that an open ball in a metric space is an open set. 3

(b) For sets A and B in a metric space X, prove that $\overline{A \cup B} = \overline{A} \cup \overline{B}$. 3

(c) Let f be a continuous map from a metric space X to a metric space Y and let $A \subseteq X$. Then show that $f(\overline{A}) \subseteq \overline{f(A)}$. 4

OR

(a) Prove that a set A in a metric space is colsed if and only if it contains all its limit points. 5

(b) Let X and Y be metric spaces. Then show that a map $f : X \to Y$ is continuous if and only if $f^{-1}(G)$ is an open set of X for each open set G of Y. 5

MATHEMATICS (THIRD PAPER)-2010

Time : Three hours] [Maximum Marks : 60

Note : Answer **all parts** of the Q. **No.1** and any **two parts** of each of the remaining questions. Q. No. 1 carries 15 marks, while remaining questions carry 9 marks each.

1. (a) Two forces of magnitudes 5 and 10 units and a distance 4 units apart, are acting along the lines having direction ratios 1, −1, 2 and 2, 1, 1 respectively. Find the second invariant of the system.

(b) Find the nul point of the plane x+y+z=0 for the system (X,Y,Z; L, M, N) at origin.

(c) Prove that the moment of inertia of a rectangular lamina of mass M and sides 2a and 2b about a diagonal is $\frac{M}{3}\cdot\frac{2a^2b^2}{a^2+b^2}$.

(d) Find an expression for the kinetic energy of a body rotating about a fixed axis.

(e) Prove that the velocity potential of an incompressible fluid whose velocity at the point (x, y, z) is given by $\frac{1}{r^5}\left[3xz\hat{i}+3yz\hat{j}+\left(3z^2-r^2\right)\hat{k}\right]$, will be $\frac{z}{r^3}$, where $r^2=x^2+y^2+z^2$.

2. (a) Define central axis of a system of forces. A given force acts along the axis of x and another given force along a generator of the cylinder $x^2+y^2=a^2$. Prove that the central axes of the system lies on an elliptic cylinder.

(b) Define wrench of a given system of forces. Two wrenches of pitches p and q, whose axes are at a distance 2a from each other, have a resultant wrench of pitch w, whose axes intersects the shortest distance between the axes of the given wrenches at a distance n from its middle point prove that the angle between the axes of the given wrenches is equal to $\tan^{-1}\frac{x(p-q)-a(2w-p-q)}{x^2-a^2+(w-p)(w-q)}$

(c) Define conjugate lines of a given system of forces. Find the mutual moment of the conjugate lines $(\hat{e},\bar{a})$ and $(\hat{e}',\bar{a}')$ with respect to the system $(\overline{R},\overline{M})$ at origin.

3. (a) Prove that the moment of inertia of a right circular cone about a line passing through the vertex and perpendicular to the axis of the cone is $\frac{3M}{20}\left(a^2+4th^2\right)$

Where M, a and h are respectively mass, radius and height of the cone.

(b) Define momental ellipsord of a body at a point. Find the equation of the momental ellipsoid at a corner of a cube with respect to the principal axes.

(c) What do you mean by a principal axis of a body at a point ? Find the condition that a line is, at a point of its length, a principal axis of a body.

4. (a) State D' Alembert's principle and derive the equations of motion with the help of this principle.

(b) A rod of length 2a, is suspended by a string of length l, attached to one end. If the string and rod revolve about the vertical with uniform angular velocity ω, and their inclinations to the vertical be 30^0 and 45^0 respectively, then show that $\omega^2\frac{6g}{3l+4\sqrt{2a}}$

(c) A thin uniform rod has one end attached to a smooth hinge and is allowed to fall from a horizontal position. Show that the horizontal strain on the hinge is greatest when the rod is inclined at an angle of 45^0 to the vertical, and then obtain the ratio of the vertical strain and weight of the rod.

5. (a) Each particle of mass of liquid moves in a plane through the axis of Z, prove that the equation of continuity is

$$\frac{\partial\rho}{\partial t}+\frac{\rho v_r}{r}v_r+\frac{\partial\rho}{\partial z}v_z+\rho\left(\frac{\partial v_r}{\partial r}+\frac{\partial v_z}{\partial z}\right)=0,$$

Where symbols have their usual meanings.

(b) Obtain the Bernoulli's equation for the steady motion of a fluid.

(c) An infinite fluid in which a spherical hollow of radius a is initially at rest under the action of no forces. If a constant pressure p_0 is applied at infinity, show that the time of filling up the cavity is

$$\pi^2 a\left(\frac{\rho}{p_0}\right)^{\frac{1}{2}}2^{\frac{5}{6}}\left(\prod\frac{1}{3}\right)^{-3}$$

6. (a) Show that the rate per unit of time at which work is done by the internal pressure between the parts of a compressible fluid is

$$\iiint P\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)dx\,dy\,dz$$

Where p is the pressure, and (u, v, w) the velocity at any point (x, y, z) and the integration extends through the volume of the fluid.

(b) Using circle theorem prove that the image of a source in a cricle is an equal source at the inverse point together with an equal sink at the centre of the circle.

(c) A source S and a sink T of equal strengths m are situated within the space bounded by a circle whose centre is O. If S and T are at equal distances from O on opposite sides of it and on the same diameter AOB, Show that the velocity of the liquid at a point P is :

$$2m\frac{OS^2+OA^2}{OS}\cdot\frac{PA.PB}{PS.PS'.PT.PT'}$$

Where S' and T' are the inverse points of S and T respectively with respect to the circle.

MATHEMATICS (FOURTH PAPER)-2010

(NUMERICAL METHODS)

Time : Three hours] [Maximum Marks : 45

Note : Attempt **all** parts of the Q. **no. 1** and any **two parts** of each Q. no. 2 to 6. Q. 1 is of 10 marks and each of Q. 2 to 6 is of 7 marks.

1. (a) To find a root of f(x)=0, which one out of the methods of the Newton Raphson and Regula Falsi, will give a particular accuracy in lesser number of interations and why ?

(b) Interpolate a function in (x,y) plane through three points (–1,1), (0,2) and (2,3).

(c) In which situation the solution of a system of three linear equations in three variables, in non-homogenous form, can not be obtained ?

(d) Find f'(0) of a function through two points : (–1,2) & (0, 1).

(e) Evaluate $\int_1^2 (1+x)\frac{dx}{x^2}$ using Simpson's one-third rule.

2. (a) Find a real root of the equation $x \log_{10} x = 1.2$ by Regula Falsi method.

(b) Discuss the Newton Raphson method to find a root of $x^5=10$.

(c) Performing three iterations find the interval in which a root of tanx + tanhx = 0 lies. Use bisection method.

3. (a) If δ and μ are central difference and average difference operators then prove that $\mu = \sqrt{1+\delta^2/4}$.

(b) Derive Divided difference interpolation formula.

(c) Given the table :

x	3	5	8	12
y	5	–5	–3	–13

Find the value of x when y = 0 by inverse interpolation using Lagrange formula.

4. (a) Find the second order derivative of f(x) at x= 1.0 using following table :

x	:	0.5	0.7	1.0
f(x)	:	1	3	7

(b) Derive Trapezoidal rule of integration for $\int_a^b f(x)dx$.

(c) Using Simpson–one third rule integrate $\int_0^1 (1+x+x^2)dx$ by dividing the interval (0,1) into 4 equal sub-intervals.

5. (a) Discuss Milne's predictor corrector formula to solve an initial value problem of first order.

(b) Solve $\frac{dy}{dx} = y^2$, $y(0) = 1$, $x \in (0,1)$, to find y (2.0) using step-size=1.0 and Im-proved Euler method.

(c) Solve $\frac{dy}{dx} = 2xy$, $y(1) = 0$, $x \in (1, 2)$, to find y (2.0) by using step size 1.0 and Runge-Kutta fourth order method.

6. (a) Solve the equations by using Gausselimination method :
$x + 2y - z = 2$; $3x + 6y + z = 1$ & $3x + 3y + 2z = 3$.

(b) Using Cholesky method solve the equations :
$x + 2y + 3z = 5$; $2x + 8y + 22z = 6$; $3x + 22y + 82z = -10$

(c) Using the power method determine the numerical largest eigen value and a corresponding eigenvector of unit length, $\begin{bmatrix} 2 & 3 & 2 \\ 4 & 3 & 5 \\ 3 & 2 & 9 \end{bmatrix}$

B.A./B.Sc. (Part III) Examination, 2011

Three-year Degree Course

MATHEMATICS

(FIRST PAPER)

Time : Three hours] [Maximum Marks : 60

Note : Attempt **all** questions. Marks are indicated against each question.

1. (a) Show that a normal subgroup of order 2 of a group G is contained in the centre of G. 2

(b) List all the elements of $\mathbb{Z}_8/H$, together with their respective orders, where H = {[0], [4]}. 2

(c) Find the least positive incongruent solutions of $12x \equiv 15 \pmod g$. 2

(d) Let A be a matrix such that $A^k = 0$, for some positive integer k. Find all possible eigenvalues of A. 2

(e) Let P be an n × n real matrix such that $x \mapsto P_x$ is a linear operator on $\mathbb{R}^n$, where $x \in \mathbb{R}^n$ is being regarded as a column matrix. Show that $\|x\| = \|P_x\|$, $\forall x \in \mathbb{R}^n$ iff $\langle x, y\rangle = \langle P_x, P_y\rangle$, $\forall x, y \in \mathbb{R}^n$. 2

2. (a) (i) Explain how the normality of a subgroup H of a group G guarantees that writing aH * bH = abH, $\forall$ aH, bH $\in$ G/H, gives rise to a binary operation* on G/H. 2

(ii) For a subgroup H of a group G, consider the subgroup N(H) = {X $\in$ G/Hx=xH} of G. Show that N(H) is the largest subgroup of G in which H is normal. 3

OR

(i) Let H be a cyclic subgroup of a group G. Show that if H is normal in G, then so is every subgroup of H. 2

(ii) If $G = S_3$, then show that $G' = A_3$ where G' denotes the commutator subgroup of G. 3

(b) Let H_1, H_2 be normal subgroups of a group G, with $H_1 \subseteq H_2$. Show that $(G/H_1)/(H_2/H_1) \simeq G/H_2$. 5

OR

Let H be a normal subgroup of A_n, $n \geq 5$. If H contains a 3-cycle, then show that $H=A_n$. If H contains a product of two distinct transpositions, then also show that $H = A_n$.

3. (a) Let G be any group with n elements and p is any positive prime dividing n, show that G must have an element of order p. 5

OR

If a is an element of a finite group G, then show that $o(d(a)) = \frac{o(G)}{o(N(a))}$, where c(a) and N(a) respectively de-note the conjugacy class of a and the normalizer of a. Derive from it the socalled 'class equation' of G. 5

(b) (i) Find Z(Sn) for $n \geq 3$ 3

(ii) Let a group G out on set X on the left. If $a \in X$, then show that the set $\{g \in G/ga=a\}$ is a subgroup of G. By what name is this group called ? 2

OR

Let S_3 out on the set $X = \{1, 2, 3\}$ on the left via the action $\sigma a = \sigma(a)$, $\forall\ \sigma \in S_3$ and $a \in X$. Find all distinct orbits of S_3 and the isotropy groups of 1, 2 and 3. 5

4. (a) Define $\sigma(n)$ and show that for any positive integer $n>1$, $\sigma(n)$ is odd iff n is either a perfect square or a multiple of 2 by a perfect square. 5

OR

Show that the quadratic residue $x^2 \equiv (-1) \pmod p$, where p is a prime, has a solution iff $p \equiv 1 \pmod 4$. 5

(b) (i) For the Legendre symbol $\left(\frac{11}{23}\right)$, find n (of the Gauss lemma) such that $\left(\frac{11}{23}\right) = (-1)^n$. 3

(ii) Evaluate the Legendre symbol $\left(\frac{1372}{29}\right)$. 2

OR

If p is a prime then prove that $(p-1)! \equiv (-1) \pmod p$. Show also that if a positive integer m divides $(m-1)!+1$, then m must be a prime. 5

5. (a) If $\lambda_1, \lambda_2, \ldots\ldots, \lambda_n$ are the distinct eigenvalues of a linear operator T on a finite dimensional vector space V with $V_1, V_2, \ldots\ldots, V_n$ being their corresponding eigen vectors, then show that these eigen vectors must be linearly independent. If in particular, dim V = n, then deduce that the matrix of T must be similar to a diagonal matrix. 5

OR

Let A be a $n \times n$ unitery matrix. Show that all the eigenvalues of A are unimodular and eigenvectors corresponding to distinct eigenvalues are mutually orthogonal. 5

(b) Let ρ_2 denote the vector space of all polynomials in x of degree atmost 2, equipped with the inner product defined by

$$\langle p,q \rangle = \int_0^1 p(x)q(x)dx.$$

Show that the basis $\{1, x, x^2\}$ of ρ_2 is not orthogonal. Convert it to a normal orthogonal basis using the Gram-Schmidt process. 5

OR

Let T be a linear operator on a finite dimensional inner product space V over $\mathbb{C}$. Show that for each $w \in V$, there exists a unique $\hat{w} \in V$, such that $\langle T(v), w \rangle = \langle v, \hat{w} \rangle$, for all $v \in V$. Show further that defining $S : V \to V$ by $S(w) = \hat{w}$, for each $w \in V$, produces a linear transformation S. 5

6. (a) Diagonalize the quadratic form that arises from the matrix $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$.

OR

(i) Let $\xi = x_1e_1 + x_2e_2 + x_3e_3$ and $\eta = y_1e_1 + y_2e_2 + y_3e_3$ in $\mathbb{R}^3$ (e_1, e_2, e_3) being the standard basis of $\mathbb{R}^3$.

For the bilinear form f given by

$x_1y_1 + 2x_1y_2 + 6x_1y_3 - x_2y_2 + 4x_1y_3 - 3x_3y_3$

find a symmetric matrix A such that $f(\xi, \eta) = Y^tAX$. 2

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(ii) Transform the quadratic form associated with the above bilinear form to diagonal form by congruent reduction. 3

(b) Show that for any real symmetric matrix A, there exists an orthogonal matrix P such that both $P^{-1}AP$ and $P^{t}AP$ are equal to the same diagonal matrix D, such that D has the eigen values of a along its diagonal. 5

OR

Let f be a symmetric bilinear form on a vector space V over $\mathbb{R}$. Show that there exists a basis of V in which f is represented by a diagonal matrix and that every other diagonal representation of f has the same number of positive entries and the same number of negative entries. 5

MATHEMATICS (SECOND PAPER)-2011

(New Course)

Time : Three hours] [Maximum Marks : 60

Note : Attempt **all** questions, symbols used have their usual meanings.

1. Prove or disprove :

(a) $\sum_{n=0}^{\infty} x^n(1-x)$ is uniformly convergent in [0, 1]. 2

(b) The improper integral $\int_0^1 \frac{dx}{\sqrt{x(1-x)}}$ is convergent. 2

(c) The multiplication map from $R \times R$ to R is differentiable. 2

(d) The Jacobian matrix of $f : R^2 \to R^2$ defined BY f (x, y) = (x+y, 2x + ay) is invertible if and only if a = 2. 2

(e) A bijection continuous map has continuous inverse. 2

2. (a) State and prove Abel's Test for uniform convergence. 7

(b) Test for the uniform convergence of the sequence of functions $\left(\frac{x}{n+x}\right)$ in $[0,\infty]$. 3

OR

2. (a) Let $\Sigma u_n(x)$ be uniformly convergent series of continuous functions on [a, b]. Then show that $\sum_{n=1}^{\infty}\int_a^b u_n(x)dx = \int_a^b \left(\sum_{n=1}^{\infty} u_n(x)\right)dx$ 7

(b) Show that the series $\frac{1}{1+x^2} - \frac{1}{2+x^2} + \frac{1}{3+x^2} \ldots\ldots\ldots$ converges uniformly in $[0,\infty]$. 3

3. (a) State and prove the μ-test. 5

(b) Show that $\int_0^{\infty} \frac{\sin x}{x} dx$ converges but not absolutely. 5

OR

3. (a) Show that $\int_0^1 x^{m-1}(1-x)^{n-1}dx$ converges if and only if both m and n are positive. 7

(b) Examine for the convergence of $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$. 3

4. (a) Show that the function $f : R^2 \to R$ defined by $f(x,y) = \sqrt{|xy|}$, where $(x, y) \in R^2$, is continuous at the origin but not differentiable there. 5

(b) Let $f : R^2 \to R$ be a function possessing partial derivatives $D_1 f$ and $D_2 f$ at $(a,b) \in R^2$. If $D_1 f$ is continuous, then show that f is differentiable at (a,b). 5

OR

4. (a) Let $f : S \to R$ be a differentiable map where $S \subseteq R^n$ is an open set. If for each $x \in S$, $x.\nabla f(x) = pf(x)$, where $p \in N$, then show that f is a homogeneous function of degree p. 5

(b) State Taylor's Theorem and use it to approximate log (1+x+2y) by a polynomial of degree one. 5

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5. (a) Let $f : R^m \to R^n$ be differentiable at $a \in R^m$. Then for $V \in R^m$, show that $\|f'(a)(v)\| \leq M\|v\|$, for some $M>0$ 5

(b) Let $f : R^3 \to R^2$ be defined by $f(x, y, z) = (x+y+z, x-y-xz)$, Then show that there is an open interval G containing the origin and a C^1-map $g : G \to R^2$ such that for $x \in G$, $f(x, g(x)) = 0$ 5

OR

5. (a) Let $f : R^2 \to R$ be a function. Suppose $D_1 f$ and $D_2 f$ exist and are differentiable at (0, 0), then show that $D_1 D_2 f(0, 0) = D_2 D_1 f(0, 0)$. 5

(b) By means of an example show that for a differentiable map $f : R \to R^2$ and $x, y \in R$, $f(y) - f(x) = f'(z)(y - x)$, where $z \in L(x,y)$ is not necessarily true. 5

6. (a) Prove that a closed ball in a metric space is a closed set. 3

(b) Prove that the union of a family of open sets in a metric space is an open set. 3

(c) Prove that a map f from a metric space X to a metric space Y is continuous if and only if the inverse image of a closed set in Y is a closed set in X. 4

OR

6. (a) By means of an example show that the closure of an open ball with centre x and radius r need not be the closed ball with the same centre and the same radius. 5

(b) Let $f : X \to Y$ be a mapping, and X and Y are metric spaces. Let $x_0 \in X$. Prove that f is continuous at x_0 if and only if whenever a sequence (x_n) converges to x_0 in X, then the sequence $(f(x_n))$ converges to $f(x_0)$ in Y. 5

MATHEMATICS (THIRD PAPER)-2011

Time : Three hours] [Maximum Marks : 60

Note : Answer **all parts** of the Questions **no.1** and any **two parts** of each of the remaining questions. Question No. 1 carries 15 marks, while remaining questions carry 9 marks each.

1. (a) If a nul line relative to a given system of forces intersects a given line, it also intersects its conjugate line with respect to the system; Prove it.

(b) Show that the axis of the resultant wrench of two given wrenches intersects the line of shortest distance between the wrenches at right angle.

(c) ABCD is a uniform parallelogram, of mass M. At the middle points of the four sides are placed particles each equal to $\frac{M}{6}$, and at the intersection of the diagonals a particle of mass $\frac{M}{3}$. Show that these five particle and the parallelogram are kinetically equivalent.

(d) If a rigid body swing under gravity from a fixed horizontal axis, show that its centre of suspension and centre of oscillation are convertible.

(e) Define velocity potential of a fluid motion. Show that the velocity potential of a fluid motion exists, iff, it is irrotational.

2. (a) Show that the equation of conjugate line, to the line having line coordinates $(\hat{e}, \bar{a})$ and passing through origin, with respect to the system $(\overline{R}, \overline{M})$ at O is $\bar{r} \times \left(\overline{R} - \frac{\overline{R}.\overline{M}}{\overline{M}.\hat{e}}\hat{e}\right) = \overline{M}$

(b) Show that the minimum distance between the two forces which are equivalent to a given system (wrench) (R, K) and which are inclined at a given angle 2α, is $\frac{2K}{R}\cot\alpha$, and that the forces are then each equal to $\frac{R}{2}\sec\alpha$.

(c) Forces $\hat{i}-3\hat{j}+2\hat{k}, -2\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}-\hat{j}+4\hat{k}$ are acting at the points whose position vectors are $\hat{i}+\hat{k}, -\hat{i}+\hat{j}$ and $\hat{j}+2\hat{k}$ respectively. Find the equation of the nul plane of the point (1, 1, 1) with respect to the system of given forces.

3. (a) A uniform lamina is bounded by a parabolic arc, of latus rectum 4a and a double ordinate at a distance b from the vertex. If $b=\frac{a}{3}\left(7+4\sqrt{7}\right)$ show that two of the principal axes at the end of a latus rectum are the tangent and normal there.

(b) A plank, of mass M, is initially at rest along a line of greatest slope of a smooth plane, inclined at an angle α to the horizon, and a man of mass M', starting from the upper end walks down the plank so that it does not move. Show that he gets to the other end in time $\sqrt{\frac{2M'a}{(M+M')g\sin\alpha}}$ where a is the length of the plank.

(c) Two unequal masses M and M' rest on two rough planes inclined at angles α and β to the horizon; they are connected by a fine string passing over a small pulley of mass m and radius a which is placed at the common vertex of the two planes. Show that the acceleration of either mass is $\frac{g[M(\sin\alpha-\mu\cos\alpha)-M'(\sin\beta+\mu'\cos\beta)]}{(M+M'+m\,k^2/a^2}$ where μ, μ' are the coefficients of friction, k is radius of gyration of the pulley about its axis and M is the mass which moves downwards.

4. (a) A bent lever, whose arms are of equal length a, the angle between them being α makes small oscillations in its own plane about the fulcrum. Show that the length of the corresponding simple pendulum is

$$\frac{2\sqrt{2}\,a}{3\sqrt{1+\cos\alpha}}$$

(b) A circular area can turn freely about a horizontal axis which passes through a point O of its circumference and is perpendicular to its plane. If motion Commences when the diameter through O is vertically above O, show that when the diameter has turned through an angle θ, the components of the strain at O along, and perpendicular to, this diameter are respectively $\frac{W}{3}(7\cos\theta-4)$ and $\frac{W}{3}\sin\theta$.

(c) Show that the moment of inertia of a right solid cane, whose height is h and the radius of whose base is a, is $\frac{3Ma^2}{20}\left(\frac{6h^2+a^2}{h^2+a^2}\right)$ about a slant side.

5. (a) Prove that liquid motion is possible when velocity at (x, y, z) is given by $u=\frac{3x^2-r^2}{r^5}, v=\frac{3xy}{r^5}, w=\frac{3xz}{r^5}$ where $r^2=x^2+y^2+z^2$ and the stream lines are intersection of the surfaces $(x^2+y^2+z^2)^3=c\,(y^2+z^2)^2$ by the planes passing through OX.

(b) Obtain the equation of continuity for spherical symmetrical fluid motion. Discuss it in case of incompressible fluid.

(c) Air, obeying Boyle's law, is in motion in a uniform tube of small section. Prove that if ρ be the density and v the velocity at a distance x from a fixed point at the time t $\frac{\partial^2\rho}{\partial t^2}=\frac{\partial^2}{\partial x^2}\{(v^2+k)\rho\}$

6. (a) A mass of liquid of density ρ whose external surface is a long circular cylinder of radius a, which is subject to a constant pressure p_0, surrounds a Coaxial long Circular Cylinder of radius b. The internal cylinder is suddenly destroyed. Show that if v is the velocity at the internal surface when the radius is r. then.

$$v^2=\frac{2p_0(b^2-r^2)}{\rho r^2\log\left(\frac{r^2+a^2-b^2}{r^2}\right)}$$

(b) Prove that the image of a doublet of strength μ at A, relative to a circle of radius a, is a doublet of strength $\mu' = \frac{\mu a^2}{(OA)^2}$ at B (the inverse point of A) and the axis of doublet makes supplementary angle. O is the centre of the circle.

(c) In the two-dimensional motion, find the stream lines due to uniform sources of strength (+m) at the points A (-c, o), B (c, o) and a uniform sink of strength (-2m) at the origin.

MATHEMATICS (FOURTH PAPER)-2011

(NUMERICAL METHODS)

Time : Three hours] [Maximum Marks : 45

Note : Attempt **all** parts of the question **no. 1** and any **two** parts of each of the remaining questions. Question no. 1carries 10 marks and remaining questions carry 7 marks each.

1. (a) Explain rounding off rules with the help of suitable examples.

(b) Using Bisection method, find the interval of second interation, in which a root of $3x^2 - 4x - 5 = 0$ lies.

(c) Prove that $(1+\Delta)(1-\nabla) = 1$.

(d) Find the polynomial which passes through the points (-2, 1), (-1.5, 2.5) and (0, 3).

(e) Describe Euler's method to solve $y' = f(x, y)$ with initial condition $(x_0) = y_0$.

2. (a) Explain Newton-Raphson method to find the root of an equation $f(x)=0$.

(b) Find a real root of the equation $x^2 - \log_e x - 12 = 0$, using Regula-Falsi method.

(c) Use fixed point iteration method to find the root of the equation $2x = \cos x + 3$ correct to three decimal places.

3. (a) Estimate the population for the year 1945 from the following table :

Year	1941	1951	1961	1971	1981
Population (in thousands)	46	66	81	93	101

(b) Using Lagrange's interpolation formula, find the form of the function y(x) from the following table :

x	0	1	3	4
y	-12	0	12	24

(c) Find the value of y when x = 1.22 from the following table :

x	1.0	1.1	1.2	1.3	1.4
y	0.841	0.891	0.932	0.963	0.985

by using Stirling's formula.

4. (a) Use Trapezoidal rule to evaluate $\int_0^1 \frac{1}{1+x} dx$.

(b) Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\cos x}\, dx$, using Simpson's 1/3-rule with $h = \frac{\pi}{12}$.

(c) Derive Milne's formula for solution of first order ordinary differential equation.

5. (a) Solve $y^1 = x - y^2$, $y(0) = 1$ and find y (0.1) correct to four decimal places, by using Taylor's series method.

(b) Given $y^1 = 1 + y^2$, $y(0) = 0$, find y (0.2) and y(0.4), by using Runge-Kutta method.

(c) Use Adams-Moulton method to find y(0.8) from the differential equation $\frac{dy}{dx} = 1 + y^2$, given that $y(o) = 0$, $y(0.2) = 0.2027$, $y(0.4) = 0.4228$ and $y(0.6) = 0.6841$.

6. (a) Solve the equations by using Gauss-Seidel method :

$4x + 2z = 4, 5y + 2z = -3, 5x + 4y + 10z = 2.$

(b) Decompose the matrix.

$$A = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 5 & 4 \\ 2 & 4 & 14 \end{bmatrix}$$

in the form of LL^T, where L is a lower triangular matrix.

(c) Find the numerically largest eigen value and the corresponding eigenvector of unit length of the matrix

$$\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$ by the power method.

B.A./B.Sc. (Part III) Examination, 2012

Three-year Degree Course

MATHEMATICS

(FIRST PAPER)

Time : Three hours] [Maximum Marks : 60

Note : Attempt **all** questions. They all carry equal marks.

1. (a) If H and K are subgroups of a group G, prove that $HK = \{hk \mid h \in H, k \in K\}$ is a subgroup of G if and only if HK = KH. Under this condition, what is the subgroup generated by the set $H \cup K$? Justify your answer. 6

OR

(i) Prove that a finite semigroup with cancellation laws is a group. 3

(ii) For a group G of order n, prove that $a^n = e$ for every $a \in G$. 3

(b) Prove that a cyclic group G is either isomorphic to $(\mathbb{Z}, +)$, or to $(\mathbb{Z}_n, +)$ for some integer $n \geq 1$.

OR

Prove that $G = \{ \bar{a} \in \mathbb{Z}_n \mid (a, n) = a\}$ is a group under the operation of multiplication of residue classes modulo n and

$G = U_n = \{\bar{a} \in \mathbb{Z}_n \mid \bar{a}.\bar{a}' = \bar{a}'.\bar{a} = \bar{1}$ for some $\bar{a}' \in \mathbb{Z}_n\}$.

2. (a) Define the commutator subgroup G' of a group G and prove that every subgroup H of G, for which $G' \subseteq H$, is normal in G. Find S'_3 and Q'_8.

OR

State and prove the Second (Noether) isomorphism Theorem concerning two subgroups H and K of a group G, at least one of which in normal in G.

(b) Prove that the set A_n of all even permutations of the symmetric group S_n, is a normal subgroup of S_n. Describe the elements of A_4 and also of a proper normal subgroup of A_4 as products of disjoint cycles.

OR

Establish a formula for the order of a permutation σ in S_n in terms of lengths of the cycles appearing in a disjoint cycle decomposition of σ. Find an element p of order 6 in the group S_5 and describe the elements of the cyclic group generated by p.

3. (a) State a necessary and sufficient condition for two elements σ and σ' of S_6, which is conjugate to $\sigma = (1526)(34)$ and also find an element g of S_6 for which $\sigma' = g\,\sigma\,g^{-1}$. Justify your answer.

OR

Define an inner automorphism of a group G. Prove that the set of all inner automorphisms of a group G forms a normal subgroup of the automorphism group A(G) of G.

(b) Prove that the subgroup generated by all the cycles of length 3 in S_n is the alternating group An. Also prove that all cycles of length 3 in A_n (when $n \geq 5$) are conjugate in A_n.

OR

When is an action of a group G on a set S said to be (i) effective (ii) transitive ? Give an example of each of the above type of actions. Describe the action of S_n on the set $S = \{1, 2, ...n\}$ by $\sigma.i = \sigma(i)$ for every $\sigma \in S_n$. What are the orbits of this action ?

4. (a) (i) Define the characteristic of an integral domain and show that it is either 0 or a prime. 3

(ii) Show that $\mathbb{Z}[i] = \{m + in \mid m, n \in \mathbb{Z}\}$ is a subring of the field $\mathbb{C}$ of all complex numbers. Is $\mathbb{Z}[i]$ an integral domain ? If so, find its field of quotients. 3

OR

For ideals I and J of a ring R, describe the ideals $I + J$, $I \cap J$ and IJ. For $R = \mathbb{Z}$, $I = m\mathbb{Z}$, $J = n\mathbb{Z}$, describe the elements of $I + J$, $I \cap J$ and IJ. 6

(b) Prove that every ideal I of the polynomial ring F[x] over a field F, is principal. Verify that $I = \{f \in \mathbb{R}[x] \mid f(6) = 0\}$ is an ideal of $\mathbb{R}[x]$ and find a generator of I.

OR

Prove that a polynomial f[x] of degree n over a field F has at most n distinct roots in F. Give an example of a field F and a polynomial f[x] over F of degree n having at least one but less than n roots in F(including multiplicities).

5. (a) If $(a, m) = 1$ and $\{r_1, r_2, ... r_n\}$ is a reduced residue system modulo m, prove that $\{ar_1, ... ar_n\}$ is also a reduced residue system modulo m. How are m and n related? Prove that $a^{Q(m)} \equiv 1 \pmod{m}$.

OR

State and prove Wilson's Theorem.

(b) State Quadratic Reciprocity Theorem. Using this result calculate the values of the Legendre symbols

$$\left(\frac{523}{227}\right) \text{ and } \left(\frac{226}{173}\right)$$

OR

Define the convolution product of two arithmetical functions. Define the functions τ, σ, and μ. Prove that

$$\sigma(n) = \sum_{d/n} \phi(d)\,\tau\left(\frac{n}{d}\right) \text{ for every } n \in \mathbb{N}$$

MATHEMATICS (SECOND PAPER)-2012

(New Course)

Time : Three hours] [Maximum Marks : 60

Note : Attempt all questions. Symbols used have their usual meanings.

1. (a) Prove or disprove : A bounded set in a metric space is totally bounded. 2

(b) Prove that an infinite set in a compact metric space has at least one limit point. 5

(c) Let $f : X \to Y$ be a continuous map from a metric space X to a metric space Y. If X is compact, then show that f is uniformly continuous. 5

OR

(a) Prove or disprove : The inverse of a bijective continuous map from a metric space to a metric space is continuous. 2

(b) Prove that the unit closed interval [0, 1] in the real line is a compact set. 5

(c) Prove that a sequentially compact metric space is totally bounded. 5

2. (a) Prove or disprove : The map $f : R^2 \to R$ defined by $f(x,y) = x$, $(x, y) \in R^2$, is differentiable. 5

(b) Let $f : R^m \to R^n$ be differentiable at $a \in R^m$, and $g : R^n \to R^p$ be differentiable at f(a). Then prove that $gof : R^m \to R^p$ is differentiable at a, and 5

$$(gof)'(a) = g'(f(a))of'(a).$$

(c) Let $f : R^m \to R^n$ be a differentiable map. Let $x, y \in R^m$ and $a \in R^n$. Then show that

$a.(f(y)-f(x)) = a.(f'(z)\,(y-x))$,

for some $z = (1 - \theta)\,x + \theta y$,

where $\theta \in (0,1)$. 5

OR

(a) Prove or disprove : For the multiplication m of real numbers

$m'(3, 2)\,(2, 3) = 13$. 2

(b) Prove that the function $f : R^2 \to R$ defined by

$$f(x,y) = \begin{cases} \dfrac{xy^2}{x^2+y^2}, & (x,y) \neq (0,0), \\ 0, & (x,y) = (0,0), \end{cases}$$

where $(x,y) \in R^2$ is not differentiable at the origin. Is it continuous at the origin ? Justify your answer. 5

(c) Show that for the map $f : R \to R^2$ defined by $f(t) = (\cos t, \sin t)$, $t \in R$,

$$f(y)-f(x)=f'(z)\,(y-x)$$

where $z \in L\,(x,y)$ is not necessarily true for $x, y \in R$. 5

3. (a) Prove or disprove : The origin is a saddle point of $f : R^2 \to R$ defined by

$f(x,y) = (y - x^2)\,(y - 2x^2)$, $(x,y) \in R^2$. 2

(b) Let $f : R^m \to R$ be a differentiable map. If for each $x \in R^m, x.\nabla f(x) = pf(x)$ for some $p \in N$, then show that f is homogeneous of degree p. 5

(c) State the 'Inverse Function Theorem'.

Show that $f : R^3 \to R^3$ defined by

$f(x,y,z) = (x, \sin y, e^z)$,

where $(x, y, z) \in R^3$, is locally invertible at the origin but f is not one-one. 5

OR

(a) Prove or disprove : The map $f : R^2 \to R^2$ defind by $f(x,y) = (x+y, 2x+ay)$, $(x,y) \in R^2$ is invertible around (x,y) iff a = 2. 2

(b) Describe the Taylor's formula and use it to approximate $\log(1+x+2y)$ by a polynomial of degree one. 5

(c) State the 'Implicit Function Theorem'. Use it show that for $f : R^2 \to R$ defined by $f(x,y) = x^2 + y^2 - 1$, $(x,y) \in R^2$, there exists an open interval G containing $\frac{\sqrt{3}}{2}$ and a unique map $g : G \to R$ satisfying

$$g\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{2}, \text{ and } f(x, g(x)) = 0, \text{ where } x \in G. \quad 5$$

4. (a) Prove or disprove : The series $\sum \frac{\sin nx}{n^4}$ is uniformly convergent in R. 2

(b) Let $\Sigma u_n(x)$ be a series of continuous functions defined on an interval I of R. If it converges uniformly to s(x), then show that s(x) is a continuous function. 5

(c) Define the map 'exp' and prove that

$\exp(x + y) = \exp x . \exp y$,

where $x, y \in R$. 5

OR

(a) Prove or disprove : The series

$$x + \frac{x^2}{2^2} + \frac{x^3}{3^2} + \ldots + \frac{x^n}{n^2} + \ldots$$

is uniformly convergent in [0,1]. 2

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(b) State and prove the 'Dirichlet's Test' for the uniform convergence of a series. 5

(c) Let $(s_n(x))_{n\in N}$ be a sequence of functions defined on an interval (a,b) of R which converges to a function s(x) on (a,b). Suppose that each $s_n(x)$ is differentiable and has continuous differential $s'_n(x)$. If $(s'_n(x))_{n\in N}$ converges uniformly to t(x) on (a,b), then show that differential of s(x) exists at each $x \in (a,b)$, and for $x_0 \in (a,b)$, $s'(x_0) = t(x_0)$. 5

5. (a) Prove or disprove : For $x, y \in R$, $x^2 + y^2 - 2ixy$ is analytic. 2

(b) Suppose that f_x and f_y of a complex function f exist in a neighbourhood of $z \in \mathbb{C}$ If f_x and f_y are continuous at z and $f_y = if_x$, then show that f is differentible at z. 5

(c) Determine an analytic function f satisfying

(i) $f(z_1 + z_2) = f(z_1) \cdot f(z_2)$, $z_1, z_2 \in \mathbb{C}$, and

(ii) $f(x) = e^x$, where $x \in R$. 5

OR

(a) Prove or disprove : The function $f(z) = |z|^2$, is analytic at $z = 0$. 2

(b) Suppose f is analytic in a region and |f| is constant there. Then show that f is constant. 5

(c) Suppose $\sum_{n=0}^{\infty} c_n z^n$ is zero at all points of a nonzero sequence (z_k) which converges to zero. Then show the power series $\sum_{n=0}^{\infty} c_n z^n$ is identically zero. 5

MATHEMATICS (THIRD PAPER)-2012

(Also for Ex-Students)

Time : Three hours] [Maximum Marks : 60

Note : Answer **all** questions. Marks allotted to each question are indicated against it.

1. (a) Define a wrench and mutual moment of two lines. Two wrenches $(\overline{P}, p\overline{P})$ and $(\overline{Q}, q\overline{Q})$ acting on a body have axes whose mutual moment is M, and whose shortest distance apart is 2h. Show that the central axis of their resultant wrench intersects perpendicularly the common perpendicular to their axes, at a distance from its middle point equal to

$$\frac{2h^2(P^2 - Q^2) - MPQ(p-q)}{2h(\overline{P} + \overline{Q})^2}$$ 2+2+3

(b) Find the line coordinates of the conjugate line of the line having line coordinates $(\hat{i}, \hat{j} - \hat{k})$ with respect to the system of forces $(2\hat{i} - 2\hat{j} + 3\hat{k}, 3\hat{i} + 3\hat{j} - 4\hat{k})$. Also obtain the conjugate forces. 3+2

OR

(a) Define central axis of a given system of forces. Two wrenches have pitches p, q and their axes intersect perpendicularly. Prove that the central axis of the resultant wrench lines on the surface $(x^2+y^2)z = (q - p)xy$ 1+4

(b) Define a nul line relative to a given system of forces. Obtain the condition that the line $(\hat{e}, \overline{a})$ should be a nul line relative to the system $(\overline{R}, \overline{M})$ at origin. Also prove that, if a nul line of a given system of forces intersects a given line, then it will intersect its conjugate line with respect to the system. 1+2+4

2. (a) Find the moment of inertia of a solid right circular cone of mass M, height h and radius of base a about a slant side of the cone. 5

(b) Define momental ellipsoid of a body at a point. Show that the momental ellipsoid at a point on the edge of the circular base of a thin hemispherical shell is $2x^2 + 5(y^2 + z^2) - 3zx = \text{constant}$. 2+5

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OR

(a) Prove that the moment of inertia of a triangular plate ABC about the perpendicular to the plate through A is $\frac{M}{12}(3b^2+3c^2-a^2)$, where M is the mass of the plate and a, b, c are sides of the triangle ABC. 6

(b) A wire is in the from of a semi-circle; show that at an end of its diameter the principal axes in its plane are inclined to the diameter at angles $\frac{1}{2}\tan^{-1}\frac{4}{\pi}$ and $\frac{\pi}{2}+\frac{1}{2}\tan^{-1}\frac{4}{\pi}$. 6

3. (a) State D'Alembert's principle. From this principle derive the equations of motion of a rigid body and show that the motion of a body about its centre of inertia is the same as it would be if the centre of inertia were fixed and the same forces acted on the body. 1+2+2

(b) If a rod of length 2a, revolves with uniform angular velocity ω about a vertical axis through a smooth joint at one extremity of the rod so that it describes a cone of semi-vertical angle 30^0, then show that $2a\omega^2=\sqrt{3}g$. Prove also that the direction of the reaction at the hinge makes with the vertical an angle $\tan^{-1}\frac{\sqrt{3}}{4}$. 5+2

OR

(a) A rod of length 2a, is suspended by a string, of length ℓ, attached to one end, if the string and rod revolve about the vertical with uniform angular velocity, and their inclinations to the vertical be θ and φ respectively, show that

$$\frac{3\ell}{a}=\frac{(4\tan\theta-3\tan\phi).\sin\phi}{(\tan\phi-\tan\theta).\sin\theta}$$ 7

(b) A solid homogeneous right circular cone, of height h and semi-vertical angle α, oscillates about a diameter of its base; show that the length of the simple equivalent pendulum is

$$\frac{1}{5}h\left(2+3\tan^2\alpha\right).$$ 5

4. (a) Prove that liquid motion is possible when velocity at (x, y, z) is given by $\frac{3x^2-r^2}{r^5}\hat{i}+\frac{3xy}{r^5}\hat{j}+\frac{3xz}{r^5}\hat{k}$, where $r^2=x^2+y^2+z^2$, and the stream lines are the intersection of the surfaces $(x^2+y^2+z^2)^3=c(y^2+z^2)^2$ by the planes passing through x-axis. 2+4

(b) An infinite mass of fluid is acted on by a force $\mu r^{-\frac{3}{2}}$ per unit mass directed to the origin. If initially the fluid in at rest and there is a cavity to the form of the sphere r = c in it, show that the cavity will be filled after an interval of time $\left(\frac{2}{5\mu}\right)^{\frac{1}{2}}.c^{\frac{5}{4}}$. 6

OR

(a) If the lines of motion are curves on the surfaces of cones having their vertices at the origin and the axis of x for common axis, prove that the equation of continuity is 6

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial r}(\rho q_r)+\frac{2\rho q_r}{r}+\frac{\cos ec\theta}{r}\frac{\partial}{\partial\phi}(\rho q_\phi)=0$$

(b) Derive the Bernoulli's equation for the unsteady irrotational motion of a fluid. 6

5. (a) A velocity field is given by

$$\overline{q}=-x\hat{i}+(y+t)\hat{j}$$

Find the stream function and prove that the stream lines for this field at t = 2 are rectangular hyperbolas. 4+2

(b) What do you mean by two dimensional doublet ? State circle theorem and use it to obtain the image of a doublet with regard to a circle. 1+1+4

OR

(a) Prove that for the complex potential $\tan^{-1}z$ the stream lines and equi-potentials are orthogonal systems of circles. 5

(b) find the stream function of the two dimensional motion due to two equal sources of strength m at distance 2a apart and sink of strength 2m between them. Also determine the stream line. If A and B be the positions of sources and O is mid point of AB, then prove that fluid speed at any point P will be $\frac{2ma^2}{OP.AP.BP}$ 3+2+2

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MATHEMATICS (FOURTH PAPER)-2012
(NUMERICAL METHODS)

Time : Three hours] [Maximum Marks : 45

Note : Attempt **all** parts of the question. Marks are indicated against each question. Symbols have their usual meaning.

1. (a) Explain the graphical method to find an initial approximation of a root of the equation f(x)=0. 2

OR

Giving suitable examples, describe the rounding off rules in numerical computation.

(b) Find the root of the equation $x^2-3x+2=0$ in the vicinity of x=0, using Newton-Raphson method. Do three iterations. 3

OR

Find the negative root of the equation $x^2-4x-10=0$, using bisection method. Do three iterations.

(c) Define the convergence of an iterative method and show that Newton-Raphson method converges quadratically. 4

OR

Give geometrical illustration of Regula-Falsi method and obtain a formula for the successive approximations to the root of the equation f(x)=0 by the method.

2. (a) Show that the operators μ and E commute.

OR

Prove that $E^{\frac{1}{2}} = \mu + \delta/2$.

(b) Define the operator ∇ and derive the Newton's backward differences interpolation formula. 3

OR

Find the missing values in the following table :

x	0	1	2	3	4	5	6
y	-4	-2	-	-	220	546	1148

(c) Find Newton's forward differences interpolating polynomial for the following data : 4

x	0	1	2	3	4	5
f(x)	-3	3	11	27	57	107

Hence, or otherwise, find f(1.5).

OR

Find the interpolating polynomial by (i) Lagrange's formula, and (ii) Newton's divided difference formula for the following data : y(1) = -3, y(3) = 9, y(4) = 30 and y(6) = 132.

3. (a) Define a cubic spline interpolating function. 2

OR

Write Simpson's $\frac{1}{3}$ rule for numerical integration $\int_a^b f(x)dx$. and explain its geometrical significance.

(b) Derive trapezoidal rule of integration for $\int_a^b f(x)dx$. 3

OR

Find f'(0,4) from the following table :

x	0.1	0.2	0.3	0.4
f(x)	1.10517	1.22140	1.34986	1.49182

(c) Using the natural cubic spline interpolation, find the value of y at $x = \frac{1}{2}$, for the following data :

x	0	1	2	3
y	1	2	9	28

OR

Fit a clamped cubic spline curve, in the intervals [0, 1] and [2, 3], that passes through the points (0, 0.0), (1, 0.5), (2, 2.0) and (3, 1.5) with the end conditions S' (0) = 0.2 and S' (3) = -1.0.

4. (a) Describe Taylor series method to solve an IVP of first order. 2

OR

Describe Euler's method to solve an IVP of first order.

(b) Given $\frac{dy}{dx} = x^2 + y, y(0) = 1$, find y (0.1) using modified Euler's method. (take h = 0.05) 3

OR

Solve $\frac{dy}{dx} = y - x, y(0) = 2$ to find y (0.1) using Runge-Kutta fourth order method. (Take h = 0.1)

(c) Derive Milne's predictor - corrector formulae for solving an IVP of first order. Also, explain how to use the method. 4

OR

Given $y'' = \frac{1}{2}(x + y + y' + 2), y(0) = 0, y'(0) = 0$, find y(0.2) by using Runge-Kutta-Nystrom method. (Taking h=0.2).

5. (a) Explain the concept of pivoting in the Gaussian Elimination method. 2

OR

Describe the Gauss-Jacobi iteration mrthod to solve a system of linear algebraic equations.

(b) State Gerschgorin Theorem and find the interval which contains the eigen values of the matrix 3

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 5 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$

OR

Explain the concept of curve fitting by the method of least squares. Discuss the fitting of parabola by this method to a given set of values.

(c) Describe Power method to find the Numerically largest eigen value and a corresponding eigen vector of a square matrix A of order n. Using the method, find the largest eigenvalue of the matrix

$$\begin{bmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 0 & 4 \end{bmatrix}$$

and a corresponding eigen vector after second iteration starting with the initial vector $\overline{V}^0 = [0,0,1]^T$. 4

OR

Explain the method of decomposing a square matrix as the product of a lower and an upper triangular matrix. Using the method, solve the system of equations.

$2x + 3y + z = 9$; $x + 2y + 3z = 6$; $3x + y + 2x = 8$

B.A./B.Sc. (Part III) Examination, 2013

MATHEMATICS (FIRST PAPER)

Note : Attempt **all** questions. They all carry equal marks.

1. (a) Prove or disprove : $(G,*)$, where $G = (0,1) \cup (1,\infty) \subset \mathbb{R}$ and for $a, \in b\ G$, $a * b = a^{\log_e b}$ is a group 4

(b) Prove that a finite subgroup of the multiplicative group $\mathbb{C}^*$ of nonzero complex numbers in cyclic. 4

(c) Find the centre of the group of rigid motions of a regular n-gon, where $n \geq 3$. 4

OR

1. (a) Prove or disprove : The group of symmetrics of a rectangle of unequal sides is isomorphic to the Klein's four group. 4

(b) Compute Art $\mathbb{Z}_n$. 4

(c) Find the centre of GL $(2,\mathbb{R})$ 4

2. (a) Prove or disprove : There are exactly eight 3-cycles in A_4. 4

(b) Prove that the orthogonal group O(2) is not a normal subgroup of GL, $(2,\mathbb{R})$. 4

(c) Prove that S_n is generated by (1 2) and (1 2 .....n). 4

OR

2. (a) Prove or disprove : $\mathbb{Q}/\mathbb{Z}$ is an infinite group each of whose elements has finite order. 4

(b) Is $S^1 \times \mathbb{Z}$ isomorphic of $\mathbb{R}$, where S^1 is the multiplicative group of complex numbers with modulus 1 ? Justify your answer. 4

(c) Let G_1 and G_2 be two non-abelian groups each of order 6. Then prove that G_1 is isomorphic to G_2. 4

3. (a) Prove or disprove : Cycles (2 4 5 7) and (3 4 5 6) in S_7 are conjugate. 4

(b) State and establish the class formula for a finite group . 4

(c) Prove that a subgroup H of a group G is normal if and only if it is a union of conjugacy classes. 4

OR

3. Classify groups of Order 8. 12

4. (a) Show that $R = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a,b,c,d \in \mathbb{Z}_2 \right\}$ forms a ring with the matrix addition and multiplication modulo 2. Is

$$A = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} r : r \in R \right\}$$

an ideal of R? Support your answer. 4

(b) Prove or disprove : There are exactly two self-inverses in the filed $\mathbb{Z}_p$.

4 (c) Show that $4x^2 + 6x + 3$ is a unit in $\mathbb{Z}_8[x]$

4. (a) Let $R=\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} : a \in \mathbb{Z}\right\}$. Then show that R is a ring with matrix addition and multiplication. Define $\varphi: R \to \mathbb{Z}$ by

$$\varphi\begin{bmatrix} a & b \\ b & a \end{bmatrix} = a - b.$$

Show that φ is a ring homomorphism and R/ Ker φ is isomorphic to $\mathbb{Z}$. 6

(b) Sate and prove the 'Division Algorithm' for F [x], where F is a field. 6

5. (a) Define the Möbius function μ and show that it is multiplicative. 4

(b) Let p be an odd prime and a be an integer such that p does not divide a. Then a is quaratic residue modulo p if and only if

$$a^{\frac{p-1}{2}} = 1 \pmod p$$ 4

(c) For an odd prime p, show that $\left(\frac{a}{p}\right) = \left(\frac{r}{p}\right)$ where is the remainder obtained when the integer a is divided by p. 4

OR

5. (a) State and prove the Chinese remainder theorem. 6

(b) For an odd prime, show that 6

$$\left(\frac{2}{p}\right) = (-1)^{\frac{p^2-1}{8}}$$

MATHEMATICS (SECOND PAPER)-2013

Note : Attempt all questions. Symbols used have their usual meanings.

1. (a) Prove or disprove : For metric spaces (X,d_1) and (Y,d_2), a uniformly continuous function $f: X \to Y$ is continuous. 2

(b) Let X be a complete metric space and F be a subset of X. Prove that F is closed iff F is complete. 5

(c) Prove that every open cover of a sequentially metric space has a Lebesgue number. 5

OR

1. (a) Prove or disprove : Every continuous function defined on a metric space is uniformly continuous. 2

(b) Prove that a closed set in a compact metric space X is a compact set. 5

(c) Prove that a compart metric space is complete. 5

2. (a) Prove or disprove : A constant map is differentiable everywhere and its differential at each point is the zero map. 2

(b) If $f: \mathbb{R}^2 \to R$ be defined by

$$f(x,y) = \begin{cases} 0, & xy \neq 0 \\ y+1; & \text{otherwise} \end{cases}$$

then Show that f is not differentiable at (0,0) and directional derivative $D_u f(0)$ exist only for $u_1, u_2 = 0$ where $u = (u_1, u_2)$. 5

(c) Prove that if S be an open set of R^n and $f: S \to R$ is differentiable then $f(y) - f(x) = f'(z)(y-x)$ where $Z \in L(x,y)$ and $L(x,y) = \{(1-t)x + ty : t \in [0,1]\}$. 5

OR

(a) Prove or disprove, partial derivative $D_j f(x)$ is the directional derivation of f along. e_j(unit coordinate vectors). 2

(b) Let where $f: S \to R^m$, where $S \subset R^n$, be differentiable at $x \in S$ and $g: R^m \to R^p$ be a linear map. Then prove that the map $g_0 f'(x)$. Also prove that each f_i is differentiable if $f = \{f_1, f_2, \ldots f_n\}$ is differentiable. 5

(c) If $f: S \to R$, where open $S \subset R^n$, is differentiable at x, then partial derivative $D_j f(x)$ exists for each j, $1 \leq j \leq n$ and $f'(x)(v) = \nabla f(x).v$ for every $v \in \mathbb{R}^m$. 5

3. (a) Prove or disprove that (0,0) is neither a local maximum nor a local minimum for the function $f: R^2 \to R$ defined by $f(x,y) = y^2 - x^3$. 2

(b) If $f: R^2 \to R$ has continuous second order derivatives and $f(t_x, t_y) = t^n f(x,y)$ for all $t \in \mathbb{R}$, where n is an integer s.t. then prove $n \geq 1$ then prove that

$$x^2\frac{\partial^2 f}{\partial x^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} + y^2\frac{\partial^2 f}{\partial y^2} = n(n-1) + (x,y)$$ 5

(c) If x and y be open subsets of R^n and $f: x \to y$ is an onto map such that- $\|f(x+v)\| - f(x)\| \geq k\|v\|$ for all $x, x+v$ in x and $k>0$, real number then prove that f is a bijection and inverse map $g = f^{-1}: y \to x$ is continuous. 5

OR

(a) Prove or disprove that the function defined by $f(x,y) = y^2 + x^2y + x^4$ has a maximum at (0,0). 2

(b) State Taylor's expansion for a function and $f: R^m \to R$ express $x^2 + xy + y^2$ in powers of x-1 and y -2. 5

(c) State implicit function theorem and use it to show that the equations $t^2 + x^3 + y^3 + z^3 = 0$, $t + x^2 + y^2 + z^2 = 0$; $x + y + z = 0$ have solutions around $(t, x, y, z) = (0, -1, 1, 0)$. 5

4. (a) Prove or disprove : the sequence $\{f_n\}$ where $f_n(x) = x^n$ is uniformly convergent on [0,1]. 2

(b) If a series Σf_n converges uniformly to f on [a,b] and each term $f_n(x)$ is integrable, then prove that f is integrable on [a,b] an the series $\Sigma\left(\int_a^x f_n dt\right)$; converges uniformly to $\int_a^x f dt$ on [a,b] is

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$$\int_a^x f dt = \sum_{n=1}^{\infty}\left(\int_a^x f_n dt\right) \text{ for } \forall x \in [a,b]$$ 5

(c) If a power series $\sum a_n x^n$ converges at each point x of the interval of convergence (-R,R) then prove that it is uniformaly convergent in the closed interval [0,R]. 5

OR

(a) Prove or disprove : The power series $\sum \frac{x^n}{n!}$ is absolutely convergent for every value of x. 2

(b) If $b_n(x)$ is a positive, monotonic decreasing function of n for each fixed value of x in the interval [a,b] and $b_n(x)$ is bounded for all n and x concerned, and if the series $\sum u_n(x)$ s uniformly convergent then show that the series $\sum b_n(x)u_n(x)$ is uniformly convergent over [a,b]. 5

(c) Show by using WeierstrassM$_n$-Test that 'O'is a point of non-uniform convergence of the sequence $\{f_n\}$ where $f_n(x) = nxe^{-nx^2}$; $x \in R$.

5. (a) Prove or disprove : the function $f(z) = c_0 + c_1 z + c_2 z^2 + \ldots + c_n z^n$ Where $c_0, c_1, \ldots, c_n$ are complex constants is analytic in the entire complex plane. 2

(b) If f(z) is an analytic function of z, then prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$$ 5

(c) Show that the function $f(z) = |(z)|^2$ is not analytic at any point $z = z_0$ except at $z = 0$ 5

OR

(a) Prove or disprove : The function $fz = \overline{Z}$ where $\overline{Z}$ is conjugate of Z (Complex number), is not analytic at any Z. 2

(b) State Cauchy Riemann equations for function $f(z) = u + iv$ and use it to show that the derivative $f'(z)$ exists every where $f(z) = z^2$. 5

(c) Prove that a power-series with non-zero radius of convergence R represents an analytic function over $|z| < R$ and the derived series has the same radius of convergence as the original series. 5

MATHEMATICS (THIRD PAPER)-2013

(Also for Ex-Students)

Note : Answer **all** questions. All the questions carry equal marks.

1. (a) Prove that any force system is equivalent to a single force passing tnrough a prescribed point and a couple. A system of forces is equivalent to a single force $\overline{R} = \hat{i} + \hat{j} + \hat{k}$ and a single couple of moment $\overline{M} = \hat{i} - 2\hat{j} + 3\hat{k}$ at the point (1,2,3). Find the symmetrical form of equations of the cental axis.

OR

Define a wrench and pitch of the wrench of a given system of force. Two wrenches of pitches p,q have axes at a distance '2a' from one another. If the resultant wrench is of pitch w and its axis is equidistant from the axes of the component wrenches, show that the angle between them is

$$\tan^{-1}\left[\frac{a(2w-p-q)}{a^2-(w-p)(w-q)}\right]$$

(b) Define conjugate forces and conjugate lines of a given system of forces.Agiven system of force $(\overline{R},\overline{M})$ at origin is reduce to two non-zero finite forces. show that the lines of action of these two forces do not intersect. 5

OR

Obtain the condition that the line $a(\hat{e},\underline{a})$ should be a nul line relative to the system at origin. Forces $(\overline{R},\overline{M})$ at origin. Forces $\hat{i}-3\hat{j}+2\hat{k}, -2\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}-\hat{j}+4\hat{k}$ are acting at the points (1,0,1),(-1,1,0), and (0,1,2) respectively.Find the equation of the nul plane of the point (1,1,1) with respect to the system of given forces. 5

2. (a) Define the principal axes of a body at a point. Show that one of the principal axes at the node of a half-loop of the curve $r^2 = a^2 \cos 2\theta$ is inclined to the initial line at an angle $\frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right)$.

OR

A Uniform lamina is bounded by a parabolic arc, of latus rectum 4a, and a double ordinate at a distance b from the vertex. If $b = \frac{a}{3}(7+4\sqrt{7})$, show that

(b) State the perpendicular axes theorem. Show that the moment of inertia of an elliptic area of mass M and whose semiaxes and b about a diameter of length 2r is $\frac{1}{4}\frac{Ma^2b^2}{r^2}$

OR

Find the equation of the momental ellipsoid at a point on the rim of a hemisphere.

(a) Define the compound pendulum and find its time period. A solid homogeneous cube of side 2a oscillates about and edge which is horizontal ; determine the length of the simple equivalent pendulum.

OR

A fine string has two masses, M an M', tied to its ends and passes over a rough pulley, of mass m, whose centre is fixed . If the string does not slip over the pulley, show that M will descend with acceleration

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$$\frac{(M-M^1)g}{\left(M+M^1+m\frac{k^2}{a^2}\right)},$$

where a is the radius and k the radius of gyration of the pulley.

(b) A thin uniform rod has one end attached to a smooth hinge and is allowed to fall from a horizontal position; show that the horizontal strain on the hinge is greatest when the rod is inclined at an angle of 45^0 to the vertical, and that the vertical, strain is then 11/8 times the weight of the rod .

OR

State and prove the D' Alembert's principle. Also, prove that the centre of inertia of a body moves as if all the mass of the body were collected at it, and as if all the external forces were acting at it in directions parallel to those in which they act.

4. Prove that the Lagrangian and the Eulerian forms of the equation of continuty are equivalent. **OR**

If the velocity $\bar{q}$ at a point (r,θ,φ) of a liquid is given by

$\bar{q} = (2Ar^{-3}\cos\theta, Ar^{-3}\sin\theta, 0)$

where A is a constant, then show that the liquid motion is irrotational. Find the velocity potential and the equations of the stream lines.

(b) Write the components of accelration of a moving fluid particle in the cylindrical polar coordinates. Steam is rushing from a boiler through a conical pipe, the diameters of the ends of which are D and d; if V and v be the corresponding velocities of the steam, and if the motion be supposed to be that of divergence from the vertex of the cone, show that

$$\frac{v}{V}=\frac{D^2}{d^2}\left(\frac{v^2-v^2}{2k}\right),$$

where k is the pressure divided by the density and being constant.

OR

A sphere of radius a is surrounded by infinite liquid of density ρ, the pressure at infinity being π The sphere is suddenly annihilated. Show that the pressure at distance r from the centre immediately falls to $\pi\left(1-\frac{a}{r}\right)$

5.(a) What arrangement of sources and sinks will give rise to the complex potential function cot $h^{-1}z$? Also, prove that the stream lines and equipotentials are orthogonal system of circles in this case.

OR

State the Milne-Thomson circle theorem. A source and a sink of equal strength are placed at the points $\left(\pm\frac{1}{2}a,0\right)$ within a fixed circular boundary $x^2+y^2=a^2$. Show that the stream lines are given by

$$\left(r^2-\frac{1}{4}a^2\right)\left(r^2-4a^2\right)-4a^2y^2=\lambda y(r^2-a^2),$$

(b) Define a two dimensional doublet. Within a rigid boundary in the form of the circle $(x+a)^2+(y-4a)^2=8a^2$, there is liquid motion due to a doublet of strength μ at the point (0, 3a), with its axis along the axis along the axis of y. Find the velocity potential of liquid motion.

OR

Show that the image system for a source outside a circle consists of an equal source at the inverse point and an equal sink at the centre of the circle.

MATHEMATICS (FOURTH PAPER)-2013

(Numerical Methods)

Note : Attempt **all** the questions. Marks are indicated against each question. Symbols have their usual meaning.

1. (a) Compare the relatvie accuracies of the approximations (13/19)~ 0.684 and $\sqrt{52}$ ~ 7.21. **OR** 4

(a') In the method of false position (regulafalsi), derive the iterative steps of finding x_{n+1} from x_n when finding a root of an equation f (x)=0.What must be the condition on the function f for success of this method ?

(b) Estimate a root of $x^3-3x+1=0$ in the interval (0,1) by Newton Raphson's method. **OR** 5

(b') Find a positive root near 0 of the equation $5x^3-20x+3=0$ by fixed point iteration method. Justify your procedure.

2. (a) State and prove Stirting's imterpolation formula. 4

OR

(a') Find the interpolation polynomial and estimate the value of f (2.4):

x	-1	0	1	2	3
f(x)	10	2	0	10	62

(b) State the interpolation formulae applicable to the following data and estimate the value of f (0) : 5

x	-1	2	4	5
f(x)	5	3	8	5

OR

(b') Estimate the missing value of f(4) from the following data :

x	1	2	3	4	5
f(x)	2	5	7	-	32

3. (a) Find the clamped cubic spline S(x) passing through the points (0,0.0), (1,0.5),(2,2.0),(3,1.5) and satisfying S'(0) = 0.2, S'(3) = -1 4

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OR

(a) Estimate the values of f' (0.4) and f''(0.4) from the data

x	0.1	0.2	0.3	0.4
f(x)	1.105	1.221	1.350	1.492

(b) State and prove Simpson's 1/3 rule for evaluation of $\int_{x_0}^{x_{2m}} f(x)dx$ where $x_i - x_{i-1} = h$ for each $1 \le i \le 2m$ **OR**

(b) Estimate the value of $\int_1^4 f(x)dx$ by simpson's (3/8)-rule using the data :

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
f(x)	0.00	0.41	0.69	0.92	1.10	1.25	1.39

4. (a) Estimate the values of y (0.1) and y (0.2) and y (0.2) by Improved Euler's method for the initial value problem 4

$yy' = x$; $y(0) = 1.5$ **OR**

(a') Estimate the value of y (1.2) in one step for the initial value problem: $y' = y^2 + xy$; $y(1) = 2$ by IV order Runge-Kutta method. State clearly the formulas being used.

(b) State and prove Adams-Moulton corrector formula for estimating corrected estimate of $y(x_{n+1})$ from given approximations of $y(x_{n+1}), y(x_n), y(x_{n-1})$ $(x_n = x_0 \to nh)$ for the initial value problem : $y' = f(x,y)$, $y(x_0) = y_0$. 5

(b) Explain the second order Taylor series method for finding approximate values of $y(x_n), (xn = x_0 + nh)$ for the second order initial value problem: $y'' = f(x,y,y')$; $y(x_0) = y_0$, $y'_{(x0)} = y'_0$

5. (a) Solve the system of equations by LU - decomposition of the coefficient matrix with proper justifications : 4

$x + y + z = 3$ $\quad 2x - y + 3z = 16$ $\quad 3x + y - z = -3$

OR

(a) Solve the following system by Gauss jacobi iteration method, justifying its use. Carry out three iterations starting with (1,1,1) :

$5x - 2y + z = 13$ $\quad x + 20y - 2z = 8$ $\quad 3x + 7y - 11z = 2$

(b) Estimate an eigen value and a corresponding normalized eigen vector by power method for matrix

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$ **OR** 5

(b') Find the Gerschgorin circles for estimating the eigen values of the matrix

$$\begin{bmatrix} 1 & 2 & -1 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

B.A./B.Sc. (Part III) Examination, 2014

Three-year Degree Course

MATHEMATICS

(FIRST PAPER)

Time : Three hours] [Maximum Marks : 60

Note : Attempt **all** questions. Symbols used have their usual meaning.

1. (a) Define the subgroup of a group generated by a subset of the group. Determine the subgroup of S_4 generated by $\{(12), (1234)\}$. 4

(b) Determine all homomorphisms from $(Q_8, \cdot)$ to $(\mathbb{Z}_8, +)$

(c) Prove that $\mathbb{Z}_{12}$ has subgroups H and K such that $H \cap K = \{\bar{0}\}$ and $H + K = \mathbb{Z}_{12}$. 4

OR

1. (a) Define automorphism group of a group G. Determine all automorphisms of $(V_4, +)$. 4

(b) Define order of an element in a group (G, ·). Let $M \in \mathbb{N}$. Prove that for each divisor d of m, all elements of order d in $\mathbb{Z}_m$ are in the same subgroup of $\mathbb{Z}_m$. 4

(c) State and prove Lagrange's Theorem. 4

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2. (a) State the subgroup Correspondence Theorem. Use it to determine the number of subgroups of $2\mathbb{Z}/18\mathbb{Z}$

(b) Determine the commutator subgroup of D_{12}.

(c) Find a permutation p of S_8 such that $p\,(1357)p^{-1} = (3268)$ and $p(24)p^{-1} = (57)$. 4

OR

2. (a) Give examples of two groups G_1, G_2 and $H_1 \trianglelefteq G_1, H_2 \trianglelefteq G_2$ such that G_1/H_1 and G_2/H_2 are isomorphic but G_1 and G_2 are non-isomorphic. 4

(b) Let $G = \left\{\begin{bmatrix} \bar{1} & \bar{a} & \bar{b} \\ 0 & \bar{1} & \bar{c} \\ 0 & 0 & \bar{1} \end{bmatrix} : \bar{a}, \bar{b}, \bar{c} \in \mathbb{Z}_3\right\}$

Prove that G is a group under the binary operation as the multiplication of matrices. Find Z (G). 4

(c) Let C be a cycle of length d in S_n. Prove that C has order d. 4

3. (a) Let G be an abelian group of order 8. Prove that G is isomorphic to either $\mathbb{Z}_8$ or $\mathbb{Z}_4 \times \mathbb{Z}_2$ or $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ 6

(b) Define normalizer of a subgroup H of a group G. Determine all conjugates of the subgroup.

$H = \{I, (12), (13), (23), (123), (132)\}$ of S_4 in S_4. 6

OR

3. (a) Let G be a group of order p^2, where p is a prime. Prove that either $G \cong \mathbb{Z}_{p^2}$ or $G \cong \mathbb{Z}_p \times \mathbb{Z}_p$. 6

(b) Define the centralizer $C_G(x)$ of an element x in a group G. Determine $C_G(x)$, for $G = A_4$ and $x = (123)$; and for $G = S_5$, $x = (12)\ (34)$. 6

4. (a) Let R be a commutative intergral domain with identity. Assume that Char (R) = p > 0. Prove that $\forall\, a, b \in R$, $(a+b)^p = a^p + b^p$ 4

(b) Determine all ring homomorphisms from $\mathbb{Z}_6$ to itself. 4

(c) Let $f(x) \in \mathbb{Z}_7[x]$. Prove that there exists $q(x) \in \mathbb{Z}_7[x]$. such that $f(x) = (x - \bar{1})q(x) + f(\bar{1})$. 4

OR

4. (a) Assuming that a group of order 15 is cylic, prove that an intergral domain R consisting of 16 elements is a field. 4

(b) Let R be a commutative integral domain of characteristic zero. Prove that there is an injective ring homomorphism

$f : Q \to R$. 4

(c) Let R be a non-zero commutative integral domain with identity. Let f(f), g(x) be non-zero polynomials in R[x].

Prove that

deg (f(x).g(x)=deg(f(x))+deg(g(x)). 4

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5. (a) Let $n \in \mathbb{N}$. Prove that $\sum_{d/n} \phi(d) = n$, where ϕ is the Euler's phi-function.

(b) Determine the smallest positive integer n which when divided by 3 and 5 leaves remainders 2 and 3 respectively. 4

(c) Determine all odd primes $p \neq 3$ such that $\left(\frac{3}{p}\right) = 1$

OR

5. (a) Prove that if f is a multiplicative function, then the arithmetic function F defined by $F(n) = \sum_{d/n} \mu(d) f(d)$ $(n \in \mathbb{N})$ is also multiplicative. 4

OR

(b) Determine the legendre symbols $\left(\frac{31}{59}\right)$ and $\left(\frac{-1}{37}\right)$. 4

(c) Find the smallest non-negative solution of $x \equiv 6 \pmod{22}$. and $x \equiv 3 \pmod{5}$. 4

MATHEMATICS (SECOND PAPER)-2013

(New Course)

Time : Three hours] [Maximum Marks : 60

Note : Attempt all questions. Symbols used have their usual meanings.

1. (a) Prove or disprove : The set [0,1) is a closed set in the real line. 2

(b) Prove that a closed set of a compact metric space is compact. 5

(c) Let X be a complete metric space and $F \subseteq X$ Then show that F is complete if and only if it is closed in X. 5

OR

1. (a) Prove or disprove : A bounded set in a metric space is compact. 2

(b) Define a continous map from a metric space to a metric space. State and prove a characterization of a continuous map. 5

(c) Prove that a compact metric space is complete. 5

2. (a) Prove or disprove : The multiplication map $m : R \times R \to R$ 2

(b) Give an example of a function $f : R^2 \to R$ which is continuous at (0,0) but not differentiable there will complete arguments. 5

(c) Let $f : R^m \to R^n$ be a differentiable map. Let $x, y \in R^m$ and $a \in R^n$. Then show that

a. $(f(y) - f(x)) = a \cdot (f'(z))(y-x))$,

for some $z = (1-\theta)x + \theta y$,

where $\theta \in (0,1)$. 5

OR

2. (a) Prove or disprove : The derivative of the map $f : R^2 \to R$ defined by $f(x,y) = x + y$, $(x, y) \in R^2$, at (0, 0) is f itself.

(b) By means of an example show that the partial derivatives need not commute.

(c) State and prove the Chain rule. 5

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3. (a) Prove of disprove : The function $f : R^2 \to R^2$ defined by $f(x,y) = (e^x + e^{-y}, e^x + e^{-y},), (x,y) \in R^2$, is locally invertible at each point of R^2. 2

(b) Let both the partial derivatives $D_1 f$ and $D_2 f$ of a function $f : R^2 \to R$ exist and be differentiable, then show that for $(x, y) \in R^2$, $D_1 D_2 f(x,y) = D_2 D_1 f(x,y)$. 5

(c) Determine extrema of $f(x,y) = xy$ subjected to the constraint $x+y=C$. 5

OR

3. (a) Prove or disprove : The map $f : R^n \to R$ defined by $f(x) =$ is differentiable $f(x) = \|x\|^2, x \in R^n$, is differentiable. 2

(b) Let $f : R^m \to R$ be a differentiable map. If for each $x \in R^m, x.\nabla f(x) = pf(x)$, where $p \in \mathbb{N}$. Then show that f is a homogeneous function of degree p. 5

(c) Investigate maxima and minima of $f(x,y) = x^3+y^3-3x+12y+20$ 5

4. (a) Proved or disprove : The series $\sum \frac{\cos nx}{n^2}$ is uniformly convergent in R. 2

(b) State and prove the Dirichlet's test for uniform convergence of a series. 5

(c) State and establish the conditions under which the series $\sum u_n(x)$ is term by term differentiable.

OR

4. (a) Prove or disprove : The series $\sum_{n=0}^{\infty} x^n(1-x)$ is uniformly convergent in [0,1]. 2

(b) Let $\sum u_n(x)$ be a series of continuous functions defined on an interval I of R. If it converges uniformly to s (x) on I, then prove that s(x) is a continuous fucntion. 5

(c) Introduce the function exp. Determine its range. 5

5. (a) Prove or disprove : If $f : \mathbb{C} \to \mathbb{C}$ is differentiable at $z \in \mathbb{C}$ then f is analytic at z. 2

(b) Show that
$$f(z) \equiv f(x,y) = \begin{cases} \frac{xy(x+iy)}{x^2+y^2} & , z \neq 0 \\ 0 & , z = 0 \end{cases}$$
is not differentiable at $z = 0$, although it satisfies the C-R equations. 5

(c) Show that an analytic function $f : \mathbb{C} \to \mathbb{C}$ which satisfies. 5

(i) $f(z_1+z_2) = f(z_1).f(z_2), z_1, z_2 \in \mathbb{C}$, and

(ii) $f(x) = e^x, x \in R$ exists.

OR

5. (a) Prove or disprove : The series $\sum_{n=1}^{\infty} nz^n$ converges for $|z| < 1$.

(b) Suppose f_x and f_y exist in a neighbourhood of z, whe... $f : \mathbb{C} \to \mathbb{C}$ is function . If f_x and f_y are continuous and $f_y = if_x$ at z, then show that f is differetiable at z. 5

(c) If f is analytic in region and if $|f|$ is constant there, then show that f is constant. 5

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MATHEMATICS (THIRD PAPER)-2013

(New Course)

Time : Three hours] [Maximum Marks : 60

Note : Attempt **all** questions. They carry equal marks.

1. (a) A given system of forces is reduced to a wrench. Show that the pitch of the wrench is an invariant for the given system of forces.

The distance between the lines of action of two forces of magnitudes 10 and 15 units is given to be 24 units. If the direction cosines of the lines are $\sqrt{2/3}, \frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}$ and $\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \sqrt{2/3}$, respectively, then obtain in the pitch of the equivalent wrench of the above forces.

OR

The axes of two given wrenches $(\overline{P}, p\overline{P})$ and $(\overline{Q}, q\overline{Q})$ pass through the points A,B respectively, where AB is the common perpendicular to their axes. If O is the middle point of the line AB, then obtain an equation of the axis of the wrench, which is equivalent to the given wrenches, referring to O as the origin (base of the reduction).

(b) Obtain the vector equation of the nul plane, of a point P whose position vector is, $\overline{p}$ with respect to the system at the origin O and hence obtain the nul point of the plane.

$2x + 3y + 5z + 6 = 0$,

with respect to the above system of forces.

If ℓ is any line through P, what can you say about the position of its conjugate line ℓ' ? Explain.

OR

If a nul line, relative to a given system of forces , intersects a given line, prove that it also intersects its conjugate line with respect to the given system.

A system of forces is equivalent to a single force $\overline{R} = \hat{i} - \hat{j} + \hat{k}$ together with a couple of moment $\overline{M} = \hat{i} - 2\hat{j} + 3\hat{k}$ at the point A(1,2,3). Show that the line, $\bar{r} = t\hat{i} + 4\hat{j} + 2\hat{k}$ is a nul line relative to the given system.

2. (a) Define momental ellipsoid of a body at a point . Also find the moment of inertia of a lamina of mass M bounded by the curve $y = \sin x$ and the x-axis, lying between $x=0$ and $x=\pi$, about the y-axis.

OR

Show that the sum of the moments of inertia of an elliptic lamina, about any two perpendicular tangents, is constant.

(b) Define the principal axes of a body at a point . Also find the equation of momental ellipsoid at the corner of a solid cube of side '2a', referred to its principal axes.

OR

A wire is in the form of a semi-circle of radius 'a' ; show that at an end of its diameter, the principal axes in its plane are inclined to the diameter at angles

$$\frac{1}{2}\tan^{-1}\left(\frac{4}{\pi}\right) \text{and} \frac{\pi}{2} + \frac{1}{2}\tan^{-1}\left(\frac{4}{\pi}\right).$$

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3. (a) State D' Alembert's principle and using it write the general equations of motion of a rigid body. A solid homogeneous cube of side '2a' oscillates about a diagonal of one of its faces, which is horizontal; determine the length of the simple equivalent pendulum and the time period.

OR

(b) A plank of mass M and length '2a' is initially at rest along a line of greatest slope of a smooth plane, inclined at an angle 30^0 to the horizon, and a man of mass M', starting from the upper end, walks down the plank so that it does not move. Show that the man gets to the other end in time,

$$\sqrt{\frac{8aM^1}{(M+M^1)g}}$$

(b) Find expressions for velocity and acceleration of a particle, moving on the surface of a sphere of radius 'a' .

OR

A solid right circular cone of vertical angle 90^0 can turm freely about a horizontal axis, passing through the centre of its base and perpendicular to its axis. It the cone starts from rest with its axis horizontal, show that when the axis is vertical, the ratio of the thrust on the fixed axis to the weight of the cone is 3 : 2.

4. (a) The velocity components of a liquid in cylindrical polar coordinates (r,θ,z) are given by

$$V_r=\left(1-\frac{A}{r^2}\right)\cos\theta, V_\theta=-\left(1+\frac{A}{r^2}\right)\sin\theta, V_z=0,$$

Where A is a constant . Show that the liquid motion is possible. Is the motion irrotational ? Explain.

Also obtain the equations of the stream lines.

OR

(a) Write the condition that the surface $F(x,y,z,t)=0$, may be a boundary surface of a moving liquid.

If the velocity of an incompressible fluid at the point (x,y,z) is given by $\bar{V}=\frac{2(-y\hat{i}+x\hat{j})}{r^2}$

where r denotes the distance from the z-axis, find surfaces which are orthogonal to the stream lines.

(b) Derive the Bernoulli's equation for the unsteady irrotational motion of a fluid. Also deduce from it the corresponding equation for the steady motion.

OR

A mass of gravitating fluid is at rest under its own attraction only; the free surface being a sphere of radius 'b' and the inner surface a rigid concentric shell of radius 'a'. Show that if this shell suddenly disappears, the initial pressure at any point of the fluid, at a distance r from the centre, is $\frac{2}{3}\pi\gamma\rho^2(b-r)(r-a)\left(\frac{a+b}{r}+t\right)$

where ρ is a fluid density and is the gravitational constant.

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5. (a) Define the strength of a simple source and find the complex potential due to a simple source of strength m, placed at the origin. Which arrangement of sources and sinks will give rise to the complex potential function $\log\left(z^2-\frac{1}{z^3}\right)$?

OR

A velocity field of a liquid is given by, $\overline{V}=x\hat{i}-(y+t^2)\hat{j}$. Find the velocity potential and prove that the stream lines for this field at $t=4$ are rectangular hyperbolas.

(b) What do you understand by two dimensional doublet? Obtain the image of doublet with regard to a circle, using the circle theorem.

OR

State Milne-Thomson circle theorem. Find the velocity potential and the stream function of a two dimensional liquid motion, when there is a source and an equal sink, within a circular cavity.

MATHEMATICS (FOURTH PAPER)-2013

(NUMERICAL METHODS)

Time : Three hours] [Maximum Marks : 45

Note : Attempt all questions. Marks are indicated against each question. Symbols have their usual meanings.

1. (a) Giving suitable examples, describe the rounding off rules in numerical calculations.

Prove that the equation x-cosx=0 has a root and estimate the root by graphical method. 4

(b) Do three iterations, taking the initial approximation $x_0=1$, to evaluate the square root of 5 by applying the fixed point iteration algorithm. Justify your procedure.

(c) What do you understand by the rate of convergence of an iterative method ? Explain . Further, prove that Newton Raphson method converges quadratically.

OR

Derive the Newton-Raphson iterative formula using the Taylor series expansion. Also, give the geometrical interpretation of the Newton-Raphson method.

2. (a) Define the operators ∇, μ, δ and E and Prove that $E^{\frac{1}{2}}=\mu+\frac{\delta}{2}$. Also, show that the operators μ and E commute. 4

OR

Define the forward difference operator Δ and prove that the n^{th} forward differences of a polynomial of degree n are constant, when the values of the independent variable are given at equal intervals. Also, find $\Delta^3[(x+1)(x+2)]$.

(b) Derive Gauss's Forward Interpolation formula, Also, write Bessel's and Everett's interpolation formulae. 5

OR

Find the interpolating polynomial by

(i) Newton's divided difference formula, and

(ii) Lagrange's formula, for the following data :

$y(1)=0, y(2)=7, y(3)=26, y(5)=124.$

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3. (a) Explain the concept of numerical integration. Derive the trapezoidal formula for the approximate evaluation of $\int_a^b f(x)dx$. Also, explain its geometrical significance. 4

OR

Using backward difference operator, derive the formula to compute the second derivative of a tabular function at points not found in the table.

(b) Using cubic spline interpolation method, find y(2.25) for the following data :

$y(0) = 1, y(1) = 4, y(2) = 0, y(3) = -2$, with the boundary conditions $y'(0) = 2 = y'(3)$.

OR

Fit a natural cubic spline curve that passes through the points (0,0.0),(1,0.5),(2,2.0) and (3,1.5).

4. (a) Apply the improved Euler's method to the IVP $\frac{dy}{dx} = x^2 + y, y(0) = 1$, to get y(0.1). Take h = 0.05. 4

OR

Find the value of y(0.1) in one step for the initial value problem $\frac{dy}{dx} = y - x, y(0) = 2$, by Runge-Kutta fourth order method. State clearly the formulas being used.

(b) What do you understand by predictor corrector methods to solved an IVP of first order.

Derive Milne's predictor-corrector formulae for solving an IVP of first order. Also, explain how to use this pair of formulae. 5

OR

Describe the Taylor series method to solve an IVP of second order differential equation.

Given $y'' = 0.5(x+y+y'+2), y(0) = 0, y'(0) = 0$; find y (0.2) and y'(0.2) by using the method. (Take h = 0.2).

5. (a) Solve the system of equations :

$2x_1 + x_2 + x_3 - 2x_4 = -10$

$4x_1 + 2x_3 + x_4 = 8$

$3x_1 + 2x_2 + 2x_3 = 7$

$x_1 + 3x_2 + 2x_3 - x_4 = -5$

using the Gauss elimination method with partial pivoting. 4

OR

Explain the concept of curve fitting by the method of least squares. Also, discuss fitting a parabola, by the method

$y = ax^2 + bx + c$

to a given set of n observations

$(x_i, y_i), i = 1, 2 \ldots\ldots, n.$

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(b) Describe Gauss-Seidel iterative method to solve a system of linear algebraic equations.

Also, use the method to solved the equations.

$6x + y + z = 20$

$x + 4y - z = 6$

$x - y + 5z = 7$

Do two iterations taking the initial approximation as (0,0,0). 5

OR

Find the eigenvalue of largest modulus and associated eigenvector of the matrix.

$$\begin{pmatrix} 2 & 3 & 2 \\ 4 & 3 & 5 \\ 3 & 2 & 9 \end{pmatrix}$$

by power method after third iteration starting with the initial vector $\underline{v}^{(0)} = (1,1,1)^T$.

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B.A./B.Sc. (Part III) Examination, 2015

Three-year Degree Course

MATHEMATICS

(FIRST PAPER)

Time : Three hours] [Maximum Marks : 60

Note : Attempt **all** questions. They all carry equal marks. Symbols used have their usual meaning.

1. (a) Define a homomorphism between two groups. Determine all homomorphisms from $(\mathbb{Z}_{15}, +)$ to $(\mathbb{Z}_{25}, +)$.

OR

Let $(G, .)$ be a finite abelian group and $n = \max \{0(a) : a \in G\}$. Prove that the order of each element of G divides n.

(b) Let H be a subgroup of group generated by {a, b}. If a and b commute, prove that H is abelian . Is H also finite ? Justify your answer.

OR

Prove that the group GL $(2, \mathbb{Z}_7)$ is non abelian. Determine the order of this group and the number of lest cosets of the subgroup SL in $(2, \mathbb{Z}_7)$ in GL $(2, \mathbb{Z}_7)$.

2. (a) State and prove second isomorphism theorem for groups. Use it to prove that if m, $n \in \mathbb{N}$, then m.n = [m,n] (m,n).

OR

Define a normal subgroup of a group. Let G_1 and G_2 be groups and $H_1 \triangleq G_1$, $H_2 \triangleq G_2$. Prove that $H = H_1 \times H_2 \triangleq G = G_1 \times G_2$ and the group G/H is isomorphic to $G_1/H_1 \times G_2/H_2$.

(b) Define the commutator subgroup of a group .Determine the commutator sub groups of S_4 and $S_3 \times S_3$.

OR

Prove that if $n \geq 5$, then any two 3-cycles in An are conjugate in An. Also prove that two disjoint cycles in Sn generate an abelian group.

3. (a) Let $(G, .)$ be a non-abelian group of order p^3, where p is a prime. Prove that $G / Z(G) \simeq \mathbb{Z}_p \times \mathbb{Z}_p$

OR

Establish the class formula for an action of a group on a finite set X. Write the class formula for the conjugation action of A_4 on itself

(b) Let G be a non-abelian group of order 10. Determine the centralizer of an element of order 2 in G and hance find the number of elements of order 2 in G.

OR

Let G be 'a non-abelian group of order 8. Prove that either $G \simeq Q_8$ or $G \simeq D_8$.

4. (a) Define the characteristic of a non-zero integral domain. Let R be a finite non-zero integral domain. Prove that it is a division ring and if $p = \text{char}(R)$, then R contains a subring isomorphic to $\mathbb{Z}_p$.

OR

Let R be a finite integral domain of characteristic p. Let $a \in R$. Prove that there exists a unique $b \in R$ such that $a = b^p$.

(b) Define a maximal ideal in a ring . Determine all maximal ideals of:

(i) $\mathbb{R}$

(ii) $\mathbb{Z}$

(iii) $\mathbb{Z}_{50}$

OR

Define a unit in a ring with identity. Determine the units of :

(i) $\mathbb{Z}_m$

(ii) $\mathbb{Z}[i] = \{a+ib : a,b \in \mathbb{Z}\}$

(iii) the polynomial ring Q [x].

5. (a) Find the set of all integers which leave remainders 1, 2 and 5 when divided by 5, 7 and 11 respectively.

OR

Let $a, b \in \mathbb{Z}$ and $m \in \mathbb{N}$. If $a \neq 0$ and if $d = (a, m)$ divides b, prove that the linear Congruence $ax \equiv b \pmod m$ has d incongruent solutions.

(b) Let p be an odd prime and $a \in \mathbb{Z}$ such that $p \times a$. Prove that $x^2 \equiv a \pmod p$ is solvable if and only if $a^{\frac{p-1}{2}} \equiv 1 \pmod p$.

OR

Let p be an odd prime . Prove that

$$\left(\frac{2}{p}\right) = \begin{cases} 1 & \text{if } p \equiv \pm 1 \pmod 8 \\ -1 & \text{if } p \equiv \pm 3 \pmod 8 \end{cases}$$

Conclude that $\left(\frac{2}{p}\right) = (-1)^{\frac{p^2-1}{8}}$

MATHEMATICS (SECOND PAPER)-2015

(New Course)

Time : Three hours] [Maximum Marks : 60

Note : Attempt **all** questions. which carry equal marks. Symbols used have their usual meaning.

1. (a) Let A be a subset of a metric space (x,d) and $f, g : A \to \mathbb{R}^n$ be maps. If p is a limit point of A and $\lim_{x \to p} f(x) = a$ and $\lim_{x \to p} g(x) = b$, then show that $\lim_{x \to p} f(x).g(x) = a.b$

(b) Prove that continuous image of a compact metric space is compact.

OR

(a) Prove that the map $S:\mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$, defined by $S(x,y) = x.y, x, y \in \mathbb{R}^n$, is continuous.

(b) Let f be a continuous map from a metric space (x,d_1) to a metric space (Y,d_2). If X is compact, then show that f is uniformly continuous.

OR

2. (a) Show that the map $f: \mathbb{R}^2 \to \mathbb{R}$ defined

$$\text{by } f(x,y) = \begin{cases} \frac{xy^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

where $(x,y) \in \mathbb{R}^2$, is not differentiable at (0,0).

(b) Let S be an open subset of $\mathbb{R}^n$. If $f:S \to \mathbb{R}$ is differentiable, then show that $f(y) - f(x) = f'(z)(y-x)$ for some $z \in L(x,y) \subseteq S$ and $L(x,y) = \{(1-t)x + ty; t \in [0,1]\}$.

OR

(a) Show that a map $f: \mathbb{R}^n \to \mathbb{R}^m$ is differentiable at $a \in \mathbb{R}^n$ if and only if each component of f is differentiable at a.

(b) By means of an example, show that for a differentiable map $f: \mathbb{R} \to \mathbb{R}^2$ and $x,y \in \mathbb{R}$ $f(y) - f(x) = f'(z)(y-x)$, where $z \in L(x,y)$, is not necessarily true.

3. (a) State and prove Euler's theorem for homogeneous functions.

(b) State the inverse function theorem. Let $f:\mathbb{R}^2 \to \mathbb{R}^2$ be defined by $f(x,y) = (e^x \cos y, e^x \sin y)$. Then show that is locally inertible at each point of $\mathbb{R}^2$. but is not injective in $\mathbb{R}^2$

OR

(a) State and prove the implicit function theorem.

(b) Let $f:\mathbb{R}^2 \to \mathbb{R}$ be defined by :

$f(x,y) = x^3+y^3-3x-3y-1, (x,y) \in \mathbb{R}^2$

Find the critical points, points of local minima and points of local maxima of the function f.

4. (a) Show that the sequence of functions $\{f_n^{\infty}\}_{n=1}$ define on a subset S of $\mathbb{R}$, converges uniformly on S if and only if for every $\epsilon > 0$, there exists a positive integer K such that $|f_n(x) - f_m(x)| < \epsilon$, $\forall m,n \geq K$ and $\forall x \in S$

(b) State and prove Abel's limit theorem use it to show that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} \ldots\ldots$

OR

(a) Show that the sequence $\{f_n^{-\infty}\}_{n=1}$, where $f_n(x) = nxe^{-nx2}; n \in \mathbb{N}$ converges pointwise, but not uniformly in $[0,\infty]$.

(b) State and prove that Abel's test for uniform convergence of an infinites series.

5. (a) Find the region of convergence of the power series $\sum_{n=1}^{\infty} \frac{(z+2)^{n-1}}{(n+1)^3 4^n}$.

(b) State and prove a necessary condition for differentiability of a map $f:D \to \mathbb{C}$, where $D \subseteq \mathbb{C}$.

OR

(a) Show that the map $f(z) = |z|^2$ is not differentiable at each point in the complex plane, except at $z = 0$.

(b) Define the complex exponential function exp(z). Prove that :

(i) $\exp(z_1 + z_2) = \exp(z_1).\exp(z_2)$.

(ii) $\exp(\bar{z})$ is nowhere analytic.

MATHEMATICS (THIRD PAPER)-2015

(New Course)

Time : Three hours] [Maximum Marks : 60

Note : Attempt **all** questions. All the questions carry equal marks.

1. (a) Find the vector equation of the conjugate liner to the line $\bar{r} = t(\hat{i} + \hat{j} + \hat{k})$ with respect to the system $(\bar{R},\bar{M})$ at origin where $\bar{R} = 2\hat{i} - 2\hat{j} + 3\hat{k}$ and $\bar{M} = 3\hat{i} + 3\hat{j} - 4\hat{k}$. Also, find the magnitudes of the conjugate forces.

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OR

Define a nul line relative to a given system of forces. Also, Prove that if a nulline relative to a given system of force intersects a given line, then it will intersect its conjugate line with respect to the system.

(b) Show that pitch of the wrench is an invariant of the force system. If forces $\vec{F}_1 = 2\hat{i} + 3\hat{j} - 4\hat{k}, \vec{F}_2 = -3\hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{F}_3 = \hat{i} - \hat{j} - 2\hat{k}$ are acting at points (1,0,1), (-1,1,0) and (0,1,2), respectively, then find the pitch of equivalent wrench and the equation of central axis.

OR

Two equal forces act one along each of the straight lines

$$\frac{x \pm a\cos\theta}{a\sin\theta} = \frac{y - b\sin\theta}{\pm b\cos\theta} = \frac{z}{c};$$

show that their central axis must, for all values of θ line on the surface

$$y\left(\frac{x}{z} + \frac{z}{x}\right) = b\left(\frac{a}{c} + \frac{c}{a}\right)$$

2. (a) Find the moment of inertia of a solid right circular cylinder of mass M, height h and radius a, about a straight line through its centre of gravity and perpendicular to its axis.

OR

Show that the moment of inertia of an elliptic lamina of mass M and semi-axes a and b about a diameter of length 2r is $Ma^2b^2/4r^2$.

(b) A uniform lamina is bounded by a parabolic arc of latus rectum 4a and a double ordinate at distance be from the vertex. If $b = \frac{a}{3}(7 + 4\sqrt{7})$, then show that two of the principal axes at an end of the latus rectum are the tangent and the normal there.

OR

Define the principal axes of a body at a point. Find the condition that a line is, at a point of its length, a principal axis of a body.

3. (a) Derive expressions for velocity and acceleration of a moving particle in the cylindrical polar co-ordinates (r, θ z).

OR

If a rod of leangth 2a, revolves with uniform angular velocity w about a vertical axis through a smooth joint at one extremity of the rod so that it describes a cone of semi-vertical angle 60^0, then show that

$$2aw^2 = 3g$$

(b) If a rigid body swings under gravity from a fixed horizontal axis, then show that its centre of suspension and centre of oscillation are convertible.

OR

A circular area can turn freely about a horizontal axis which passes through a point 0 of its circumference and is perpendicular to its plane. If motion commences when the diameter through 0 is vertically above 0, then show that when the diameter has turn through an angle 60^0, the components of strain at 0 along, and pependicular to, this diameter are w/6 and $w/2\sqrt{3}$ respectively.

4. (a) What physical significance is implied in the equation of continuity? Obtain the equation of continuity for a homogeneous liquid. For irrotational liquid motion, show that velocity potential satisfies the Laplace equation.

OR

Show that

$$\frac{x^2}{a^2}\tan^2 t + \frac{y^2}{b^2}\cot^2 t = 10$$

is a possible form for the bounding surface of a liquid. and find an expression for the normal velocity.

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(b) Steam is rushing from a boiler through a conical pipe, the diameters of the ends of which are D and d. If V and v are the corresponding velocities of the steam, and if the motion is supposed to be that of divergence from the vertex of the cone, then prove that

$$\frac{v}{V}=\frac{D^2}{d^2}\exp\left(\frac{v^2-V^2}{2k}\right),$$

where K is the pressure divided by the density and supposed to be constant.

OR

An infinite mass of fluid is acted on by a force $\mu / r^{3/2}$ per unit mass directed to the origin. If initially the fluid is at rest and here is cavity in the form of sphere r = a in it, then show that the cavity will be filled up after an interval of time $\left(\frac{2}{5\mu}\right)^{1/2} a^{5/4}$

5. (a) What is the physical significance of stream function for two dimensional fluid motion ? Also, find the stream function of the two dimensional motion due to two equal sources and an equal sink midway between them.

OR

Find the complex potential due to a simple source of strength m, placed at a point (1,2). Which arrangement of sources and sinks will give rise to the complex potential function $\cot h^{-1}(z)$?

(b) Within a rigid boundary in the form of the circle

$(x + a)^2 + (y - 4a)^2 = 8a^2$,

there is liquid motion due to a doublet of strength μ at the point (0,3a), with its axis along the axis of y. Find the velocity potential of the liquid motion.

OR

A source S and a sink T of equal strengths m are situated within the space bounded by a circle whose centre is 0. If S and T are at equal distances from 0 on opposite side of it and on the same diameter AOB, then show that the velocity of the liquid at any point P is

$$2m\left[\frac{(OS)^2+(OA)^2}{OS}\right]\cdot\frac{(PA).(PB)}{(PS).(PS').(PT).(PT')}$$

where S' and T' are the inverse points of S and T with respect to the circle respectively.

MATHEMATICS (FOURTH PAPER)-2015

(NUMERICAL METHODS)

Time : Three hours] [Maximum Marks : 45

Note : Attempt **all** the questions. Marks are indicated against each question. Symbols have their usual meanings.

1. (a) Explain, what do you understand by the convergence of an iterative method . 4

OR

Explain bisection method to find a root of an equation f (x) = 0.

(b) Derive the Newton-Raphson formula using theTaylor series expansion. Also, explain the method geometrically. 5

OR

Use the Regula-Falsi method to find a root of the equation $x^2-x-2=0$ in the range $1<x<3$. Do three iterations.

2. (a) Define the operators $\nabla\mu$ and E. Also, derive the Newton's backward difference interpolation formula . 4

OR

Find the missing values in the following table :

x	0	1	2	3	4	5	6
y	–4	–2	–	–	220	546	1148

(b) Using Newton's forward difference formula, find the sum

$S_n = 1^3 + 2^3 + 3^3 + \text{----------} + n^3$. 5

Derive the Newton's divided differences interpolation formula. Using Lagrange's interpolation formula, find the form of the function y (x): from the following table

x	0	1	3	4
y	–12	0	12	24

3. (a) Explain the concept of numerical integration. Derive the Trapezoidal formula for the approximate evaluation of $\int_a^b f(x)dx$ Also, explain it's geometrical significance. 4

OR

Fit a cubic spline curve that passes through the points (0,1), (1,4), (2,0) and (3,-2) with the boundary conditions S'(0) = 2 and S'(3)=2.

(b) Using backward difference operator, derive the formula to compute the second derivative of a tabular function at points not found in the table. 5

OR

Derive Simpson's (3/8) - rule

$$\int_{x_0}^{x_3} y\,dx = \frac{3}{8}h(y_0 + 3y_1 + 3y_2 + y_3)$$

using this rule evaluate

$$I = \int_0^1 \frac{1}{1+x}dx$$

With h = 1/6 . Evaluate the intergral by using Simpson's (1/3)-rule and compare the result .

4. (a) Describe Euler's method to Solve IVP of first order. Further, describe the steps to apply Runge-Kutta Nystrom method to solve an IVP of second order . 4

OR

Given $y'' = \frac{1}{2}(x + y + y' + 2)$, $y(0) = 0$, $y'(0) = 0$

Find y (0.2) by using Rung-Kutta Nystrom method (Taking h = 0.2).

(b) Derive Adams-Bashforth-Moulton's predictor corrector formulae to solve an IVP of first order explaining how to apply the formula. 5

OR

Describe the Taylor's series method to solve an IVP of second order differential equation.

Given, $y'' = 0.5(x+y+y'+2)$, $y'(0) = 0$, $y(0) = 0$;

find y (0.2) and y'(0.2) by using the method . (Take h = 0.2)

5. (a) Describe the Gaussian Elimination method to solve a system of linear algebraic equations.

Aslo, solve the system of equations :

$2x_1 + x_2 + x_3 - 2x_4 = -10$

$4x_1 + 2x_3 + x_4 = 8$

$3x_1 + 2x_2 + 2x_3 = 7$

$x_1 + 3x_2 + 2x_3 - x_4 = -5$

OR

Solve the equations :

$x_1 + x_2 + x_3 = 1$

$4x_1 + 3x_2 - x_3 = 6$

$3x_1 + 5x_2 + 3x_3 = 4$

by the LU decomposition method.

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(b) Describe Gauss-Seidal iterative method to solve a system of linear algebraic equations. Also, use the method to solve the equations :

$10x_1 - 2x_2 - x_3 - x_4 = 3$

$-2x_1 - 10x_2 - x_3 - x_4 = 15$

$-x_1 - x_2 + 10x_3 - 2x_4 = 27$

$-x_1 - x_2 - 2x_3 - 10x_4 = -9$

Do two iterations taking the initial approximation as (0,0,0,0). 5

OR

Describe Power method to find the numerically largest eigen value and a corresponding eigen vector of a square matrix A of order n. Using the method, find the largest eigenvalue of the matrix :

$$\begin{bmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 0 & 4 \end{bmatrix}$$

and a corresponding eigen vector after second iteration starting with the intial vector $\bar{v}^0 = [0,0,1]^T$.

B.Sc. (Part-III) Examination, 2016
MATHEMATICS
First Paper
(Abstract Algebra)

Time : 3.00 Hours] [M.M. 60

Note : Attempt all questions. They all carry equal marks. Symbols used have their usual meaning.

1. (a) Define a homomorphism between two groups. Determine all homorphisms

(i) from $(Q_8, \cdot)$ to $(\mathbb{Z}_4, +)$

(ii) from $(Q, +)$ to $(Q^+, \cdot)$.

OR

Prove that all subgroups of a cyclic group are cyclic. Is the coverse true? Justify your answer.

(b) Define the center of a group. Determine the center of (i) GL_2 (IR) (ii) $Q_8 \times S_3$.

OR

(i) State and prove Euler's Theorem.

(ii) Give an example of a finite group in which the converse of Lagrange Theorem is not true.

2. (a) Let H be a subgroup of a group G. Define a conjugate of H in G. If N is a normal subgroup of G contained in H, then prove that N is contained in each conjugate of H in G.

OR

State Fundamental Theorem of homomorphisms for groups. Use it to prove that (i) if $m, n \in N$ and if n/m, then $n\mathbb{Z}/m\mathbb{Z} \simeq \mathbb{Z}m/n$ (ii) if H is a finite subgroup of S^1, then $S^1/H \simeq S^1$.

(b) Let f be a non-identity permutation in Sn. Prove that f is a product of disjoint cycles in Sn.

OR

Let G be a group of order 2n, where n is odd. Prove that G has a subgroup of index 2.

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3. (a) Let H be a subgroup of a finite group G. Prove that the number of conjugates of H in G is the index $[G : N_G(H)]$. Determine the normalizer of {I, (12) (34)} in S_4.

OR

Let G be a non-abelian group of order 10. Write the class equation for the conjugation action of G on itself.

(b) Let G be a group of order p^2, where p is a prime. Prove that either $G \simeq \mathbb{Z}_{p^2}$ or $G \simeq \mathbb{Z}_p \times \mathbb{Z}_p$.

OR

Let G be a non-abelian group of order 8. Show that $|Z(G)| = 2$. If G has two non-isomorphic subgroups of order 4, then prove that $G \simeq D_8$.

4. (a) Define a subring of a ring. Prove that

$$S = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in \mathbb{Z} \right\}$$

is a subring of $M_2(\mathbb{C})$. Also, prove that each non-zero element of S is a unit of S.

OR

Define an integral domain. Prove that if $m \in \mathbb{N}$, $m > 1$, then the ring $(\mathbb{Z}_m, +, .)$ is an itegral domain if and only if m is a prime. Is an integral domain of order 32 a field? Justify your answer.

(b) Let R be a commutative ring with identity. Prove that R is a field if and only if R has no non-zero proper ideals.

OR

Let F be a field F[x] be the polynomial ring over F in indeterminate x. Prove that every ideal of F[x] is generated by a single polynomial in F[x]. If $a \in F$ and I is the ideal of F[x] generated by x-a, then prove that I is a maximal ideal of F[x].

5. (a) State and prove Chinese Remainder Theorem.

OR

Define a multiplicative function. Prove that the mobius function μ is multiplicative and for each $n \in \mathbb{N}$,

$$\sum_{d|n} \mu(d) = \begin{cases} 1 & \text{if } n = 1 \\ 0 & \text{if } n > 1 \end{cases}$$

(b) Let $n \in \mathbb{N}$, $n > 1$. Let $p_1, p_2, \ldots, p_r$ be all distinct prime divisors of n. Prove that

$$\varphi(n) = n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right)\ldots\left(1 - \frac{1}{p_r}\right),$$

where φ is the Euler's phi-function.

OR

Let p be an odd prime and $a \in \mathbb{Z}$ such that pXa. Define the Legendra symbol $\left(\frac{a}{p}\right)$. Prove that

(i) $\left(\frac{-1}{p}\right) = 1$ if $p \equiv 1 \pmod 4$

(ii) $\sum_{r=1}^{p-1} \left(\frac{r}{p}\right) = 0$

MATHEMATICS
Second Paper, 2016
(Advanced Analysis)

Time : 3.00 Hours] [M.M. 60

Note : Attempt all questions which carry equal marks. Symbols used have their usual meaning.

1. (a) If $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear map, then show that $\| Tx \| \leq M \| x \|, x \in \mathbb{R}^n$, where M is positive real number. Use it to prove that T is continuous.

(b) Prove that a compact subset of a metric space is closed and bounded. Is the converse true? Justify your answer.

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OR

(a) Prove that a map $f : IR^n \to IR^m$ is continuous if and only if each of its component is continuous.

(b) Prove that a compact metric space is complete. Is the converse true? Justify your answer.

2. (a) Let $f, g : IR^n \to IR^m$ be differentiable maps. Then prove that the map $f, g : IR^n \to IR$, defined by
$(f \cdot g)(x) = f(x).g(x)$, $x \in IR^n$,
is also differentiable.

(b) Let $f : IR^2 \to IR$ be a map possessing partial derivatives $D_1 f$ and $D_2 f$ with $D_1 f$ continuous at $(a,b) \in IR^2$. Show that f is differentiable at (a,b).

OR

(a) Give an example of map $f : IR^2 \to IR$ which is continuous at (0,0) but not differentiable there, with complete arguments.

(b) Let S be an open subset of IR^n, $x, y \in S$ and
$L(x,y) \equiv \{(1-t)x + ty : t \in [0,1]\} \subseteq S$
If $f : S^2 \to IR^m$ is differentiable, then show that for each non-zero a in IR^m, there is a $z \in L(x, y)$ such that
a. $(f(y)-f(x)) = a.f'(z)\,(y-x)$.

3. (a) State and prove Schwarz's theorem.

(b) Determine extrema of $f(x,y) = xy$, subject to the constraint $x + y = 5$

OR

(a) Use Taylor's theorem to express $x^2y + 3y - 2$ in powers of x-1 and y+2.

(b) State the Implicit Function Theorem Let $f : IR^3 \to IR^2$ be defined by $f(x,y,z) = (x + y + z, x - y - xz)$, $(x,y,z) \in IR^3$. Then show that there is an open interval U containing the origin and a C^1-map $g : U \to \mathbb{R}^2$ such that for $x \in U$, $f(x,g(x)) = 0$.

4. (a) Test the series $\sum_{n=1}^{\infty} \frac{1}{1+n^2x}$ for uniform covergence on

(i) an interval containing 0,

(ii) an interval not containing 0.

(b) State and prove Dirichlet's test for uniform convergence of an infinite series.

OR

(a) Let $\{f_n\}_{n=1}^{\infty}$ be a sequence of functions defined on a set $S \subseteq IR$, which converge uniformly on S. If each f_n is continuous at a point a_0 of S, then show that
$\lim_{x \to a_0} \lim_{n \to \infty_0} f_n(x) = \lim_{n \to \infty} \lim_{x \to a_0} f_n(x)$

(b) State Abel's test for uniform convergence of an infinite series. Prove that the series $\sum_{n=1}^{\infty} (-1)^n \frac{x^2+n}{n^2}$ converges uniformly in [0,1], but does not converge absolutely there.

5. (a) Show that

$$f(z) = \begin{cases} \frac{xy(x+iy)}{x^2+y^2} & , z \equiv (x+iy) \neq 0 \\ 0 & , z = 0 \end{cases}$$

is not differentiable at z = 0, although it satisfies the Cauchy-Riemann equations there.

(b) Introduce the complex exponential map exp (z). Prove that

(i) $\exp(z_1 + z_2) = \exp(z_1) \exp(z_2)$,

(ii) $\exp(\bar{z})$ is nowhere analytic.

OR

(a) Find an analytic map of which the real part is the map
$u(x, y) = y^3 - 3x^2y$, $x, y \in \mathbb{R}$

(b) Let f be an analytic map is a domain $D \subseteq \mathbb{C}$. If $f'(z) = 0, \forall z \in D$, then show that f is a constant map.

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MATHEMATICS
Third Paper, 20016
(Advanced Mechanics)

Time : 3.00 Hours] [M.M. 60

Note : Attempt all questions. All the questions carry equal marks. Symbols used have their usual meaning.

1. (a) Define mutual moments of two lines. Show that the two conjugate lines of a given system of forces do not intersect each other.

OR

Two system of forces are equivalent to $(\overline{R}_1.\overline{M}_2)$ and $(\overline{R}_2.\overline{M}_2)$ respectively at origin. Show that the condition that the axes of the equivalent wrenches intersect is

$$\overline{R}_1.\overline{M}_2 + \overline{R}_2.\overline{M}_1 = (p_1 + p_2)(\overline{R}_1.\overline{R}_2)$$

where p_1, p_2 are their pitches.

(b) The axes of two wrenches $(\overline{P}, p\overline{P})$ and $(\overline{Q}, q\overline{Q})$ are at right angle and the shortest distance between them is 2a. Prove that the axis of resultant wrench divides the line of shortest distance,

(i) Orthogonally

(ii) In the ratio;

$$Q\{2aQ + (p-q)P\} : P\{2aP - (p-q)Q\}$$

OR

Define nul point and nul plane of a given system of forces. Find the nul point of the plane $x + 2y - 3z = 5$ with respect to the system $(\overline{R}, \overline{M})$ at origin, where, $\overline{R} = 3\hat{i} + 2\hat{j} + \hat{k}$ and $\overline{M} = \hat{i} - 2\hat{k}$.

2. (a) Show that the moment of inertia of a solid sphere of Mass M and radius a about its diameter, when the mass density

of any point is proportional to the cube of its distance from the centre, is $\frac{1}{2}Ma^2$.

OR

Show that the sum of the moment of inertia of an elliptic area about any two mutually perpendicullar tangents, is always constant.

(b) Let a uniform rod of length 2a, turning about its end O, start from the position in which it was vertically above O. Show that the vertical reaction of the fixed axis Vanishes when $\theta = \cos^{-1}\left(-\frac{1}{3}\right)$.

OR

A bent lever, whose arms are of lengths a and b the angle between them being α, makes small Oscillations in its own plane about the fulcrum. Show that the length of the Corresponding simple pendulum is

$$\frac{2}{3}\frac{a^3 + b^3}{\sqrt{a^4 + b^4 + 2a^2b^2\cos\alpha}}.$$

3. (a) Show that the acceleration of a moving particle in Cylindrical Coordinate system (ρ, φ, z) is

$$(\ddot{\rho} - \rho\dot{\varphi}^2)\hat{\rho} + \frac{1}{\rho}\frac{d}{dt}(\rho^2\dot{\varphi})\hat{\varphi} + \ddot{z}\hat{z}$$

where '.' denote the differentiation with respect to time.

OR

A plank of mass M is intially at rest along a line of greatest slope of smooth plane inclined at an angle α to the horizon, and a man, of mass M' starting from the upper end walks down the plank so that it does not move. Show that he gets to the other end in time $\sqrt{\frac{2M'a}{(M+M')g\sin\alpha}}$

Where a is the length of the plank.

(b) Define momental ellipsoid. Find the momental ellipsoid at a point on the rim of a hemisphere.

OR

Find the time of oscillation of a compound pendulum. Consisting of a rod, of mass m_1 and length a, carrying at one end a sphere of mass m_2 and diameter 2b, the other end of the rod being fixed.

4. (a) Find the value of n show that the ellipsoid,

$$\frac{x^2}{a^2k^2t^{2n}}+kt^n\left(\frac{y^2}{b^2}+\frac{z^2}{c^2}\right)=1$$

is a prossible from of boundary surface of a liquid at any time t.

OR

If every particle moves on the surface of a sphere, prove that the equation of continuity is

$$\frac{\partial\rho}{\partial t}\cos\theta+\frac{\partial}{\partial\theta}(\rho w\cos\theta)+\frac{\partial}{\partial\varphi}(\partial w'\cos\theta)=0,$$ ρ being the density, θ, φ the latitude and longitude respectively of an element and w, w' the angular velocities of any element in latitude and longitude respectively.

(b) A mass of liquid of density ρ whose external surface is a long Circular Cylinder of radius a, which is subject to a constant pressure II surrounds a coaxial long circular cylinder of radius b. The internal cylinder is suddenly annihilated. Show that if v is the velocity at the internal surface when the radius is r, then

$$v^2=\frac{2\pi(b^2-r^2)}{\rho r^2\log(r^2+a^2-b^2)/r^2}$$

OR

If the external forces are conservative and density is a function of pressure p only then

$$\frac{d}{dt}\left(\frac{\overline{W}}{\rho}\right)=\left(\frac{\overline{W}}{\rho}\bullet\nabla\right)\bar{q}$$

where $\bar{q}$ is velocity and $\overline{W}=Curl\bar{q}$, the vorticity vector.

5. (a) Between the fixed boundaries $\theta=\frac{\pi}{4}$ and $\theta=-\frac{\pi}{4}$ there is a two dimensional liquid motion due to a surece of strength a at the point (r = a, θ = 0) and an equal sink at the point (r = b, θ = 0). Show that the stream function is

$$-m\tan^{-1}\left\{\frac{r^4(a^4-b^4)\sin4\theta}{r^8-r^4(a^4+b^4)\cos4\theta+a^4b^4}\right\}$$

OR

In the case of the two-dimensional fluid motion produced by a source of strength m placed at a point S outside a rigid circular disc of radius a whose centre is O, show that the velocity of Slip of the fluid in contact with the disc is greatest at the points where the line joining the ends of the diametr at right angles to OS. Cut the circle and prove that the magnitude at these points is $\frac{2m.OS}{(OS^2-a^2)}$.

(b) Show that the image of a doublet of strength μ, at a distance f, from the centre of a circle, relative to the circle is a doublet of strength $\mu a^2/f^2$, where a is radius of the circle.

OR

State and prove the Mione-Thomson Circle theorem. Hence, find the image of a source with respect to a circle of radius a.

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MATHEMATICS
Fourth Paper, 2016
(Numerical Methods)

Time : 3.00 Hours] [M.M. 45

Note : Attempt all questions. Marks are indicated against each question. Symbols used have their usual meaning.

1. (a) Describe Newton-Raphson method for numerically solving an equation involving one unknown. Discuss briefly the
   (i) Geometrical significance of the method, and
   (ii) Convergence of the method. 5

   OR

   Find a real root of the equation $x^3 + x - 1 = 0$ correct to three decimal places, using the fixed point iteration method.

   (b) Derive the formula

$$x_{n+1} = \frac{1}{3}\left[2x_n + \frac{a}{x_n^2}\right]$$

   for computing successive approximations for the cube root of the number a by Newton-Raphson method and use it to obtain $\sqrt[3]{2}$. Do three iterations.

   OR

   Use Regula-Falsi method to find the real root of the equation
   $x^3 - x - 1 = 0$
   which lies between 1 and 2. Do three iterations.

2. (a) Define central difference operator, shift operator and average operator. Also derive Newton's forward difference interpolation formula. 4

   OR

   Find the missing value in the following table :

x	0	5	10	15	20	25
y	6	10	-	17	-	31

   (b) Use Newton's divided difference formula to find the interpolating polynomial from the given data :

x	7	8	9	10
y	3	1	1	9

   and hence evaluate y(9.5).

   OR

   State, without proof, Stirling's formula for central interpolation. Fuuther, use the formula to find out the value of y at x = 1.91 from the data given below :

x	1.7	1.8	1.9	2.0	2.1	2.2
$y=e^x$	5.4739	6.0496	6.6859	7.3891	8.1662	9.0250

3. (a) In the interval $[x_i, x_{i+1}]$, the cubic spline $S_i(x)$ may be expressed as :
   $S_i(x) = a_i + b_i (x-x_i) + c_i (x-x_i)^2 + d_i (x-x_i)^3$,
   $i = 0, 1, 2, \ldots, n-1$.
   Determine the constants a_i, b_i, c_i and d_j using the conditions for a natural cubic spline.

   OR

   Evaluate,

$$\int_0^6 \frac{dx}{1+x^2}$$

   , by using
   (i) Simpson's one-third rule
   (ii) Trapezoidal rule

   (b) If $y = A + Bx + Cx^2$ and y_0, y_1, y_2 are the values of y corresponding to x = a, a+h, a+2h respectively, prove that

$$\int_a^{a+2h} y\,dx = \frac{h}{3}(y_0 + 4y_1 + y_2)$$ 5

   OR

   Fit a cubic spline curve that passes through the points (0,1), (1,4), (2,0) and (3,-2) with the boundary conditions S '(0) = 2 and S '(3) = 2.

4. (a) Derive Milne's predictor-corrector method for solution of an ordinary differential equation numerically. 4

   OR

Solve,

$\frac{dy}{dx} = 3x + \frac{1}{2}y^2$ with $y = 1$, when $x = 0$,

by Runge-Kutta forth-order method to find $y(0.1)$ and $y(0.2)$, taking $h = 0.1$.

(b) Solve by Taylor's series method the following differential equation,

$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$,

with $y = 0$ when $x = 1$, to find $f(1.1)$ and $y(1.2)$ with $h = 0.1$, correct to three decimal places. 5

OR

Use Euler's modified method to solve

$dy/dx = x + y$,

with initial condition $x_0 = 0$, $y_0 = 1$ (in steps of 0.2) for obtaining $y(0.2)$ and $y(0.4)$.

5. (a) Apply Gauss-Seidal method to slove the system, 4

$3x + y + z = 3$

$2x + y + 5z = 5$

$x + 4y + z = 2$

OR

Solve the system of equations :

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ -10 \end{bmatrix}$$

using Cholesky's method.

(b) Describe the Gauss Elimination method to solve a system of linear algebraic equations. Also, discuss partial and full pivoting giving suitable examples.

OR

Describe Power method to find the numerically largest eigenvalue and a corresponding eigenvector of a square matrix A of order n.

B.A./B.Sc. (Part III) Examination, 2017

MATHEMATICS (FIRST PAPER)

(ABSTRACT ALGEBRA)

Time : Three hours] [Maximum Marks : 60

Note : Attempt **all** questions. They all carry equal marks. Symbols used have their usual meaning.

1. (a) For a, b in $\mathbb{R}$, let $T_{ab} : \mathbb{R} \to \mathbb{R}$ be defined by Tab $T_{ab}(x) = ax + b$ for all x in $\mathbb{R}$. Show that $H = \{T_{ab} | a, b \text{ in } \mathbb{R} \text{ with } a \neq 0\}$ is a sub-group of Sym (IR). Which T_{ab} are isometries of $\mathbb{R}$? Support your answer.

OR

Define dihedral group D_{2n} of order 2n for $n \geq 3$. Prove that every subgroup of D_{2n} of odd order is cyclic.

(b) Define order of an element x in a group G. If the order of x in a group G is n, show that the order of x^k is $\frac{n}{\text{ged}(k,n)}$ for any integer k. Find the number of generators of $\mathbb{Z}_{36}$. OR

Let H and K be finite subgroups of a group G. Prove that $|HK|.|H \cap K| = |H|.|K|$. Is HK a subgroup of G? Support your answer.

2. (a) Define an automorphism of a group. Determine Aut ($\mathbb{Z}$) and Aut (S_3)

OR

Prove that A_4 has no subgroup of index 2. Does A_5 have subgroup of index 5? Support your answer.

(b) Define the commutator subgroup [G,G] of a group G. Prove that [G,G] is a normal subgroup of G and find $[Q_8, Q_8]$. OR

Prove that S_n is generated by (1 2) and (1 2 3 . . .n). Is S_4 generated by (1 3) and (1 2 3 4)? Support your answer.

3. (a) Establish the conjugacy class equation of a finite group. Write conjugacy class equation of $S_3 \times \mathbb{Z}_2$. OR

Let G be a group of order p^n, where p is a prime and n is a positive integer. If $N \neq \{e\}$ is a normal subgroup of G, prove that $N \cap Z(G) \neq \{e\}$ and deduce that Z(G), the center of G, is nontrivial.

(b) Prove that an action of a group G on a set X determines a permutation representation of G on X. OR

Let H be a subgroup of a group G. Show that there is a transitive action of G on G/H the set of left cosets of H in G. Give an example

of a group action with exactly two fixed points.

4. (a) Define an ideal of a ring. Let I be an ideal of a commutative ring R. Show that $J = \{a \in R \mid a^k \in I$ for some positive integer k$\}$ is an ideal of R. Is I=J? Support your answer. **OR**
Define an integral doman with an example. Prove that every non-zero finite integral domain is a division ring.
(b) Give an example of a ring without identity. Prove that every ring can be embedded in a ring with identity. **OR**
Let F be a field. Define an irreducible polynomial over F. Prove that the ideal of F[x] generated by an irreducible polynomial over F is maximal.

5. (a) Let f and g be multiplicative arithmetic functions and $h(n)=\sum_{d\mid n} f(d)g\left(\frac{n}{d}\right)$ for all positive integers n. Show that h is multiplicative and deduce that sum of divisor function σ is multiplicative. Is σ completely multiplicative ? Support your answer. **OR**
Define the Euler phi-function ϕ. Prove that $n=\sum_{d\mid n}\phi(d)$ for all positive integers n and using Mobius inversion formula find an expression for ϕn.
(b) Let p be an odd prime, a and b be integers not divisible by p. Prove that $\left(\frac{ab}{p}\right)=\left(\frac{a}{p}\right)\left(\frac{b}{p}\right)$. Is 23 a qudratic residue modulo 53? Support your answer State Gauss' Lemma. Determine all odd primes $p \neq 3$ such that $x^2 = 3 \pmod p$ has a solution.

MATHEMATICS (SECOND PAPER)-2017
(Advanced Analysis)

Time : Three hours] [Maximum Marks : 60

Note : Attempt **all** questions. Symbols used have their usual meanings.

1. (a) Define a continuous map from a metric space to a metric space. State and prove a characterization of a continuous map between metric spaces.
(b) Show that every open cover of a compact metric space has a Lebesgue number, Let x=(0, 1) with usual metric and $U_n=\left(\frac{1}{n},1\right), n\geq 2$. Does

a Lebesgue number exist for the cover $\{U_n : n \geq 2\}$ of x ? **OR**

(a) Let $T: \mathbb{R}^n \to \mathbb{R}^m$ be a linear map such that $\lim_{v\to 0}\frac{T(v)}{\|v\|}=0$. Show that T=0.
(b) Let x be a complete metric space and $F \subseteq X$. Show that F is complete if and only if it is closed in X.

2. (a) Define differentiability of a map $f: \mathbb{R}^m \to \mathbb{R}^n$. Let a map $f: \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $f(x, y)=(x^2+y^2, xy), (x,y)\in \mathbb{R}^2$. Compute the total derivative of f at (1,2).
(b) State and prove the chain rule of differentiation for functions of several variables. **OR**
(a) Show that the map $f: \mathbb{R}^2 \to \mathbb{R}$, defined by $f(x,y)=\sqrt{|xy|}(x,y)\in \mathbb{R}^2$, is not differentiable at the origin.
(b) By means of an example, show that for a differentiable map $f: \mathbb{R} \to \mathbb{R}^2$ abd $x, y \in \mathbb{R}, f(y)-f'(x)=f(z)(y-x)$, where $z\in\{(1-t)x|+ty : t\in[0,1]\}$, is not necessarily true.

3. (a) Let $f: \mathbb{R}^2 \to \mathbb{R}$ be a differentiable map. If for each $x \in \mathbb{R}^n, x\nabla f(x)=pf(x)$, where $p\in\mathbb{N}$, then show that f is a homogeneous function of degree p.
(b) State the implicit function theorem. Let $f: \mathbb{R}^3 \to \mathbb{R}^2$ be defined by f(x,y,z)= (x+y+z, x-y-2xz), $(x,y,z)\in \mathbb{R}^3$.
Show that there is an open interval U containing the origin and c^1-map $g: U \to \mathbb{R}^2$ such that for $x \in U, f(x, g(x))=0$. **OR**
(a) State a set of sufficient conditions for a real valued map of two variables to have a local maximum or minimum value at a given point. Examine the map $f(x,y)=2x^2-xy-3y^2-3x+7y$; $(x,y)\in \mathbb{R}^2$, for local minima and maxima.
(b) State Taylor's theorem for a map $f: \mathbb{R}^n \to \mathbb{R}$. Use it to approximate log(1+x+2y) by a polynomial of degree 1.

4. (a) Show that the series whose n^{th} partial sum is $\frac{x}{1+nx^2}$ converges uniformly on any finite closed interval [a,b].
(b) State and prove weierstrass' M-test for the uniform convergence of a series. **OR**

(a) Show that the sequence $\left\{\frac{nx}{1+n^2x^2}\right\}_{n=1}^{\infty}$ is not uniformly convergent on [-1,1]. Is it uniformly convergent on $\left[\frac{1}{2},1\right]$? Justify your answer.

(b) State and prove Abel'e test for uniform convergence of an infinite series.

5. (a) State and prove a sufficient condition for differentiability of a map $f:D\to\mathbb{C}$ Where $D\subseteq\mathbb{C}$

(b) If f is an analytic map in a domain $D\subseteq C$ and $|f|$ is constant there, then show that f is a constant map. **OR**

(a) Show that the map $f(z)=|z|^2$ is no where analytic in the complex plane.

(b) Suppose $f(z)=\sum_{n=0}^{\infty}a_n z^n$ converges for $|z|<R$. Show that f'(z) exists and is equal to $\sum_{n=1}^{\infty}n a_n z^{n-1}$ throughout $|z|<R$.

MATHEMATICS (THIRD PAPER)-2017

(Advanced Mechanics)

Time : Three hours] [Maximum Marks : 60

Note : Answer **all** questions. All the questions carry equal marks. Symbols have their usual meaning.

1. (a) Three forces P, Q and R act along the straight lines x=a, y=b; y=b, z=c and z=c, x=a, respectively. Show that they can be reduced to a single force.

OR

Define central axis of a given system of forces. Forces $P\hat{i}+Q\hat{j}, Q\hat{j}+R\hat{k}$ and $R\hat{k}+P\hat{i}$ are acting at the points whose position vectors are $-\hat{i}+\hat{j}+\hat{k}, \hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}-\hat{k}$, respectively. Find the vector equation of their central axis.

(b) Find the vector equation of conjugate line to the line $\bar{r}=\hat{i}-\hat{j}+t(\hat{i}+\hat{j})$

with respect to the system $(\bar{R},\bar{M})$ at origin. where $\bar{R}=2\hat{i}-4\hat{j}+\hat{k}$ and $\bar{M}=\hat{i}-\hat{j}+2\hat{k}$. Also obtain conjugate forces. **OR**

Define nul line and conjugate lines for a system of forces. Also, prove that if a nul line relative to a given system of forces intersects a given line, then it will intersect it conjugate line with respect to the system.

2. (a) Find the moment of inertia of a lamina of mass M bounded by the curve y=sinx and the x-axis, lying between x=0 and x=π, about the x-axis. **OR**

Find the moment of inertia of a solid right circular cylinder of mass M, height h and radius 'a' about

(i) its axis, and

(ii) a line passing through its centre of gravity and perpendicular to its axis.

(b) Define the principal axes. Show that the principal axes of a square at a corner are the diagonal through the corner and a line parallel to the other diagonal. **OR**

Show that at an extremity of the bounding diameter of a semi-circular lamina, the principal axis makes an angle $\frac{1}{2}\tan^{-1}(8/3\pi)$ to the diameter.

3. (a) Obtain expressions for velocity and acceleration of a particle, moving on the surface of a right circular cylinder of radius 'b'. **OR**

Find the moment of the effective forces of a right body rotating about a fixed axis.

(b) Obtain the time period of a compound pendulum and show that if the axis of suspension of a compound pendulum either passes through the centre of inertia or be at infinity; its time of oscillation is infinite.

OR

A thin uniform rod has one end attached to a smooth hinge and is allowed to fall from a horizontal position. Show that when the horizontal strain on the hinge is a maximum, the verticle strain is then 11/8 times the weight of the rod.

4. (a) Consider the steady, two dimensional flow field given in Eulerian description by the expression:

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$$\bar{q} = \frac{x\hat{i} + y\hat{j}}{x^2 + y^2}.$$

Determine whether the flow is :

(i) Incompressible or compressible,

(ii) Irrotational or rotational, **OR**

What physical significance is implied in the equation of continuity ? Derive the equation of continuity in the Lagrangian form. Also, write the equation of continuity in the Eulerian form for a compressible fluid.

(b) A sphere of radius 'a' is surrounded by infinite liquid of density P, the pressure at infinity being π. The sphere is suddenly annihilated. Show that the pressure at a distance 'r' from the centre immediately falls to

$$\pi\left(1 - \frac{a}{r}\right).$$

OR

Derive the Euler's equation of motion in vector form for an inviscid fluid, and hence express it in the cylindrical polar co-ordinates.

5. (a) Define a two dimensional doublet. Obtain the image of a doublet with regard to a circle by using circle theorem. **OR**

What do you understand by two dimensional source and sink ? Find the complex potential due to a simple source of strength +m placed at the point (0,1).

(b) Find the velocity potential of the liquid motion when there is a source and an equal sink inside a circular cavity of radius 'a'. **OR**

Find the stream function of the two dimensional liquid motion due to two equal sources and an equal sink midway between them. Also, find the speed of the liquid at any point.

MATHEMATICS (FOURTH PAPER)-2017

(Numerical Methods)

Time : Three hours] [Maximum Marks : 45

Note : Attempt **all** the questions. All question carry equal marks. Symbols have their usual meaning.

1. (a) Estimate a root of $x^3-3x+1=0$ near $x=0$ by the fixed point iteration method. Do three iterations. **OR**

(a') Estimate a root of $x^3-3x+1=0$ in the interval (1,2) by Newton-Raphson method. Do three iterations.

(b) Estimate a root of $x^3-x-2=0$ by bisection method. Do three iterations.

OR

(b') Explain the method of Regula Falsi and estimate a root of $x^3-9x+1=0$ between 2 and 3 by this method. Do three iterations.

2. (a) Prove the Lagrange's interpolation formula for a function f satisfying $f(x_i) = y_i$ for $i = 0,1,2,3 (x_0 < x_1 < x_2 < x_3)$. **OR**

(a') Find the polynomial interpolating the following data by Newton' divided difference interpolation formula :

x	0	1	3	4
f	-12	0	12	24

and estimate the value of y(-2).

(b) Prove the stirling's interpolation formula. **OR**

(b') Estimate the missing value of f(4) from the following data :

x	1	2	3	4	5
f(x)	2	5	7	-	32

3. (a) Estimate the values f' (0,4) and f''(0.4) from the data :

x	0.1	0.2	0.3	0.4
f(x)	1.105	1.221	1.350	1.492

OR

(a) Write and prove the Simpson's (3/8) rule for numerical integration of a functions f.

(b) Using cubic splines, estimate the value of y(1.4) from the data : $y(0)=1, y(1)=4, y(2)=0, y(3)=-2$ with the end conditions : $y'(0)=2=y'(3)$. **OR**

(b) Using natural cubic splines estimate y(1.5) from the data :

x	0	1	2	3
y	1	2	9	28

4. (a) Estimate y(0.2) in two steps buy second order Taylor series method where $y'=x-y^2$, $y(0)=2$ **OR**

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(a') Estimate y(0.2) and y(0.4) taking step size 0.2 for the I.V.P.

$\frac{dy}{dx} = \frac{y^2 - x}{y^2 + x}$ with y(0)=1 by Runge-Kutta fourth order method.

(b) Use Adams-Bashforth predictor method to find y(1.4) for the I.V.P.:
$x^2y'+xy=1$, y(1)=1
where it is given that
y(1.1)=0.996, y(1.2)=0.986, y(1.3)=0.972.

OR

(b') Estimate y(0.2) in one step for the second order initial value problem:
y'=0.5(x+y+y'+2), Y(0)=0, y'(0)=0 by Runge-Kutta-Nystrom method.

5. (a) Carry out three iterations towards solving the system :
$5x-2y+z=9$
$x+20y-2z=4$
$3x+7y-11z=3$
by Gauss-Jacobi iteration method starting from (0, 0, 0).

OR

(a) Solve the system of linear equations by Choleshy's method.

$x_1 + 2x_2 + 3x_3 = 2$
$2x_1 + 8x_2 + 22x_3 = 5$
$3x_1 + 22x_2 + 82x_3 = -6$

(b) Estimate the numerically largest eigen value and a corresponding eigenvector of unit length of the matrix :

$$\begin{bmatrix} 2 & 3 & 2 \\ 4 & 3 & 5 \\ 3 & 2 & 9 \end{bmatrix}$$

OR

(b') Using the method of least squares, fit a straight line to the following data :

x	0.5	1.0	1.5	2.0	2.5	3.0
y	15	17	19	14	10	7

B.A./B.Sc. (Part-III) Examination, 2018

Three-Year Degree Course

MATHEMATICS (First Paper)
(ABSTRACT ALGEBRA)

Time : Three Hours] [Maximum Marks : 60

Note : Attempt all questions. They all carry equal marks. Symbols used have their usual meaning.

1. (a) Define the subgroup of a group generated by a subset of it. Prove that (Q, +) cannot be generated by a finit subset.

OR

Let H and K be subgroups of a group (G, .). Prove that H.K. is a subgroup of G if and only if H.K=K.H

(b) State and prove Wilson's Theorem.

OR

Let (G,.) be a group of order 2n, $n \in N$.
Prove that the number of elements of order 2 in g is odd.

2. (a) State and prove first isomorphism theorem and give one application of it.

OR

Let H be a subgroup of a group (G, ,) containing the commutator subgroup [G, G] of G. Prove that $H \triangleq G$. Determine the commutator subgroup of $GL(2, \mathbb{Z}_2)$.

(b) Prove that S_n is generated by S={(12), (23), ...(n-1, n)}.

OR

Let $f \in S_n, f \neq I$. Prove that f can be written as a product of disjoint cycles in S_n.

3. (a) Let θ be an action of a group (G, .) on a non-empty set X, Let $x \in X$. Define the isotrophy subgroup Gx and the orbit Gθx of x. Prove that if $g \in G$ then $G_{g\theta x} = g.Gx.g^{-1}$

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OR

Let $(G, .)$ be a non abelian group of order p^3, where p is a prime. Prove that $G/Z(G) \simeq Z_p \times Z_p$.

(b) Prove that any two cyclic subgroups of order 4 of S_4 are conjugate.

OR

Determine the number of conjugates of a subgrop of order 2 of a non abelian group or order 2 of a non abelian group or order 14.

4. (a) Define a ring homomorphism. Determine all ring homomorphisms from

(i) $\mathbb{Z}_6$ to $\mathbb{Z}_9$

(ii) IR to the polynomial ring $\mathbb{Z}_2$ [X].

OR

Define a subring of a ring. Prove that $S = \left\{\begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in \mathbb{C}\right\}$ is a subring of $M_2(\mathbb{C})$ which is a non-commutative division ring.

(b) Prove that ideal I of IR [x] generated by $x^2+1 \in \mathbb{R}$ [x] is maximal and $\mathbb{R}$ [x]/I is a field isomorphic to $\mathbb{C}$

OR

Let F be a finite of characteristic 3. Prove that for each $a \in F$ there exists a unique $b \in F$ such that $b^3 = a$.

5. (a) Determine the set of all integers which leave remainders 1, 2 and 6 when divided by 6,7 and 11 respectively.

OR

Define a multiplicative function. Prove that Mobius function m is multiplicative and for each $n \in N, \sum_{d|n} |\mu(d)| = 2^r$ where r is the number of distinct prime divisors of n.

(b) Let p be an odd prime. Prove that $\left(\frac{2}{p}\right) = (-1)^{\frac{p^2-1}{8}}$

OR

Prove that there are infinitely many primes of the form 4n+1 $(n \in \mathbb{N})$.

MATHEMATICS (Paper - Second)- 2018
(Advanced Analysis)

Time : Three Hours] [Maximum Marks : 60

Note : Attempt all questions which carry equal marks. Symbols used have their usual meaning

1. (a) Let IR be the set of all real numbers. Define a 'convergent sequence' in a metric space. Using definition, show that $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ is convergent in $(\mathbb{R}, d)$ where d is the usual metric on $\mathbb{R}$. Prove that every convergent sequence in a metric space is bounded but the converse of it is not necessarily true.

(b) Let (X,d) and (Y, ρ)be two metric spaces and $f : (X,d) \to (Y,\rho)$ be a function. Prove that f is continuous on X into Y if and only if $f^{-1}(F)$. is a closed subset of X for every closed subset F of Y.

OR

(a') Prove that every compact subset of a metric space is boundded as well as closed.

(b') Prove that the closed interval [a,b] in $\mathbb{R}$, where $a, b \in IR$ and a<b is a compact subset of $\mathbb{R}$ under usual metric on IR.

2. (a) Let N be the set of all natural numbers and n, $m \in \mathbb{N}$. Define directional derivative of a function $f : S(\subseteq IR^n) \to IR^m$ at a point $C \in$ in the direction $\hat{u}$.

Define a function $f : IR^2 \to IR$ by

$$f(x,y) = \begin{cases} \frac{xy^2}{x^2+y^4}, & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$$

Show that the directional derivative of f at (0,0) exists in all directions but it is not continuous at (0,0).

(b) Let S be an open connected subset of $\mathbb{R}^n$. If a function $f:S \to \mathbb{R}^m$ is differentiable at a point $C \in S$ such that $f'(c)=0 \ \forall C \in S$ then prove that f is continuous at $C \in S$ it is constant on S.

OR

(a') Let S be an open subset of $\mathbb{R}^n$. If $f:S \to \mathbb{R}^m$ is a function such that $D_k f, 1 \le k \le n.$, exists and continuous in a neighbourhood of $C \in S$, then prove that f is differentiable at C.

(b) Define the 'partial derivative' of a function $f: S (\subseteq \mathbb{R}^n) \to \mathbb{R}^m$ with respect to x_1 where $x=(x_1, x_2, \ldots, x_i, \ldots, x_n) \in S$, Define a function $f: \mathbb{R}^2 \to \mathbb{R}$ by

$$f(x,y)=\begin{cases} \dfrac{xy(x^2-y^2)}{x^2+y^2}, & \text{if } (x,y)\neq(0,0) \\ 0 & \text{if } (x,y)=(0,0) \end{cases}$$

Show that $\dfrac{\partial^2 f}{\partial x \partial y}(0,0) \neq \dfrac{\partial^2 f}{\partial y \partial x}(0,0)$.

3. (a) State and prove 'Young's Theorem' for reversing the order of partial derivatives

(b) Use Taylor's Theorem to express the following functions in powers of (x–1) and (y–2) :

(i) $f(x,y) = x^3 + y^3 + xy^2 \ \forall (x,y) \in \mathbb{R}^2$

(ii) $f(x,y) = x^2 + xy + y^2 \ \forall (x,y) \in \mathbb{R}^2$

OR

(a') State and prove 'Euler's Theorem' for a homogeneous function of degree m in n variables $x_1, x_2, \ldots, x_n$.

(b') Explain 'Lagrange's Multipliers Method' to obtain the maximum or minimum value of a function $f: \mathbb{R}^3 \to \mathbb{R}$. Using this method, find the maximum volume of a parallelopiped inscribed in the ellipsoid.

$$\frac{x^2}{a}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1.$$

4. (a) Sate and prove 'Weierstrass M-Test' for the uniform convergence of the series

$$\sum_{n=1}^{\infty} u_n(x)$$

defined in [a, b], where $0<a<b<\infty$. Hence or otherwise, show that the series.

$$\sum_{n=1}^{\infty} \frac{1}{n^2+nx} \forall x \in [0,1]$$

is uniformly convergent in [0, 1].

(b) State and prove 'Abel's Test' for uniform convergence of the series $\sum_{n=1}^{\infty} U_n(x)v_n(x)$ in a closed interval [a, b].

OR

(a') Let $f(x) = \sum_{n=1}^{\infty} a_n x^n$

for $|x|<1$. If $f(x) \to s$ as $x \to 1$ and $\lim_{n\to\infty} n a_n = 0$, then prove that $\sum_{n=0}^{\infty} a_n$ converges to the sum s.

(b') Prove that the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} \sin\left(1+\frac{x}{n}\right)$

converges uniformly $\forall x \in [0,1]$ but the series

$$\sum_{n=0}^{\infty}\left(\frac{nx}{1+n^2x^2}-\frac{(n-1)x}{(n-1)^2x^2}\right)$$

is not uniformly convergent in [0, 1].

5. (a) If the four partial derivatives exist, continuous and satisfy the Cauchy-Riemann equations in domain Q, then prove that f(z) is continuous as well as analytic in the domain Q, where $f(z) = u(x, y) + iv(x,y)$.

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(b) If $u(x,y) = e^x(x\cos y - y\sin y)$, then find $v(x,y)$ and the analytic function $f(z) = u(x,y) + iv(x,y)$.

OR

(a') if $f(z) = u(x,y) + iv(x,y)$ and $u - v = e^x(\cos y - \sin y)$ then find $f(z)$ in terms of z.

(b') Show that the function

$f(z) = 3 + 2\sqrt{|xy|}$, $z = x + iy$

is not analytic at (0, 0), although the Cauchy- Riemann equations are satisfied at (0,0).

MATHEMATICS (Third Paper) - 2018
(Advanced Mechanics)

Time : Three Hours] [Maximum Marks : 60

Note : Attempt all questions. All the questions carry equal marks. Symbols have their usual meaning.

1. (a) A given system of forces $(\bar{R}, \bar{M})$ at origin is reduced to two non-zero finite forces. Show that the lines of action of these two forces do not intersect.

OR

Define nul plane at a point with respect to a system of forces. Two forces $\hat{i} - 3\hat{j} + 2\hat{k}$ and $2\hat{i} + \hat{j} - 3\hat{k}$ are acting at the points (1, 2, 3) and (– 1, 2, –1), respectively. Find the equation of the nul plane of the point (1, 1, 1) with respect to the system of forces.

(b) Forces $X\hat{i}$, $Y\hat{j}$ and $Z\hat{k}$ act in non-intersecting edges of a rectangular parallelopiped, none of them is along the co-ordinate axes, the corresponding edges being of lengths a, b and c. Find the equation of the central axis for the above system and the condition that the system is reduced to a single force.

OR

Two forces $\bar{P}$ and $\bar{Q}$ act along non-intersecting lines. Prove that their central axis intersects the common perpendicular to the two lines, and divides it in the ratio.

$\bar{Q}.(\bar{P}+\bar{Q}) : \bar{P}.(\bar{P}+\bar{Q})$

2. (a) Find the moment of inertia of a solid right circular cone of mass 'M', height 'h' and radius of base 'a' about a line passing through the vertex and perpendicular to the axis of the cone.

OR

Show that the sum of the moments of inertia of an elliptic lamina, about any two perpendicular tangents, is constants.

(b) Define momental ellipsoid of a body at a point and obtain the equation of momental ellipsoid at a point on the rim of a hemisphere.

OR

A wire is in the form of a semi-circle; show that at an end of its diameter a principal axis in its plane is inclined to the diameter at an angle $\frac{1}{2}\tan^{-1}\left(\frac{4}{\pi}\right)$.

3. (a) Obtain expressions for velocity and acceleration of a particle, moving on the surface of a sphere of radius b.

OR

A Plank, of mass M, is initially at rest along a line of greatest slope of a smooth plane inclined at an angle 30° to the horizon, and a man, of mass M', starting from the upper end walks down the plank so that it does not move. show that he gets to the other end in time $\sqrt{\frac{4M'a}{(M+M')g}}$, where 'a' is the length of the plank.

(b) A bent lever, whose arms are of lengths a and b, the angle between them being 90°, makes small oscillations in its own plane about the fulcrum. Show that the length of the corresponding simple pendulum is $\frac{2}{3}\frac{(a^3+b^3)}{\sqrt{a^4+b^4}}$

OR

A uniform semi-circular arc, of mass 'M' and radius 'a', is fixed at its ends to two points in the same vertical line, and is rotating with constant angular velocity w. Show that the horizontal thrust on the upper end is $\frac{M(g+w^2a)}{\pi}$

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4. (a) Derive the equation of continuity for a compressible fluid in the cylindrical polar co-ordinates.

OR

If the velocity $\bar{q}$ at a point in the spherical polar co-ordinates is given by $\bar{q} = \left(2Ar^{-3}\cos\theta, Ar^{-3}\sin\theta, 0\right)$, where 'A' is a constant, then prove or disprove that the liquid motion is irrotational. Also, find the equations of stream lines.

(b) Obtain the Bernoulli's equation for the unsteady, irrotational motion of an inviscid compressible fluid.

OR

A sphere is at rest in an infinite mass of homogeneous liquid of density ρ, the pressure at infinity being p_0. If the radius R of the sphere varies in any manner, then show that the pressure at the surface of the sphere at any time 't' will be :

$$p_0 + \frac{1}{2}\rho\left[\frac{d^2}{dt^2}(R^2) + \left(\frac{dR}{dt}\right)^2\right]$$

5. (a) Test whether the velocity potential $\varphi + \frac{1}{2}\log\left[\frac{(x+a)^2+y^2}{(x-a)^2+y^2}\right]$ is a possible motion of an incomkpressible fluid. If so, determine the stream lines.

OR

If there are sources at (0, a) and (0, –a) and sinks at (a, 0) and (–a, 0), all of equal strengths, then show that the circle through these four points is a stream line.

(b) Define a two dimensional doublet. Within a rigid boundary in the form of the circle $(x + a)^2 + (y - 4a)^2 = 8a^2$, there is liquid motion due to a doublet of strength μ at the point (0, 3a), with its axis along the axis of y. Find the stream function of the liquid motion.

OR

Between the fixed boundaries $\theta = \frac{\pi}{6}$ and $\theta = -\frac{\pi}{6}$ there is a two dimensional liquid motion due to a source at the point $(r = a, \theta = \alpha)$ and a sink at the origin, absorbing water at the same rate as the source produces it. Find the stream function of the motion.

MATHEMATICS (Fourth Paper)-2018
(Numerical Methods)

Time : Three Hours] [Maximum Marks : 45

Note : Attempt **all** questions. **All** the questions carry equal marks. Symbols have their usual meaning.

1. (a) Estimate a root of the equation $x^3 - 3x + 1 = 0$ between 0 and 1 by the method of Regula Falsi to two places of decimal. Justify the applicability of the method.

OR

(a') Find a root of $x^3 - 3x + 5 = 0$ by Newton Raphson's method near $x = 2$, convert to three places of decimal.

(b) State and establish a set of sufficient conditions for the success of the method of fixed point iteration for finding a root of an equation of the form $g(x) = x$.

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OR

(b') Explain the method of estimating the value of the positive n^{th} root of a positive real number a, where n is any interger ≥ 2. Also, give an example for its illustration.

2. (a) State and prove Newton's Divided Difference Interpolation formula.

OR

(a') Estimate the value of f(4) from the data :

$f(-2) = -15$, $f(-1) = -4$, $f(1) = 0$, $f(3) = 20$, by Lagrangian interpolation. How many values of f(4) are given by this method?

(b) Estimate the missing value from the following data :

Lowing data :

x	0	1	2	3	5	6
f(x)	5.2	8.0	10.4	12.4	?	15.2

OR

(b') Derive Gauss first interpolation formula.

3. (a) Describe the method of cubic splines to estimate the value of f(x) at any point $a \in [x_0, x_0 + 3h]$ if its values at points x_0, $x_0 + h$, $x_0 + 2h$ and $x_0 + 3h$ and are specified and set up equations for finding the cofficients of the respective cubic polynomials, if the second derivatives of the whole function at the two end points x_0 and $x_0 + 3h$ are to be zero.

OR

(a') Fit a cubic spline curve that passes through points (0, 1), (1, 4), (2, 0) and (3, –2) with the given derivative boundary conditions $S'(0) = 2$ and $S'(3) = 2$.

(b) Estimate the values of f'(0.2) and f''(0.2) for the function f given by the following data :

x	0.1	0.2	0.3	0.4	5
f(x)	0.425	0.475	0.400	0.450	0.525

OR

(b') Describe the Simpson's $\frac{1}{3}$ rule for numerical integration and use it to estimate the area enclosed by x-ax i and the curve y= f(x) and the lines x = 0 and x = 2.5, where f(x) is given by the data :

x	–0.5	0	0.5	1	1.5	2.0	2.5	3
f(x)	5.0	3.5	4.0	5.5	6.0	6.5	5.0	4.5

4. (a) Find y(1) in two steps for the differential equation $y' = x^2 - y^2$ with $y(0) = 0$ by improved Euler method.

(a') State the Runge Kutta fourth order method for finding y(a+h) where y'=f(x,y) and y(a) is given. Estimate the value of y(2) for the initial value problem $y' = \frac{y}{x} - y^2, y(1) = 1$ by the above method in one step.

(b) Estimate the value of y(0.4) by Milne's predictor corrector method for the I.V.P : $y' = xy + y^2$; $y(0) = 1$ where it is given that y(0.1) = 1.117, y(0.2) = 1.287, y(1.3) = 1.504

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OR

(b') Find $y(0.3)$ in three steps for the second order I.V.P. $y'' - xy' + 3xy = 0$, $y(0) = 0$, $y'(0) = -3$, by Second order Taylor series method.

5. (a) State a sufficient condition for applicability of the Gauss'-Seidel-iteration method for solving a system of n linear equation in as many unknowns. Apply this to solve the system :

$$3x - 6y + 2z = 23$$
$$-4x + y - z = -15$$
$$x - 3y + 7z = 16$$

Starting from the initial solution $(x_0, y_0, z_0) = (1, 1, 1)$.

OR

(a') Solve the system of linear equations by LU- factorization method.

$$x + y + z = 3$$
$$2x - y + 3z = 16$$
$$3x + y - z = -3$$

(b) State Gerschgorin's theorem concerning positions of eigenvalues of a matrix. Find the circular regions containing the eigen values of

$$\begin{bmatrix} -33 & -16 & -72 \\ 24 & 10 & 57 \\ 8 & 4 & 17 \end{bmatrix}$$

OR

(b') Estimate the numerically largest eigen value and find a corresponding eigen vector by power method for the matrix

$$\begin{bmatrix} 4 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Starting from the initial vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ using at least four iterations.

B.Sc. (Part-III) Examination, 2019

Three-year Degree Course

MATHEMATICS
FIRST PAPER
(Abstract Algebra)

Time : Three Hours] **[Maximum Marks : 60**

Note : Attempt **all** questions. All are of equal marks. The symbols have their usual meanings.

1. (a) Define order of an element of a group. Let a be an element of G which is of order m. Show that $< a^r > = < a^{(m, r)} >$. Deduce that the order of a^r is $\frac{m}{(m, r)}$.

OR

Show that the group of automorphisms of $\mathbb{Z}_m$ is isomorphic to the group U_m of prime residue classes modulo m.

(b) Show that the order of a subgroup of a finite group divides the order of the group. Deduce that every group of prime order is cyclic.

OR

Describe the group of symmetries of a regular pentagon and find its center.

2. (a) State the first isomorphism theorem and use it to show that $\frac{\mathbb{R}}{Q}$ is isomorphic to $\frac{S^1}{P}$, where P is that group of roots of unity.

OR

Check if all subgroups of $Q_8 \times \mathbb{Z}_3$ are normal.

(b) Show that $\frac{S_4}{V_4}$ is isomorphic to S_3.

OR

Find the number of elements of order 15 in S_8.

3. (a) Let G be a group of order P^n, where p is a prime and $n \geq 1$. Let X be a finite set on which G acts. Show that :

$|X| \equiv |X^G| \pmod{p}$,

where X^G is the fixed point set of the action.

OR

Show that any group of order p^2 is isomorphic to $\mathbb{Z}_{p^2}$ or else it is isomorphic to $\mathbb{Z}_p \times \mathbb{Z}_p$, where p is a prime number.

(b) Let G be a finite group, and H be a subgroup of G. Show that the number of conjugates to H is $[G : N_G(H)]$. Let G be a group of order $p^2.q$, where p and q are distinct primes. Let H be a subgroup of order p^2. Show that H is a normal subgroup or else $N_G(H) = H$.

OR

Let G be a group of order p^n, where p is prime and $n \geq 1$. Show that G has a normal subgroup of order p^r for all $r \leq n$.

4. (a) Let R be a ring containing prime number of elements. Show that R is a zero ring or else it is a field.

OR

Determine the group of automorphisms of the field $\mathbb{R}$ of real numbers.

(b) Show that $X^3 + X + 1$ is irreducible polynomial in $\mathbb{Z}_2[x]$. Deduce that the ideal I generated by $X^3 + X + 1$ is maximal ideal of $\mathbb{Z}_2[x]$.

What is the order of the field $\frac{\mathbb{Z}_2[x]}{I}$?

OR

Let $\mathbb{F}$ be a finite field of characteristic p. Show that every element of $\mathbb{F}$ has a p th root. Is this result true for infinite field of characteristic p ? Justify your answer.

5. (a) State and prove the Chinese remainder theorem. Deduce that the Euler phi function is ϕ multiplicative.

OR

Show that $n = \sum_{\frac{d}{n}} \mu(d)\sigma\left(\frac{n}{d}\right)$, where σ is the sum of divisors function and μ is the Möbius function.

(b) Define Legendre symbol. Compute $\left(\frac{28}{29}\right)$. Is 28 quadratic residue modulo 29 ?

OR

Suppose that $X^2 + \bar{a}$ is reducible in $\mathbb{Z}_p[x]$ for all but finitely many primes p. Show that a is a perfect square.

MATHEMATICS (Second Paper) - 2019
(Advanced Analysis)

Time : Three Hours] **[Maximum Marks : 60**

Note : Attempt **all** questions which carry equal marks. Symbols used have their usual meaning.

1. (a) Define a continous map from a metric space to a metric space. Prove that a linear map $T : \mathbb{R}^n \to \mathbb{R}^m$ is continuous.

(b) Prove that a continuous image of a compact metric space is compact.

OR

(a) Prove that a map f from a metric space X to a metric space Y is continuous if and only if $f(Cl A) \subseteq Cl f(A), \forall A \subseteq X$.

(b) State and prove the Lebesque covering lemma.

2. (a) Give an example of a map $f: \mathbb{R}^2 \to \mathbb{R}$ such that the directional derivatives of f exist at a point of $\mathbb{R}^2$ in all directions but it is not continuous there, with complete arguments.

(b) State and prove the chain rule of differentiation.

OR

(a) Prove that a map $f: \mathbb{R}^n \to \mathbb{R}^m$ is differentiable if and only if each of its component is differentiable.

(b) Let s be an open subset of $\mathbb{R}^n$ and $f: s \to \mathbb{R}$ be differentiable at each point of s. If x and y are distinct points of s such that :

$L(x, y) = \{(1-t)x + ty,\ t \in [0, 1]\} \subseteq s$,

then prove that

$f(y) - f(x) = grad\, f(z).(y - x)$,

for some $z \in L(x, y)$.

3. (a) By means of an example show that the partial derivatives need not commute.

(b) State and prove the Schwarz's theorem.

OR

(a) Let $f: \mathbb{R}^n \to \mathbb{R}$ be a differentiable map. If for each $x \in \mathbb{R}^n, x.\nabla f(x) = pf(x)$, when $p \in \mathbb{N}$, then show that f is a homogeneous map of degree p.

(b) State the inverse function theorem. Let $f: \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $f(x, y) = \left(e^x \cos y,\ e^x \sin y\right), \forall (x, y) \in \mathbb{R}^2$.

Show that f is locally invertible at each point of $\mathbb{R}^2$ but it is not bijective. $\mathbb{N}$

OR

4. (a) Show that the sequence of functions $\{f_n\}_{n=1}^{\infty}$, where $f_n(x) = \frac{nx}{1+n^2x^2}, \forall n \in$ has a continuous limit function on [0, 1], but does not converge uniformly there. Does the above sequence converge uniformly on [1, 2] ? Justify your answer.

(b) State and prove Abel's limit theorem and use it to show that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \ldots\ldots$

OR

(a) State and prove the Dirichlet's test for uniform convergence of a series.

(c) Introduce the map 'exp' and determine its range. Prove that :

$\exp(x+y) = \exp(x)\exp(y),\ \forall x, y \in \mathbb{R}$.

5. (a) Prove that a real-valued map of a complex variable either has derivative zero or the derivative does not exist. Is the map $f(z) = |z|,\ z \in \mathbb{C}$, differentiable at $z = 0$? Justify your answer.

(b) Find an analytic map of which the real part is the map $u(x, y) = 4xy - x^3 + 3xy^2;\ x, y \in \mathbb{R}$.

OR

(a) If $f(z) = u(x, y) + iv(x, y)$ is differentiable at $z_0 = x_0 + iy_0$ in $\mathbb{C}$, where $z = x + iy \in \mathbb{C}$ then prove that Cauchy-Riemann equations $u_x(x_0, y_0) = V_y(x_0, y_0)$ and $u_y(x_0, y_0) = -V_x(x_0, y_0)$ hold. Is the converse true ? Support your answer.

(b) If $\sum_{n=0}^{\infty} a_n z^n$ has radius of convergence $R > 0$, then show that $f(z) = \sum_{n=0}^{\infty} a_n z^n$ is analytic in the region $|z| < R$.

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MATHEMATICS (Third Paper) - 2019
(Advanced Mechanics)

Time : Three Hours] **[Maximum Marks : 60**

Note : Attempt **all** questions. **All** questions carry equal marks. Symbols have their usual meaning.

1. (a) Define Wrench of a system of forces. Forces P, Q, R act along three straight lines $x = 0, y - z = a; y = 0, z - x = a$ and $z = 0, x - y = a$ respectively. Find the pitch of the equivalent wrench.

OR

Prove that any force system is equivalent to a single force passing through a prescribed point and a couple. If a system of forces is equivalent to $(\hat{i}+\hat{j}+\hat{k};\ \hat{i}-2\hat{j}+3\hat{k})$ at the point (1, 2, 3), find the vector equation of the central axis of the system.

(b) Obtain the vector equation of the null plane of a point $A(\bar{a})$ with respect to a system of forces $(\bar{R}, \bar{M})$ at origin. Also, find the null point of the plane $x + 2y - 3z = 4$ with respect to a system $\bar{R} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\bar{M} = 3\hat{i} - 2\hat{j} + 5\hat{k}$ at origin.

OR

Obtain the vector equation of conjugate line to the line $\bar{r} = t(\hat{i} - \hat{j} + \hat{k})$ with respect to a system $(2\hat{i} - 2\hat{j} + 3\hat{k};\ 3\hat{i} + 3\hat{j} - 4\hat{k})$ at origin. Also, find the conjugate forces.

2. (a) Find the moment of inertia of a uniform triangular lamina ABC of mass M, about an axis perpendicular to its plane through A.

OR

Find the moment of inerita of the arc of a circle of mass M, which subtends an angle 2α at the centre, about (i) the diameter bisecting the arc, and (ii) an axis through its middle point and perpendicular to its plane.

(b) Obtain the equation of momental ellipsoid at the centre of an elliptic plate $4x^2 + 5y^2 \le 20$.

OR

If A and B be the moments of inertia of a uniform lamina about two perpendicular axes, $0X$ and $0Y$, lying in its plane, and F be the product of inertia of the lamina about these lines, show that the principal moments at 0 are equal to

$$\frac{1}{2}\left[A + B \pm \sqrt{(A-B)^2 + 4F^2}\right].$$

3. (a) A uniform rod, of mass M and length 2a, can turn freely about one end which is fixed. If it is started with angular velocity ω from the position in which it hangs vertically, find its angular velocity at any instant.

OR

A particle is moving on a sphere $r = a$. Show that its velocity in the spherical polar coordinates (r, θ, φ) is given by $\bar{v} = a\,\dot{\theta}\,\hat{\theta} + (a\sin\theta)\dot{\varphi}\,\hat{\varphi}$.

(b) A solid homogeneous cone, of height h and vertical angle 90^0 oscillates about a horizontal axis through its vertex. Find the length of simple equivalent pendulum.

OR

Find the moment of the effective forces of a rigid body rotating about a fixed axis.

4. (a) Explain the Lagrangian and the Eulerian approaches for the description of fluid motion and show that these approaches are equivalent.

OR

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Prove that the liquid motion is possible when velocity at the point (x, y, z) is given by

$$\bar{q} = \left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2 - r^2}{r^5} \right),$$

where $r^2 = x^2 + y^2 + z^2$. Also, test whether the motion is of potential kind.

(b) Derive the Euler's equations of motion for a homogeneous, inviscid fluid moving under forces arising from a potential V in the form

$$\frac{\partial \bar{q}}{\partial t} - \bar{q} \times curl\ \bar{q} = -grad\left(V + \frac{1}{2}|\bar{q}|^2 + \frac{p}{\rho} \right).$$

OR

A mass of liquid of density ρ whose external surface is a long circular cylinder of radius a, which is subject to a constant pressure π, surrounds a coaxial long circular cylinder of radius b. The internal cylinder is suddenly destroyed, and if v is the velocity at the internal surface when the radius is r, then prove that

$$v^2 = \frac{2\pi(b^2 - r^2)}{\rho r^2 \log\left(\frac{r^2 + a^2 - b^2}{r^2} \right)}$$

5. (a) Prove that for an incompressible, irrotational two dimensional flow, stream function and velocity potential are harmonic functions.

OR

A velocity field of a liquid is given by.

$\bar{q} = x\hat{i} - (y + t^2)\hat{j}$

Find the velocity potential and prove that the stream lines for this field at $t = 2$ are rectangular hyperbolas.

(b) Define a two dimensional doublet. Find the complex potential at any point $P(z)$ due to a doublet of strength μ placed at the point $(-1, 1)$ and whose axis is perpendicular to X-axis.

OR

Find the velocity potential and stream function of the liquid motion at any point $P(r, \theta)$, when there is a source and an equal sink inside a circular cavity of radius a.

MATHEMATICS (Fourth Paper) - 2019
(Numerical Methods)

Time : Three Hours] **[Maximum Marks : 45**

Note : Attempt **all** questions. **All** questions carry equal marks. Symbols have their usual meaning.

1. (a) Find a real root of the equation $x^3 - 3x - 5 = 0$ correct to two decimal places by fixed point iteration method.

OR

Find a real root of the equation $3x^2 + 5x - 40 = 0$ correct to two decimal places by Regula-Falsi method using the formula three times.

(b) Develop an algorithm using Newton-Raphson method, to find the k^{th} root of a positive number N and hence evaluate $3\sqrt{24}$ correct to four decimal places.

OR

Define order of convergence of an iterative method. Show that Newton-Raphson method has second order of convergence.

2. (a) State and prove Lagrange's Interpolation formula. Also, write Newton's divided difference interpolation formula.

OR

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Find the missing values in the following table :

x	45	50	55	60	65
y	3	–	2	–	–2.4

(b) Prove that the n^{th} forward differences of a polynomial of degree n is constant, when the values of the independent variable are given at equal intervals.

OR

Find $y'(1.1)$ and $y''(1.1)$ from the following data :

x	1.0	1.1	1.2	1.3	1.4
y	7.989	8.403	8.781	9.129	9.451

3. (a) Define cubic spline function. Find a natural cubic spline function for the following data and estimate the value of $f(0.5)$:

x	0	1	2	3
$f(x)$	0.0	0.5	2.0	1.5

OR

Explain the concept of numerical integration. Derive the Trapexoidal formula for the approximate evaluation of $\int_a^b f(x)dx$. Also, explain its geometrical significance.

(b) Find the maximum and minimum value of the function $y = f(x)$ which satisfies the following data :

x	0	1	2	3	4	5
y	0	0.25	0	2.25	16	56.25

OR

Derive the Simpson's one-third formula for the approximate evaluation of $\int_a^b f(x)dx$ and use it to evaluate $\int_0^1 \frac{dx}{1+x}$ by dividing the interval into four equal subintervals.

4. (a) Derive the Adams-Moulton predictor and correcter formulae for the I.V.P. :

$y' = f(x, y), y(x_0) = y_0$ to express $y(x_4)$ in terms of $y(x_0), y(x_1), y(x_2)$ and $y(x_3)$.

OR

Describe the Taylor's series method to solve an initial value problem and second order differential equation :

$y'' = f(x, y), y(x_0) = y_0, y'(x_0) = y'_0$

Using the method, find $y(0.2)$ and $y'(0.2)$ for the I.V.P. :

$y'' = f(x + y + y' + 2)/2, y'(0) = 0, y(0) = 0$

by taking $h = 0.2$.

(b) Explain the improved Euler's method for the first order I.V.P. : $y' = f(x, y), y(x_0) = y_0$, and hence estimate the value of $y(0.1)$ and $y(0.2)$ for the I.V.P. : $y' = x^2 + y^2, y(0) = 1$.

OR

Estimate the value of $y(0.4)$ for the initial value problem $y' = 1 + xy, y(0) = 1$.

by fourth order Runge-Kutta method in two steps.

5. (a) Describe Power method to find the numerically largest eigenvalue and corresponding eigen vector of a square matrix A of order n. Using the method, estimate the largest eigenvalue and corresponding eigenvector for the matrix $\begin{bmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 1 & 4 \end{bmatrix}$, starting with the initial vector $\bar{X}^{(0)} = [100]^T$, at the end of second iteration.

OR

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Explain the method of decomposing a square matrix as a product of a lower and an upper triangular matrix. Using the method, solve the system of equations :

$$x_1 + x_2 + x_3 = 1$$
$$4x_1 + 3x_2 - x_3 = 6$$
$$3x_1 + 5x_2 + 3x_3 = 4$$

(b) State the Gerschgorin's theorem concerning the positions of the eigenvalues of a square matrix. Draw the Gerschgorin circles for the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.

OR

Perform three iterations of the Gauss-Seidel iteration method starting from the initial solution $\left(x_1^0, x_2^0, x_3^0\right) = (1, 1, 1)$ for the system of equations :

$$20x_1 + x_2 - 2x_3 = 17$$
$$2x_1 - 3x_2 + 20x_3 = 25$$
$$3x_1 + 20x_2 - x_3 = -18$$

to obtain the values of x_1, x_2 and x_3.

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